INVESTIGATION OF THE INSTABILITY OF A PLASMA IN A MIRROR TRAP

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The ion-cyclotron instability observed in the PR-5 installation differs in many respects from instabilities of the same type encountered in experiments with other mirror traps. This implies a principally different nature of this instability. In the present work the most characteristic feature which is considered is the wave structure of the unstable oscillations; in particular, an attempt is made to determine whether the oscillations are transverse ($k_{\parallel} = 0$) or not. The method employed for this purpose is based on a determination of the orientations of external magnetic fields of the wave and their phasing relative to the electric-potential wave. When $k_{\parallel} = 0$, a characteristic feature is the presence of a specific effect related to longitudinal nonuniformity of the plasmoid. The results are in accordance with those which one would expect for transverse oscillations.

The proof of the transverse character of the oscillation indicates that instability in PR-5 is not of the Harris type as in the case of mirror traps with external neutral-particle injection. Apparently one must also reject the explanation of instability predicted earlier as a result of transverse oscillations built-up by the drift mechanism or as a result of the non-Maxwellian nature of the distribution function, since the observed value of the parameter $k\rho_i$ is too small for this. The most probable seems to be the explanation proposed recently by Kadomtsev and Pogutse,[2] according to which the phenomena observed are negative mass instabilities.

1. INTRODUCTION

INVESTIGATIONS of the containment of plasma in mirror-type traps (Phoenix-II, Alice-Baseball, DCX-2, DECA-II, Ogra-II, AC, PR-5, PR-6) have revealed ion-cyclotron instabilities that serve in many cases as sources of appreciable losses. The theory indicates that the same outward features of this instability (buildup of oscillations at a frequency close to H_{Hi} or its harmonics) may mask in various cases collective interactions of different types. These may be, for example, instabilities of the Harris type, in which waves grow as a result of resonance between the longitudinal oscillations of the electrons and Larmor rotation of the ions, or instabilities of the drift type, an appreciable role in the buildup of which is played by diamagnetic current, or else "negative-mass" instability, due to the inhomogeneity of the magnetic field. Finally, this may be some other mechanism not yet revealed by the theory.

To explain the nature of the instability observed in a concrete experiment, it is necessary to compare the experiment with theory by means of a detailed investigation of the dependence of a number of characteristics of the instability—for example the threshold of its buildup, the oscillation-frequency interval, the magnitude of the increment, etc.—such factors as the plasma density, the energy of the ions and their distribution function, the electron temperature, the magnetic field intensity, etc. It seems, however, that a study of dependences of this type should logically be the second stage in the search for the nature of the investigated instability, whereas the first stage should be aimed at establishing the structure of the oscillations, taking this to mean the determination of such characteristics as the wave propagation direction (i.e., the ratio $k_{\parallel}/k_{\perp}$), the distribution of the oscillation amplitude in the space occupied by the plasma, the wavelength compared with the Larmor radius of the ions (the parameter $k\rho_i$). The urgency of investigating these characteristics is dictated by the fact that knowledge of these characteristics makes it possible to establish immediately the non-participation of various instability mechanisms in the investigated case. Such a narrowing down of the circle of possible explanations simplifies further searches.

One of the structural characteristics, which is particularly important in this respect, is the vanishing or non-vanishing of the longitudinal component $k_{\parallel}$ of the wave vector. A number of types of ion-cyclotron instability consist of a buildup of oblique electron-Langmuir waves, i.e., its $k_{\parallel}$ is in principle not equal to zero; these are instabilities of the Harris type. For other instabilities—of the drift type or of the "negative mass" type—oscillations with $k_{\parallel} = 0$ are possible. It has been established that in many experiments with mirror traps one observes precisely the first case (Phoenix, Alice-Baseball, Ogra-II). As to the discussed PR-5 setup, the instability observed here differs strongly from the foregoing ones in its outward manifestations and possibly pertain to another type. This serves as a stimulus for an experimental investigation of k.

2. PRELIMINARY ANALYSIS OF THE PROBLEM

Investigations of the oscillations of the electric field in experiments with PR-5 have shown[1] that the wave travels in the azimuthal direction, having a wave number m = 2, 3 or 4. In the longitudinal direction, on the other hand, the phase of the oscillations at a fixed instant of time is the same throughout. This can occur in two cases: either there exists a purely transverse wave ($k_{\parallel} = 0$, or else the oscillations constitute a superposition of two oblique waves having identical $k_\perp$ and equal but opposite $k_\parallel$ of such magnitude, that a standing wave is produced in a longitudinal direction, with an antinode at the center of the plasmoid and with nodes on its ends. In the former case ("zeroth longitudinal mode") there is no drop of the alternating potential along the force lines of the magnetic field inside the plasma; in the latter case ("first longitudinal mode") the alternating potential decreases smoothly from the center towards the ends. This uncovers in principle a possibility of distinguishing between the indicated cases by measuring the electric field, thereby establishing the absence or presence of a decrease in the amplitude of the oscillations along the length. However, under the conditions of PR-5 the performance of such an experiment in "pure" form is made difficult by the diffuseness of the boundaries of the plasma and by the complicated configuration of the magnetic field, as a result of which the results always leaves room for doubts.

Another way, which was chosen here for the solution of the problem, is to measure the magnitude and the phase relationships of the currents in the wave. This method has the advantage that the pictures of the currents for the cases $k_\parallel = 0$ and $k_\parallel \neq 0$ are quite different and in a certain sense are even contradictory, and therefore the result of the experiment can be interpreted with complete certainty.

Let us consider the question of the currents, i.e., of the motion of the ions and electrons, in the case of natural oscillations of a plasmoid contained in a trap. We purposely leave aside the question of the cause of the oscillations, i.e., of the nature of the instability. Such an abstraction is justified if the instability increment is much smaller than the oscillation frequency (this is usually the case for all the instabilities investigated in detail in mirror traps, including PR-5), for in this case the structure characteristics of the waves do not depend on the buildup mechanism.

A rough idea of the ordered motion of the particles in the wave can be obtained by using the conclusions deduced for an infinite homogeneous plasma. For a qualitative description, such a simplification is applicable even if the wavelength of the oscillations is comparable with the dimensions of the plasmoid. Certain refinements connected with the changeover to a bounded plasma are best carried out later.

In the linear approximation, for electrostatic oscillations without allowance for the thermal motion of the particles, the dispersion equation takes the form (in standard notation):

$$1 = \frac{\omega_{pi}^2}{\omega^2 - \omega_{Hi}^2} \frac{k_\perp^2}{k^2} + \frac{\omega_{pe}^2}{\omega^2} \frac{k_\parallel^2}{k^2}.$$

The terms in the right side describe the relative contributions of the ions and electrons to the charge produced in the wave. The particle motion itself has the following character. The ions move across the magnetic field along ellipses elongated in the $k_\perp$ direction in a ratio ω/ω_{Hi}. At the maximum of the positive potential of the wave, the positions of the ions correspond to the ends of the minor semi-axes of the ellipses, and the velocities of motion are maximal and directed along $k_\perp$, i.e., in the direction of wave propagation. The longitudinal motion of the ions can be disregarded, since the effects associated with them are always smaller by a factor m_i/m_e than the effects connected with the longitudinal motion of the electrons. The latter execute oscillations along the magnetic field, and at the maximum of the positive potential of the wave the phase of the oscillation corresponds to the undisplaced position of an electron having, however, a maximal velocity directed along $-k_\parallel$, i.e., in opposition to the wave. In addition, the electrons drift across the magnetic field perpendicular to $k_\parallel$, but this motion does not produce charges in the case of a uniform plasma.

It is seen from the dispersion equation that starting with $k_\parallel/k = 1$ and down to smaller values the charge in the wave is produced practically only by the longitudinal motion of the electrons, and the contribution of the ions can be neglected (oblique Langmuir wave). However, when $k_\parallel = 0$ or when $k_\parallel$ is sufficiently small, it is the electron charge, to the contrary, which becomes negligible, and the wave can be regarded as produced only by the above-described transverse motion of the ions. The case of exact vanishing of $k_\parallel$ is not physically distinguished, in the sense that the mechanism of the oscillations remains the same also at nonzero (but small) $k_\parallel$. Therefore, when we refer in this paper to transverse oscillations, we actually mean only a certain degree of accuracy, or specifically values of $k_\parallel/k$ so small that the oscillations become "ionic."

Having explained the character of motion of the particles in the case of oscillations of an infinite plasma, we shall use this approximation for the construction of a system of currents in a wave traveling along the periphery of a plasma cylinder. In the case of a wave with $k_\parallel = 0$, as follows from the foregoing, the currents produced only by the ions are phased relative to the potential wave in the manner shown in Fig. 1a, (for a wave traveling in the "ionic" direction and having an azimuthal number $m = 2$). The magnetic field of the wave outside the plasma cylinder can have a longitudinal direction and maxima coinciding with the maxima of the potential, the maximum of the positive potential corresponding to a current direction in the wave propagation direction.

In the case $k_\parallel \neq 0$, the system of currents produced by electrons only will have the form shown in Fig. 1b (also for a wave in the ionic direction with $m = 2$). Account is taken here of the fact that each electron takes part simultaneously in two motions corresponding to two oblique waves with symmetrical components $\pm k_\parallel$ of such a magnitude, that on the ends of the plasma cylinder there are nodes of the potential oscillations. As a result of this addition, the maxima of the longitudinal current turn out to be located not at the maxima of the potential, as in the case for a single wave, but between them, and the currents in front of the maxi-

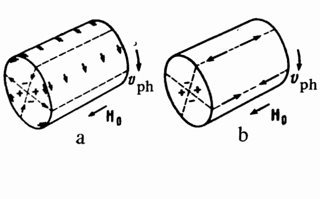


FIG. 1. Direction of currents in the wave in the case of oscillations of a cylindrical plasmoid (the unmarked arrows): a – "ionic" oscillations ($k_\parallel = 0$), b – "electronic" oscillations ($k_\parallel \neq 0$).

mum of the positive potential are directed towards the center of the cylinder, and those in front of the maximum of the negative potential are directed towards the ends. The direction of the external magnetic field of the wave will obviously be azimuthal.

At small $k_{\parallel}$, generally speaking, there is also a possible intermediate case of oscillations in which the contributions of the ions and of the electrons to the formation of the charge of the wave are comparable, i.e., the oscillations are half "ionic" and half "electronic." In this case the values of the longitudinal and transverse components of the magnetic field will also be comparable (and its vector will rotate at the frequency of the oscillations).

If we go over from the approximate analysis that uses concepts applicable to oscillations of a uniform plasma to a more rigorous solution of the problem of oscillations of a plasmoid that is bounded radially (and is detached from the sidewalls) and longitudinally, then the character of the oscillations changes. Thus, for example, for the case $k_{\parallel} = 0$ the most significant changes consist in the fact that, first, the oscillations turn into the peculiar surface waves considered by Kadomtsev and Pogutse[2], and second, a splitting of the frequencies takes place for waves propagating in opposite azimuthal directions, namely, the frequency of the wave traveling in the electronic side becomes much smaller than ω_{Hi}, so that this wave drops out of consideration. As to the character of the ordered motion of the particles in the wave, the concepts described above remain qualitatively valid, and consequently the picture of the current, obtained for oscillations of this type, remains in force. Similarly, the picture of the currents for the "electronic" oscillations also remains the same.

However, if the plasmoid has a strong longitudinal inhomogeneity, then the picture of the currents at $k_{\parallel} = 0$ must be supplemented by one more detail, which plays the decisive role in the experimental differentiation of the transverse wave from the wave with $k_{\parallel} \neq 0$, especially if one bears in mind the intermediate case of "ionic-electronic" oscillations. The point is that if the dense part of the plasmoid, containing the bulk of the plasma, has a short length (compared with the length of the trap), then longitudinal electron currents will occur, nonetheless, even in the case of transverse oscillations. This is due to the fact that during the oscillations almost the entire ionic charge is separated in the dense central part, whereas the plasma region in which the potential of the wave remains the same along the magnetic field, must have a larger length, since it extends to very low densities determined by the condition $\omega_{pe} \approx \omega_{Hi}$. Therefore, when ionic charge is separated in some magnetic tube near the center, electrons move to the same place from the lateral regions, thereby effecting, as it were, a redistribution of the positive charge in the entire length of the tube, up to the boundary indicated above. Since $\omega_{pe} \gg \omega_{Hi}$ in the greater part of this length, the redistribution takes place "instantaneously" and the phase of the longitudinal electronic current follows the phase of the separation of the ionic charge almost exactly; this charge is separated most rapidly, obviously, during that phase of the wave which leads the maximum of the positive potential by one quarter of the period. Thus, in the case of transverse oscillations, the electronic current connected with the longitudinal inhomogeneity of the plasmoid is shifted by one quarter of a period relative to the potential wave in exactly the same manner as in the case of oscillations with $k_{\parallel} \neq 0$, but its direction is opposite. All that needs to be added is that in the central part of the trap the principal role is still played by the azimuthal ionic current, whereas the longitudinal current of the electrons being secondary, must have a smaller magnitude.

3. ORGANIZATION OF EXPERIMENT

The experiments were performed for the purpose of determining whether the oscillations observed in PR-5 are transverse or whether they have $k_{\parallel} \neq 0$. The method consisted of measuring the external magnetic fields of the wave, reproducing on their basis the picture of the currents, and comparing this picture with the models discussed above.

To measure the alternating magnetic fields, a set of six loop antennas was placed inside the trap; the antennas were located near the chamber wall along a line corresponding to the center of one of the gaps between the stabilizing conductors. The outermost antennas were located 30 cm (half the distance to the mirror) away from the central section of the chamber. Some of the antennas could be arbitrarily oriented in order to measure the longitudinal or azimuthal components of the magnetic field.

Each antenna was a loop with area of several cm^2, protected by a screen made of copper tubing having a transverse cut covered with a special hood to prevent the plasma particles and the electric field from penetrating into it. Evidence of noise of this type was the fact that when the plane of the loop was rotated by 180°, the sign of the signal was reversed (relative to signals from the other antennas), and the magnitude remained the same.

The signals from the antennas were fed through matched cables to broad-band amplifiers, and from them to two-beam oscilloscopes, with which the phase shifts were recorded. The oscilloscope sweeps, which spanned several periods of the oscillations, were triggered simultaneously. In addition to the investigated signal, synchronous time markers were fed to each channel, making it possible to determine the phase shifts between the signals with accuracy not worse than 10–15°. Besides the magnetic signals, one of several channels registered also signals from electrostatic antennas (disc probes flush with the wall), located in the central section of the trap.

MEASUREMENT RESULTS

The most complete measurements of the magnetic field of the wave were carried out in a regime in which the unstable oscillations have a azimuthal number $m = 2$. For waves of $m = 3$ and $m = 4$, the observed picture is apparently analogous.

As a result of the measurements it was established that both components of the magnetic field, longitudinal and azimuthal, are present. Judging from the character of the signals, the main component is the longitud-

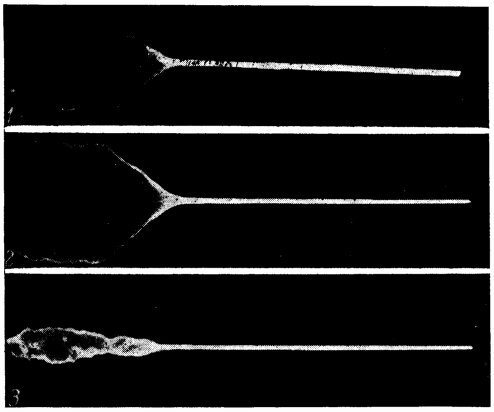


FIG. 2. Form of the packets of oscillations of the electric field of the wave (1), of the longitudinal component (2), and of the azimuthal component (3) of the magnetic field. The sweep duration is 100 μsec. The vertical scale for the signals of the magnetic field is the same.

inal one. It has the largest magnitude (amplitude of about $(2-3)\times 10^{-3}$ G in the central section of the trap) and the form of its oscillation packet duplicates exactly the form of the pocket of the oscillations of the electric field, unlike the azimuthal component, the pocket of which is more irregular (Fig. 2). In addition, the oscillations of the longitudinal component, similar to the oscillations of the electric field, constitute a sufficiently regular sinusoid (with the frequency of the fundamental harmonic), whereas the oscillations of the azimuthal component contain a second and a third harmonic, the amplitudes of which are approximately the same as the amplitude of the fundamental.

With increasing distance from the central section, the amplitude of the oscillations of the longitudinal magnetic field decreases quite rapidly, by an approximate factor 10–15 over a distance of 30 cm.

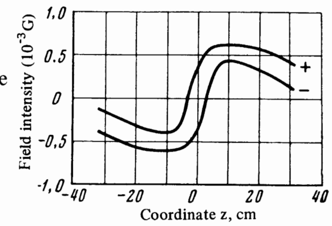
The phase relation between the oscillations of the longitudinal component corresponds to an azimuthal current having maxima at the maxima of the electric-potential wave and directed along the motion of the wave during the positive phase of the potential.

The azimuthal component of the magnetic field of the wave has a maximum amplitude of approximately 5×10^{-4} G. Near the central section of the trap, this component vanishes and assumes opposite signs on the opposite sides of the null point. The position of the null point in the interval between the stabilizing conductors does not coincide with the geometrical central section of the trap, and is always shifted towards one of the ends, depending on the sign of the stabilizing field (Fig. 3). The shift is directed towards sides on which, at a given sign of the stabilizing field, the force lines bear towards the sidewall.

At an appreciable distance from the central section, the azimuthal magnetic field begins to decrease, but this decrease is slower (by approximately a factor 1.5–2 at a distance of 30 cm from the center) compared with the decrease of the longitudinal component.

The waveform of the oscillations of the azimuthal magnetic field, as already noted, is strongly nonsinusoidal, thus making impossible a sufficiently accurate measurement of their phase shifts relative to the oscillations of the electric field. However, since the

FIG. 3. Longitudinal variation of the azimuthal component of the magnetic high frequency at the instant of its maximum at opposite polarities of the stabilizing field.



distortion of the sinusoid by higher harmonics changes from pulse to pulse in random fashion, its influence can be eliminated by averaging over a large number of oscillograms. The phase shifts obtained as a result of such an averaging turned out to be the same as the phase shifts in those individual cases when the oscillations of the azimuthal magnetic field represent undistorted sinusoids.

The phase relation of the azimuthal component established in this manner corresponds to a longitudinal current whose maxima are located in the intervals between the maxima of the electro-potential wave, the longitudinal currents being directed from the center of the plasmoid towards its ends before the positive maximum of the potential.

5. DISCUSSION OF EXPERIMENTAL RESULTS

The obtained picture of the currents indicates that the instability produced under the conditions that the PR-5 installation constitutes a buildup of waves with $k_{\parallel} = 0$. The other possibility—an oblique electron-Langmuir wave—is apparently excluded.

The concrete singularities of the observed magnetic high frequency fields (the absolute magnitude, the absence of an azimuthal field in the center, etc.) agree well with the previously developed concepts.

The absolute magnitude of the fundamental—longitudinal—component of the magnetic field of the wave, observed in the experiment, agrees with the estimate obtained by calculation on the basis of the model of transverse oscillations of the plasmoid, described in the Sec. 2. This estimate is equivalent to assuming that the amplitude of the longitudinal magnetic field at the wall should be approximately equal to the amplitude of the electric field, multiplied by the ratio of the phase velocity of the wave to the velocity of light:

$$H_{\parallel} = E v_{\text{ph}} / c.$$

The discrepancy observed when the experimental values of the magnetic and electric fields are compared in this manner, in accordance with the foregoing formula, usually does not exceed a factor of 2.

The fact that the observed azimuthal magnetic field vanishes in the central part is natural, since the longitudinal currents on both sides of the center should have opposite directions. The shift of the null point towards one of the ends is connected with the fact that in a trap with stabilizing conductors the constant magnetic field force lines that do not lie near the axis, are essentially asymmetrical with respect to the center in the interval between the conductors. On one side of the center the force lines approach the axis and go off to the end mirror; on the other side, the force lines deviate more and more radially and go off to the side wall in a region having a mirror ratio smaller than on the end. This fraction of the force lines is shorter. The shift of the null point of the longitudinal current in this direction indicates that the redistribution of the positive charge separated at the center of the wave occurs to a greater degree in the direction of the longer half of the force lines than in the direction of the shorter one, which is perfectly natural.

The slower decrease of the azimuthal high frequency magnetic field with increasing distance from the center, compared with the longitudinal field, obviously confirms the notion that the region spanned by the oscillations is much longer than the region in which the bulk of the plasma is concentrated (the total length of the main part of the plasmoid from the PR-5 does not exceed 20—30 cm).

6. CONCLUSION

Our principal result is the establishment of the transverse character of the investigated oscillations. This means that the cause of the instability in the PR-5 trap has a different character than in the case of mirror traps with external injection of fast neutral particles (Phoenix, Alice-Baseball, Ogra-II), where an oblique electron-Langmuir wave is built up as a result of resonance between the oscillations of the electrons and the Larmor rotation of a definite group of ions.

A number of theoretical papers have dealt with the possibility of buildup of ion-cyclotron oscillations with $k_{\|} = 0$ as a result of the influence of the transverse spatial inhomogeneity of the density[3], or as a result of the non-Maxwellian character of the distribution function[4], or else of the joint action of both these causes[5]. However, in all the cases under consideration, the necessary condition for the instability is the satisfaction of the relation $k\rho_i \gtrsim 1$ for the buildup oscillations; this relation is certainly violated in PR-5 ($k\rho_i \approx 0.2$, see[1]). Thus, the oscillation-buildup mechanisms considered in the cited papers should apparently not be the causes of the bursts of instability in PR-5.

Kadomtsev and Pogutse[2] proposed a theory explaining the occurrence of instability in the PR-5. This explanation is based on factors of an entirely different nature. Namely, they point to an increase of the magnetic field along the length in both sides of the center of the trap (with simultaneous steep decrease of the density), leading to the so-called "negative mass" effect. A similar instability, called "modified negative-mass instability," was investigated independently both theoretically and experimentally by an Oak Ridge group using the DCX-2 installation[6].

It seems highly probable that the explanation proposed by Kadomtsev and Pogutse is correct, since we are faced with a multilateral correlation between experiment and the theory developed by them. This correspondence, however, concerns principally the character of the oscillations, whereas the assumptions concerning the cause of their buildup has as yet found no direct confirmation.

In conclusion, the authors are grateful to M. S. Ioffe for valuable discussions.

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