INFLUENCE OF THE SURFACE ON THE CYCLOTRON RESONANCE IN METALS

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We give here a theory of cyclotron resonance in metals for an arbitrary electron scattering law at the surface. We show that the cyclotron resonance depends in an essential way on this law. In the limiting case of non-specular scattering (condition (3.38)) the results are the same as those of Azbel' and Kaner. In the limiting case of almost specular scattering of the electrons by the surface (condition (3.10)) a situation arises the possibility of which was discussed in a paper by Chambers. We show that when the electrons are specularly reflected from the metal surface the surface impedance decreases steeply due to the large contribution to the conductivity from the surface electrons. The dependence of the surface impedance on the parameters is then exactly the same as in the case, considered earlier by us, of a cylinder without a magnetic field with a radius equal to the Larmor radius.

1. INTRODUCTION

In the present paper we develop a theory of cyclotron resonance in metals, taking into account the scattering of the electrons by the surface. The original theory of cyclotron resonance[1] was developed assuming the scattering of the conduction electrons from the metal surface to be a diffuse one. Up to the present the opinion that the character of the electron scattering by the surface had little effect on various transport phenomena was widely held. Roughly speaking, such a conclusion is based on the theory of the anomalous skin-effect by Reuter and Sondheimer[2] in which it is shown that in the case of the strongly anomalous skin-effect the surface impedance in the limiting cases of specular and diffuse reflections differs only by a factor 8/9.

The static skin-effect discovered recently by Azbel'[3], various surface effects in a strong magnetic field[4,5], and also size effects (see, e.g.,[6]) have shown that there exists a large class of phenomena which depend in an essential manner on the nature of the electron reflection from the surface. On the other hand, for the strongly anomalous skin-effect, when the main contribution to the conductivity comes from electrons which skip along in the skin layer along the surface of the metal we must assume the reflection to be close to specular.[7] The existence of magnetic surface levels discovered by Khaĭkin[8] (see also[9]) is strong evidence for the specular reflection of the electrons from the surface.

We note now that the theoretically predicted form[1,10,11] of the cyclotron resonance line is not confirmed experimentally (see, e.g., the work by Khaĭkin[12] and other authors). Moreover, the oscillations of the surface impedance connected with cyclotron resonance are usually a small fraction of the average values of the impedance. Chambers[13] tried to remove these contradictions by taking the so-called non-resonance electrons more rigorously into account. The form of the cyclotron resonance line calculated by Chambers under the assumption that the non-resonance electrons give a significantly larger contribution to the conductivity than the resonance electrons turned out to be in good agreement with experiment. However, the problem of the origin of such a large contribution from the non-resonance electrons which Chambers connected with the insufficiently large magnitude of $\Omega\tau$ (Ω: the cyclotron frequency, τ: the relaxation time) remained obscure up to the present moment. In this connection it is necessary to study the influence of the surface of the sample on the cyclotron resonance in metals in much more detail than was done in Kaner's paper.[14]

We shall show here that the surface impedance of a metal in the region of cyclotron resonance depends strongly on the way the electrons are scattered from the surface. It turns out that when the scattering of the electrons from the surface is nearly specular the contribution to the conductivity from electrons which "jump along" the surface inside the skin-layer increases strongly. In the case of a weak magnetic field the presence of such electrons leads to the appearance of magnetic surface levels and in the case of cyclotron resonance to a strong increase of the non-resonance part of the conductivity. The large contribution to the conductivity from the non-resonance electrons arises thus not only owing to the insufficient magnitude of $\Omega\tau$ but also from the fact that the reflection of the electrons from the surface is nearly specular.

A peculiar situation arises if the reflection of the electrons from the surface is specular. In that case even for an electron mean free path which is large (compared to the Larmor radius) their scattering takes place inside the metal (by impurities, phonons, and so on) and not at the surface. This leads to an appreciable increase of the metal conductivity. The dependence of the surface impedance on the parameters is in that case exactly the same as for a thin cylindrical conductor whose surface radius of curvature is equal to the Larmor radius, without a magnetic field.[15]

As the main aim of the present paper is the study of the influence of the metal surface on the cyclotron resonance, the problem of the exact form of the Fermi surface is inessential. To simplify the calculations we restrict ourselves to the case of a quadratic dispersion law for the electrons in the metal. We write the collision integral in the kinetic equation in the relaxation-time approximation.

2. DERIVATION OF THE INTEGRAL EQUATION FOR THE FIELD IN THE METAL

We assume that a non-magnetic metal occupying the half-space $z > 0$ is in a uniform constant magnetic field **H** which is parallel to the y-axis, and in a high-frequency field, the electric field vector **E** of which is parallel to the $z = 0$ plane. The original set of equations consists of the wave equation

$$E_\mu''(z)=\frac{4\pi i\omega}{c^2}j_\mu(z)\qquad u=x,y,z, \tag{2.1}$$

in which we neglect the displacement current compared to the conduction current and the kinetic equation for the conduction electrons, linearized in the field **E**:

$$(i\omega+\tau^{-1})f_1+v_z\frac{\partial f_1}{\partial z}+\Omega\frac{\partial f_1}{\partial\varphi}=e\frac{\partial f_0}{\partial\varepsilon}\mathbf{vE}. \tag{2.2}$$

Here

$$f_0=\left\{1+\exp\left(\frac{\varepsilon-\mu}{T}\right)\right\}^{-1}$$

is the equilibrium electron Fermi distribution function, f_1 the correction to the equilibrium distribution function which is linear in **E**, $\Omega = eH/mc$ the electron cyclotron frequency, ω the frequency of the external field, $\epsilon = mv^2/2$ the electron energy, μ the chemical potential, T the temperature, m the electron effective mass, φ the azimuthal angle which is defined when we introduce a spherical polar system of coordinates in velocity space:

$$v_x=v\sin\theta\sin\varphi,\quad v_y=v\cos\theta;\quad v_z=v\sin\theta\cos\varphi;$$
$$0\leqslant\theta\leqslant\pi;\quad -\pi/2\leqslant\varphi\leqslant 3\pi/2.$$

We solve the kinetic equation (2.2) by the characteristics method. We write the solution of the characteristics equations,

$$d\varphi/\Omega=dz/v_z$$

in the form

$$z=z_0+r_H\sin\varphi. \tag{2.3}$$

Equation (2.3) reflects the fact that when there is a magnetic field present the projections of the electron orbits onto the xz-plane are circles with radius $r_H = v/\Omega \sin\theta$. The integration constant z_0 is the distance of the center of the circle to the metal surface (see Fig. 1). Using the characteristics (2.3) we easily obtain the general solution of the kinetic equation:

$$f_1(\varphi,z_0)=e^{-\beta\varphi}\left[A(z_0)+\frac{e}{\Omega}\frac{\partial f_0}{d\varepsilon}\int^{\varphi}\mathbf{v}(\varphi')\mathbf{E}(z_0+r_H\sin\varphi')e^{\beta\varphi'}d\varphi'\right], \tag{2.4}$$

where, in agreement with (2.3), $z_0 = z - r_H \sin\varphi$ and

$$\beta=(1+i\omega\tau)/\Omega\tau. \tag{2.5}$$

To find the arbitrary function $A(z_0)$ it is necessary to use the boundary conditions that are connected with the nature of the electron motion in the metal. The boundary condition must be given on some line $F(z, \varphi) = 0$ which intersects with all characteristics. When there is no constant magnetic field we can use the surface of the metal $z = 0$ as such a line as in that case the electrons moving along straight lines sooner or later hit the surface. One sees easily that when a magnetic field is present giving the boundary condition on the

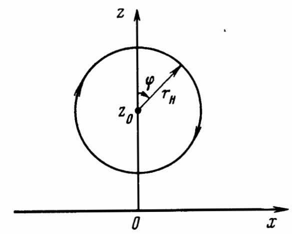

FIG. 1

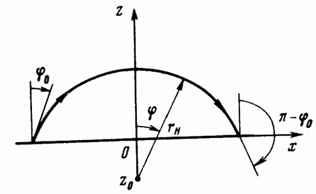

FIG. 2

FIG. 1. Projection of an orbit of an electron which does not hit the metal surface onto the xz-plane ($z_0 > r_H$).

FIG. 2. Projection of an orbit of an electron which hits the metal surface onto the xz-plane ($|z_0| < r_H$).

surface $z = 0$ determines the function $A(z_0)$ only in the region

$$z_0<r_H, \tag{2.6}$$

since only electrons satisfying Eq. (2.6) hit the surface. Such electrons leave the surface at an angle

$$\varphi_0=-\arcsin(z_0/r_H), \tag{2.7}$$

and arrive at the surface at an angle $\pi - \varphi_0$ (see Fig. 2). We use for them Reuter and Sondheimer's boundary condition:[2]

$$f_1(z=0,\varphi=\varphi_0)=pf_1(z=0,\varphi=\pi-\varphi_0), \tag{2.8}$$

in which the scattering of the electrons by the surface is characterized by a single parameter p, the specularity coefficient. In actual cases the coefficient p depends on the glancing angle ψ of the electrons. If we assume that this dependence is of the form $pf(\psi)$, where $f(\psi) \to 1$ when $\psi \ll \psi^*$, the boundary condition (2.8) will hold with a constant parameter p if $\psi^* \gg (\delta/r_H)^{1/2}$ (δ is the penetration depth of the field into the metal). Determining the function $A(z_0)$ from the condition (2.8) we find for the distribution function f_1 in the region (2.6) the following expression:

$$f_1=\frac{e}{\Omega}\frac{\partial f_0}{\partial\varepsilon}e^{-\beta\varphi}[1-pe^{-2\beta(\pi/2-\varphi_0)}]^{-1}\left\{\int_{\varphi_0}d\varphi'+pe^{-2\beta(\pi/2-\varphi_0)}\int_{\varphi}^{\pi-\varphi_0}d\varphi'\right\} \tag{2.9}$$
$$\times e^{\beta\varphi'}\mathbf{v}(\varphi')\mathbf{E}[z-r_H(\sin\varphi-\sin\varphi')],\quad z_0<r_H.$$

In the region

$$z_0>r_H \tag{2.10}$$

the boundary condition (2.8) is inapplicable since in that case the electrons do not travel up to the metal surface and rotate along circles exactly as in an unbounded sample. The distribution function of such electrons does not differ at all from the electron distribution function in an infinite space. Determining the function $A(z_0)$ in the region (2.10) from the condition that f_1 must be periodic with period 2π in φ, we find

$$f_1=\frac{e}{\Omega}\frac{\partial f_0}{\partial\varepsilon}[e^{2\pi\beta}-1]^{-1}\int_{\varphi}^{\varphi+2\pi}e^{\beta(\varphi'-\varphi)}\mathbf{v}(\varphi')\mathbf{E}[z-r_H(\sin\varphi-\sin\varphi')]\,d\varphi',$$
$$z_0>r_H. \tag{2.11}$$

Equations (2.9) and (2.11) solve the problem of finding the electron distribution function in a semi-bounded metal sample when there is a magnetic field present and for an arbitrary specularity coefficient p. At $z_0 = r_H$ the distribution function has a discontinuity.[1)]

[1)] It is necessary to bear the discontinuity of the distribution function in mind when solving the kinetic equation using Fourier transforms.

Using the distribution functions (2.9) and (2.11) we can evaluate the current density in the metal:

$$j_\mu(z)=-\frac{2em^3}{h^3}\int_0^\infty v^2\,dv\int_0^\pi \sin\theta\,d\theta\int_{-\pi/2}^{3\pi/2} v_\mu(\varphi)f_1(z,\varphi)\,d\varphi,\quad \mu=x,y,z. \tag{2.12}$$

When evaluating the current density we must bear in mind that electrons moving with speeds $v > v_0$ (the limiting velocity

$$v_0=\Omega z/\sin\theta(1+\sin\varphi) \tag{2.13}$$

is determined from the condition $z_0 = r_H$) collide with the surface while electrons moving with velocities $v < v_0$ do not hit the surface. Therefore, when integrating over the velocity from zero to v_0 we must substitute into (2.12) the distribution function (2.11) while when integrating from v_0 to infinity, the distribution function (2.9). The complete expression for the current density in the metal has thus the form[2)]

$$j_\mu(z)=-\frac{2e^2m^3}{\Omega h^3}\int_0^\pi \sin\theta\,d\theta\int_{-\pi/2}^{3\pi/2} e^{-\beta\varphi}n_\mu(\varphi)\,d\varphi\Big\{[e^{2\pi\beta}-1]^{-1}\int_0^{v_0}dv\int_\varphi^{\varphi+2\pi}d\varphi'$$
$$+\int_{v_0}^\infty dv[1-pe^{-2\beta(\pi/2-\varphi_0)}]^{-1}\Big[\int_{\varphi_0}^{\varphi}d\varphi'+pe^{-2\beta(\pi/2-\varphi_0)}\int_{\varphi}^{\pi-\varphi_0}d\varphi'\Big]\Big\}$$
$$\times e^{\beta\varphi'}n_\nu(\varphi')E_\nu[z-r_H(\sin\varphi-\sin\varphi')]v^4\frac{\partial f_0}{\partial\varepsilon}, \tag{2.14}$$

where $n_\mu = v_\mu/v$.

We obtain the equation for the field in the metal by substituting Eq. (2.14) for the current density into the wave equation (2.1). When changing to Fourier components it is convenient to continue the function $\mathbf{E}(z)$ to the region $z < 0$ in an even manner: $\mathbf{E}(-z) = \mathbf{E}(z)$. When calculating the Fourier components of the current density

$$J_\mu(k)=2\int_0^\infty j_\mu(z)\cos kz\,dz$$

it is convenient to interchange the order of integration over z and v. As a result we find

$$J_\mu(k)=\int_0^\infty \Xi_{\mu\nu}(k,k')\mathscr{E}_\nu(k')\,dk', \tag{2.15}$$

where

$$\mathscr{E}_\mu(k)=2\int_0^\infty E_\mu(z)\cos kz\,dz$$

is the Fourier component of the field in the metal while $\Xi_{\mu\nu}(k, k')$—the conductivity kernel—can be written in the form of two terms

$$\Xi_{\mu\nu}(k,k')=\sigma_{\mu\nu}(k)\delta(k-k')+\zeta_{\mu\nu}(k,k'). \tag{2.16}$$

In the first term in (2.16)

$$\sigma_{\mu\nu}(k)=-\frac{2e^2m^3}{\Omega h^3(e^{2\pi\beta}-1)}\int_0^\infty v^4\frac{\partial f_0}{\partial\varepsilon}dv\int_0^\pi \sin\theta\,d\theta\cdot \tag{2.17}$$
$$\times\int_{-\pi/2}^{3\pi/2} e^{-\beta\varphi}n_\mu(\varphi)\,d\varphi\int_\varphi^{\varphi+2\pi} e^{\beta\varphi'}n_\nu(\varphi')\cos[kr_H(\sin\varphi-\sin\varphi')]\,d\varphi'$$

is the conductivity tensor of an infinite sample. The second term

$$\zeta_{\mu\nu}(k,k')=-\frac{4e^2m^3}{\pi\Omega h^3}\int_0^\infty v^4\frac{\partial f_0}{d\varepsilon}dv\int_0^\pi \sin\theta\,d\theta\int_{-\pi/2}^{3\pi/2} e^{-\beta\varphi}n_\mu(\varphi)\,d\varphi\int_0^{z^*}\cos kz\,dz\cdot$$
$$\times\Big\{-[e^{2\pi\beta}-1]^{-1}\int_\varphi^{\varphi+2\pi}d\varphi'+[1-pe^{-2\beta(\pi/2-\varphi_0)}]^{-1}\Big[\int_{\varphi_0}^{\varphi}d\varphi'+pe^{-2\beta(\pi/2-\varphi_0)}\int_\varphi^{\pi-\varphi_0}d\varphi'\Big]\Big\}$$
$$\times e^{\beta\varphi'}n_\nu(\varphi')\cos\{k'[z-r_H(\sin\varphi-\sin\varphi')]\} \tag{2.18}$$

is that the non-difference part of the conductivity kernel, i.e., the part which is not a function of the difference $k - k'$ and which arises when we take into account the reflection of the electrons from the metal surface. The boundary coordinate

$$z^*=r_H(1+\sin\varphi) \tag{2.19}$$

is determined from the condition $z_0 = r_H$. Putting

$$\mathscr{K}_{\mu\nu}(k)\equiv\frac{4\pi i\omega}{c^2}\sigma_{\mu\nu}(k),\quad Q_{\mu\nu}(k,k')\equiv\frac{4\pi i\omega}{c^2}\zeta_{\mu\nu}(k,k'),$$

we get the following integral equation for the Fourier component of the field in the metal

$$[k^2\delta_{\mu\nu}+\mathscr{K}_{\mu\nu}(k)]\mathscr{E}_\nu(k)+\int_0^\infty Q_{\mu\nu}(k,k')\mathscr{E}_\nu(k')\,dk'=-2E_\mu'(+0). \tag{2.20}$$

Changing in Eq. (2.18) from an integration over the coordinate z to an integration over the angle of emergence φ_0 ($z = r_H(\sin\varphi - \sin\varphi_0)$; see (2.7) and (2.3)) we can write Eq. (2.18) after some simple transformations in the form

$$\zeta_{\mu\nu}(k,k')=-\frac{8e^2m^3}{\pi\Omega^2h^3}\int_0^\infty v^5\frac{\partial f_0}{\partial\varepsilon}dv\int_0^\pi \sin^2\theta\,d\theta\int_{-\pi/2}^{\pi/2}\operatorname{ch}\beta\Big(\frac{\pi}{2}-\varphi\Big)n_\mu(\varphi)\,d\varphi\cdot$$
$$\times\int_{-\pi/2}^{\varphi}\cos\varphi_0\,d\varphi_0\Big\{\int_{\varphi_0}^{\pi/2}e^{-\beta(\pi/2-\varphi')}\frac{1+pe^{-2\beta(\varphi'-\varphi_0)}}{1-pe^{-2\beta(\pi/2-\varphi_0)}}d\varphi'-\int_{-\pi/2}^{\pi/2}\frac{\operatorname{ch}\beta(\pi/2+\varphi')}{\operatorname{sh}\beta\pi}d\varphi'\Big\}$$
$$\times n_\nu(\varphi')\cos[kr_H(\sin\varphi-\sin\varphi_0)]\cos[k'r_H(\sin\varphi'-\sin\varphi_0)]. \tag{2.21}$$

If the reflection of the electrons from the surface is diffuse ($p = 0$) the non-difference part of the conductivity kernel reduces to the same expression as the one in the paper by Azbel' and Kaner (Eq. (5.10) in[1]).

3. SURFACE IMPEDANCE FOR CYCLOTRON RESONANCE CONDITIONS

Equation (2.20) enables us in principle to evaluate the surface impedance of a metal for arbitrary ratios of the Larmor radius, the electron mean free path, and the skin-layer depth. However, when studying cyclotron resonance it is sufficient to have the asymptotic expressions for $\mathscr{K}_{\mu\nu}(k)$ and $Q_{\mu\nu}(k, k')$ in the range of the strongly anomalous skin-effect when the thickness of the skin-layer is much smaller than the electron Larmor radius. The main role is then in Eq. (2.20) played by large $k \sim \delta^{-1}$ and the expansion of its coefficients can be made in terms of the parameter

$$kr_H\gg 1. \tag{3.1}$$

Condition (3.1) enables us to use the steepest descent method for the evaluation of the integrals. The saddle points are determined from the condition $\cos\varphi = \cos\varphi_0 = \cos\varphi' = 0$ while $v_z = 0$. Hence, the main contribution to the conductivity in the case of a strongly anomalous skin-effect is made by electrons which move in the skin-layer parallel to the surface. The z-component of the current density is then small and we may assume that the vector $\mathbf{E}$ lies in the xy-plane. In what follows the indices ν, μ take on the values x and y.

From all the conduction electrons only those whose

[2)]Summation over the index ν which can take on the values x, y, and z is implied if it is repeated twice.

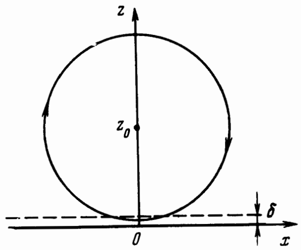

FIG. 3

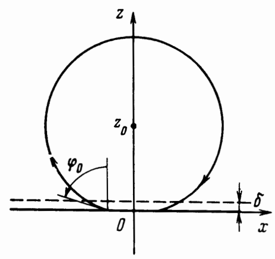

FIG. 4

FIG. 3. Trajectory of a resonance electron which does not hit the surface.

FIG. 4. Trajectory of a resonance electron which hits the surface ($\varphi_0 \to -\pi/2$).

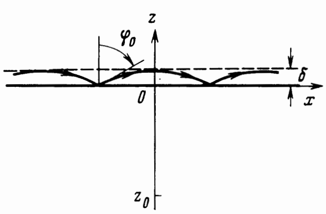

FIG. 5. Trajectory of a non-resonance electron which "jumps along" in the skin-layer ($\varphi_0 \to \pi/2$).

trajectories are shown in Figs. 3, 4, and 5 are thus important. Electrons moving along circles, shown in Figs. 3 and 4 are resonance electrons which give a contribution to the cyclotron resonance. However, electrons moving along "jumping" orbits in the skin-layer (Fig. 5) are of a non-resonance nature and give no contribution to the cyclotron resonance.

We note that if the electrons are reflected specularly from the surface (p = 1) in the integrand in Eq. (2.21) for the non-difference part of the conductivity kernel the additional singularity in the saddle-point $\varphi_0 = \pi/2$ is important. Physically this singularity is connected with the fact that for specular reflection of the electrons from the surface of the metal the conductivity increases steeply just when we take into account the electrons which "skip along" in the skin-layer along the surface. Such electrons leave the surface just at angles φ_0 which are close to $\pi/2$ (see Fig. 5).[3)] As in the cyclotron region $|\beta| \sim 1$, the exponent in the denominator of (2.21) can close to the singularity be expanded in a power series.

$$1 - pe^{-2\beta(\pi/2-\varphi_0)} = 1 - p + 2p\beta(\pi/2 -$$

In the integral over φ_0 a small neighborhood of the point $\pi/2$ of the order of $(\delta/r_H)^{1/2}$ is important. Therefore, if the scattering of the electrons which "skip along" in the surface layer by the surface dominates over the scattering inside the metal the following inequality will hold:

$$1 - p \gg |\beta| (\delta / r_H)^{1/2}. \tag{3.2}$$

If, however, the electrons which "skip" along the surface in the skin-layer are mainly scattered inside the metal the inequality with the opposite inequality sign holds. In the present section we restrict ourselves to case (3.2).[4)] The opposite limiting case will be considered in section 4.

The asymptotic expression for $\mathscr{K}_{\mu\nu}(k)$ in the region (3.1) can most simply be obtained by first evaluating the integrals over φ and φ' in (2.17) by the saddle point method. The remaining integrals over v and θ can as follows be expressed in terms of the thermodynamic potential,

$$\Omega(\varepsilon) = -T \ln\left\{1 + \exp\left(\frac{\mu - \varepsilon}{T}\right)\right\}$$

Writing[5)]

$$-\int_0^\infty v^3 \frac{\partial f_0}{\partial \varepsilon} dv \int_0^\pi n_u^2(\pm\pi/2)\, d\theta = \frac{\pi}{m^2} |\Omega(0)|. \tag{3.3}$$

$$k_r \equiv [2\omega e^2 m |\Omega(0)| / c^2\hbar^3]^{1/3}, \tag{3.4}$$

We get the following expression for $\mathscr{K}_{\mu\nu}(k)$ in the region (3.1):

$$\mathscr{K}_{\mu\nu}(k) = \frac{ik_T^3}{k} \delta_{\mu\nu} \operatorname{cth} \beta\pi. \tag{3.5}$$

The evaluation of the asymptotic expression for $\zeta_{\mu\nu}(k, k')$ is somewhat complicated. It is convenient to proceed as follows. First of all we integrate the integral over φ_0 by parts. In the region (3.2) we can then consider all exponentials to be slowly varying (compared to the cosines) functions. We then use the method of steepest descent to calculate the integrals over φ and φ'. Finally, the integrals over v and θ are again evaluated using Eq. (3.3). Noting that in the region (3.1)

$$\frac{\sin 2r_H(k - k')}{k - k'} \approx \pi\delta(k - k'),$$

we can write Eq. (2.20) in the regions (3.1), (3.2) in the form

$$(\xi^2 + \alpha/\xi) g(\xi) + 2\alpha \int_0^\infty g(\xi') \left[A \frac{\ln \xi/\xi'}{\xi^2 - \xi'^2} - \frac{B}{\sqrt{\xi\xi'}(\xi + \xi')} \right] d\xi' = 1, \tag{3.6}$$

where

$$\alpha = \frac{i}{1 - \lambda}, \quad A = -\frac{1}{4\pi^2} \frac{1 + p}{1 - p} \frac{(1 - \lambda)(3 - p + \lambda - 3p\lambda)}{1 - p\lambda^2},$$

$$B = \frac{1}{8\pi} \frac{(1 - p)(1 + \lambda)^2}{1 - p\lambda^2},$$

$$\xi = \frac{k}{k_c}, \quad k_c^3 = k_T^3 \frac{1 - p\lambda^2}{1 - p\lambda}, \quad g(\xi) = -\frac{k_\text{п}^2 \mathscr{E}_\mu(k_\text{п}\xi)}{2E_\mu'(+0)}, \quad \lambda = e^{-2\pi\beta}.$$

Hartman and Luttinger[16] gave a solution of an equation such as (3.6) and they found for the surface impedance the following expression

$$Z = \frac{32\sqrt{3}\,\pi e^{i\pi/3}\omega}{c^2 k} \exp\left\{-\frac{1}{3}\ln\frac{\alpha}{i}\right\} \left[\sin\frac{\pi z_1}{3} \sin\frac{\pi z_2}{3}\right]^2 \Big/ (1 - \rho_1)(1 - \rho_2). \tag{3.7}$$

Here $\rho_1 = \cos \pi z_1$ and $\rho_2 = \cos \pi z_2$ ($0 < \operatorname{Re} z_i < 1$, $i = 1, 2$) are the roots of the quadratic equation

[3)]In weak magnetic fields the presence of such electrons leads to the existence of quantal oscillations of the surface impedance which are connected with magnetic surface levels.[8] In the cyclotron resonance region ($\Omega \sim \omega$) quantum transitions between surface levels are semi-classical in nature. This makes it possible to study cyclotron resonance using the classical kinetic equation for the conduction electrons.

[4)]We shall show in the following that if the electrons are scattered specularly by the surface, the part of the surface impedance which oscillates (when cyclotron resonance is taken into account) is appreciably less than its average value. From the point of view of cyclotron resonance (3.2) gives thus the most important region.

[5)]When the conduction electron gas is degenerate,

$$k_T = k_0 = (e^2 m^2 v_0^2 \omega / c^2\hbar^3)^{1/3}, \tag{3.4'}$$

v_0 is the Fermi velocity.

$\rho^2 - \rho(1 - 2\pi B + \pi^2 A) - 2\pi B = 0$. We have thus obtained the complete expression for the surface impedance in the regions (3.1), (3.2). However, for practical purposes the general expression (3.7) is inconvenient to use because of its complexity. In the following we shall consider in detail some special cases.

The Azbel'-Kaner Case

We assume that the reflection of the electrons from the metal surface is sufficiently diffuse so that

$$1 - p \gg \frac{1}{\Omega\tau}. \tag{3.8}$$

Condition (3.8) means that all electrons which hit the surface and form open orbits (not lying, in general, in the skin-layer) are primarily scattered by the surface and not inside the metal. The region (3.8) comprises an appreciable band of variation in p as the existence of cyclotron resonance requires that the inequality $\Omega\tau \gg 1$ is satisfied.

We shall find an expression for the surface impedance in a small neighborhood near the cyclotron resonance. As at resonance $\omega = n\Omega$, $n = 1, 2, \ldots$ we have near the n-th resonance $\lambda = 1 - \delta n$, where

$$\delta_n = 2\pi\left(\frac{1}{\Omega\tau} + i\frac{\omega - n\Omega}{\Omega}\right),$$

while the most important δ_n are those such that $|\delta_n| \sim 1/\Omega\tau \ll 1$. In the region $|\delta_n| \ll 1 - p$ the general expression (3.7) for the surface impedance can be written in the form

$$Z = \frac{8\pi}{\sqrt{3}} e^{i\pi/3} \frac{\omega}{c^2 k_T} \delta_n^{1/3}. \tag{3.9}$$

In the immediate vicinity of a resonance the surface impedance of a metal is thus independent of the specularity coefficient p, when (3.8) is satisfied, and has the same form as in the paper by Azbel' and Kaner[1] in the case of a diffuse surface.

The Chambers Case

Let now the reflection of the electrons from the surface be so close to being specular that the inequality

$$|\beta|(\delta/r_H)^{1/2} \ll 1 - p \ll 1/\Omega\tau. \tag{3.10}$$

is satisfied. In that case the electrons which "skip along" in the skin-layer (Fig. 5) are mainly scattered by the surface while the resonance electrons (Figs. 3, 4) are scattered inside the metal. Under those conditions the main contribution to the conductivity is made by the non-resonance electrons which "skip-along" in the skin-layer. On the other hand, the resonance electrons make a substantially smaller contribution. The oscillations of the surface impedance which are connected with the cyclotron resonance are thus only a small fraction of the average value. Expanding the general expression (3.7) for the surface impedance in terms of the small parameter $1 - p$ we find the first two terms:

$$Z = \frac{2^{5/3}\sqrt{3}\,\pi\omega e^{-2\pi i/3}}{c^2 k_T}(1-p)^{1/3}\{1 - 2^{1/3}[(1-p)\operatorname{cth}\beta\pi]^{2/3}\}. \tag{3.11}$$

The first term in (3.11) is independent of the magnetic field but depends significantly on the specularity of the surface. The oscillating dependence of the surface impedance on the magnetic field which is connected with the cyclotron resonance occurs only in the second term in (3.11) which is a small fraction (of the order of $(1 - p)^{2/3}$) of the first term.

In experiments one usually measures not the surface impedance itself but its derivative with respect to the magnetic field. For the sake of a convenient comparison with experimental data we evaluate the derivative of the surface impedance with respect to the magnetic field in the vicinity of cyclotron resonance.

Differentiating (3.1) and writing

$$\gamma_n \equiv \tau(\omega - n\Omega),$$

where n is the number of the resonance, we get

$$\frac{dZ_n}{d\Omega} = -\frac{4\pi^{1/3}}{\sqrt{3}}\frac{n^{1/3}(\omega\tau)^{5/3}}{c^2 k_T}(1-p)\exp\left\{-i\left(\frac{\pi}{6} + \frac{5}{3}\operatorname{arctg}\gamma_n\right)\right\}(1+\gamma_n^2)^{-5/6}. \tag{3.12}$$

In Fig. 6 we give the graph of the resonance factor in (3.12). This form of line shape has been observed experimentally in copper.[17]

We shall here not discuss in detail the line shape of the cyclotron resonance as it depends not only on the quality of the surface but also on the electronic properties of the metal considered (in particular, on the electron dispersion law, the electron-phonon interaction, and so on). We note merely that in the region (3.10) a situation arises the possibility of which was discussed by Chambers.[13]

Cyclotron resonance depends thus significantly on the properties of the metal surface. In the limiting case (3.8) the situation occurs which is described in the paper by Azbel' and Kaner[1] while the region (3.10) corresponds to Chambers' paper.[13]

We can now note that in actual cases the Chambers case may occur because if the surface of the sample is pretty nearly a plane (the magnitude of the roughness $d \ll \lambda_0\sqrt{(r_H/\delta)}$, where λ_0 is the electron wavelength), the reflection of the electrons will be nearly specular. If, however, one tries artificially to increase the diffu-

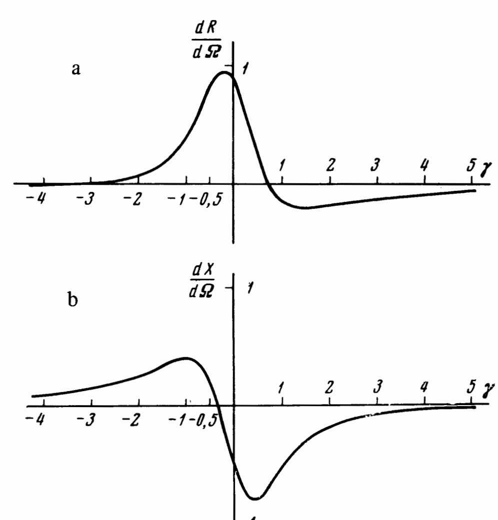

FIG. 6. Real (a) and imaginary (b) parts of the derivative of the surface impedance with respect to Ω in the vicinity of cyclotron resonance in the Chambers case.

sivity of the surface the magnitude of the roughness becomes comparable to the skin-layer thickness. This has as a consequence that the resonance electrons will be strongly scattered in the skin-layer by surface inhomogeneities and cyclotron resonance will not be observed. For the realization of the Azbel'-Kaner case two inequalities, $\delta \gg d \gg \lambda_0\sqrt{(r_H/\delta)}$, must thus be satisfied simultaneously.

4. SPECULAR REFLECTION FROM THE SURFACE

We now evaluate the surface impedance of a metal in the case when the surface is so close to being specular that even the electrons which "skip-along" in the skin-layer are scattered inside the metal notwithstanding their frequent collisions with the surface. In that case

$$1-p \ll |\beta|(\delta/r_H)^{1/2}. \tag{4.1}$$

From the point of view of cyclotron resonance this situation differs little from the Chambers case. As far as the average value of the surface impedance and its magnetic field dependence are concerned, there is an essential difference from (3.11). Apart from a constant factor we can find these quantities without in fact solving the integral equation.

The presence of a singularity at the saddle point $\varphi_0 = \pi/2$ in the integrand of (2.21) when p = 1 means that the main contribution to the conductivity when (4.1) is satisfied comes from the electrons which "skip along" in the skin-layer. The main contribution to the integrals over φ and φ' in (2.21) then comes from only a small neighborhood of the point $\pi/2$ of order $(\delta/r_H)^{1/2}$. Expanding the integrand of (2.21) in that neighborhood we get for the non-difference part of the conductivity kernel in the region (4.1) the following expression:

$$\zeta_{\mu\nu}(k,k') = -\frac{2^{9/2}e^2m^3}{\pi\beta\Omega^{1/2}h^3k'^{3/2}}F_\mu(k/k')\delta_{\mu\nu}\int_0^\infty v^{7/2}\frac{\partial f_0}{\partial\varepsilon}dv, \tag{4.2}$$

where

$$F_\mu(x) = \int_0 \sqrt{\sin\theta}\, n_\mu^2(\pi/2)d\theta\int_0^\infty d\zeta\int_0^\zeta d\eta\int_0^\zeta d\xi\cos(\zeta^2-\xi^2)\cos[x(\zeta^2-\eta^2)] \tag{4.3}$$

is a function of x. Assuming that the electron gas in the metal is degenerate we get from (4.2)

$$\zeta_{\mu\nu}(k,k') = \frac{2^{9/2}}{\pi}\delta_{\mu\nu}\frac{e^2m^2v_0^{5/2}}{\beta k'^{3/2}\Omega^{1/2}h^3}F_\mu(k/k'). \tag{4.4}$$

Equation (2.20) takes in the region (4.1) the form

$$k^2\mathscr{E}_\mu(k) + i\frac{2^{7/2}}{\pi^3\beta}\frac{k_0^3}{k^{3/2}}\sqrt{\frac{v_0}{\Omega}}\int_0^\infty\mathscr{E}_\mu(k')F_\mu(k/k')dk' = -2E_\mu'(+0). \tag{4.5}$$

Putting further

$$k = k_3\xi,\quad g_\mu(\xi) = -\frac{k_3^2\mathscr{E}_\mu(k_3\xi)}{2E_\mu'(+0)},\quad k_3^{5/2} = i\frac{2^{7/2}}{\pi^3}\frac{k_0^3}{\beta}\sqrt{\frac{v_0}{\Omega}}, \tag{4.6}$$

we can reduce Eq. (4.5) to a dimensionless equation:

$$\xi^2g_\mu(\xi) + \int_0^\infty g_\mu(\xi')F_\mu(\xi/\xi')d\xi' = 1. \tag{4.7}$$

In the region (4.1) we get for the surface impedance the following expression:

$$Z_\mu = -\frac{4\pi i\omega}{c^2}\frac{E_\mu(0)}{E_\mu'(+0)} = \frac{8i\omega}{c^2k_3}A_\mu, \tag{4.8}$$

where $A_\mu = \int_0^\infty g_\mu(\xi)d\xi$ is a constant of order unity which is determined by solving Eq. (4.7). We can also write Eq. (4.8) in the form

$$Z = \frac{\text{const}}{c}\left\{\frac{r_0\omega}{c}\left(\frac{\omega}{\omega_0}\right)^4\left(1+\frac{1}{\omega^2\tau^2}\right)\right\}^{1/5}\exp\left\{i\left(\frac{\pi}{2}-\frac{2}{5}\operatorname{arctg}\frac{1}{\omega\tau}\right)\right\}. \tag{4.9}$$

Here ω_0 is the electron plasma frequency, $r_0 = v_0/\Omega$, c the velocity of light.

The steep increase in the metal conductivity in the region (4.1) when the electrons which "skip along" in the skin layer are taken into account leads thus to a steep decrease in the surface impedance. The same situation then arises as in the case of a thin cylindrical conductor with a specular surface without a magnetic field.[15] We obtain in those two cases expressions for the surface impedance which differ by a constant factor. The role of the cylinder radius is played by the Larmor radius in the case of a plane sample in a magnetic field.

The surface impedance of a metal with a specular surface decreases monotonically with the magnetic field proportional to $H^{-1/5}$. An experimental verification of this dependence would be of great interest for characterizing the conditions of electron reflection.

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39