SELF-INDUCED TRANSPARENCY OF A GAS IN A MAGNETIC FIELD

A. I. ALEKSEEV and A. S. CHERNOV

Moscow Engineering-physics Institute

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Self-induced transparency of a gas in a magnetic field by passage of a polarized light pulse is investigated. It is shown that the external magnetic field decreases considerably the number of resonant atomic transitions for which self-induced transparency of the medium is possible. An analytic solution for a linearly polarized light pulse propagating without change of its shape or loss of energy is found in the case of atomic transitions involving a change of the total momentum $1 \rightleftarrows 0$, $1/2 \to 1/2$ and $1 \to 1$. In the presence of self-induced transparency, the direction of rotation of the light-pulse polarization is opposite that of Faraday rotation in a monochromatic wave.

THE self-induced transparency of a resonant medium in the propagation of an ultrashort intense light pulse was predicted theoretically[1] and was recently investigated experimentally in a solid[2] and in a gas[3]. Subsequently, this phenomenon was used many times in the investigation of nonlinear properties of a medium.

The physical cause of the self-induced transparency is the absorption of photons by resonant atoms on the leading front of a moving light pulse, followed by induced emission of the excited atoms on the trailing edge of the same pulse. As a result, the light pulse moves under certain conditions in a resonant medium without distortion of its profile and without energy loss.

It was noted in[4] that level degeneracy influences the self-induced transparency when the spectral line is homogeneously broadened. It turned out that the motion of a light pulse without change of profile[1,2] is possible if the resonant atomic transitions are accompanied by a change of the total angular momentum $0 \rightleftarrows 1$, $1 \to 1$, $1/2 \to 1/2$, and $3/2 \rightleftarrows 1/2$. In the case of atomic transitions $j \to j$ $(j > 1)$, self-induced transparency is possible for light pulses with several maxima, whereas for atomic transitions $j \rightleftarrows j + 1$ $(j \geq 1)$ the phenomenon of self-induced transparency does not occur at all.

In an external magnetic field **H**, the effect of self-induced transparency acquires new features. In particular, the dependence of this effect on the degeneracy of the levels and on the type of the atomic transition becomes stronger. We assume further that the degeneracy of the levels is due to the different orientations of the total angular momentum. Such a situation is realized, for example, for atoms (molecules) of a gas. The necessary condition for self-induced transparency in a gas is smallness of the duration of the light pulse compared with the time of irreversible relaxation, which is determined by the atomic collisions and by the spontaneous emission. At the same time, the time of the Doppler relaxation can be either larger or smaller than the duration of the light pulse.

Self-induced transparency of a gas in the presence of a magnetic field occurs only for resonant atomic transitions with change of total angular momentum $1 \rightleftarrows 0$, $1/2 \to 1/2$, and $1 \to 1$. In other cases, motion of the light pulse without deformation or loss (2π-pulse) is not realized. The direction of rotation of the polarization of a 2π-pulse is opposite to the usual Faraday rotation in a monochromatic wave, and the absolute magnitude of the angle of rotation of the polarization of a 2π-pulse is larger or smaller than the Faraday angle, depending on the parameters of the experiments. From the angle of rotation of the 2π-pulse polarization it is possible to calculate the g-factors of the resonant levels. The foregoing singularities of self-induced transparency has been established for linearly polarized pulses moving along the magnetic field.

Assume that the Zeeman splitting is small compared with the frequency of the atomic transition. Then the upper and lower resonant levels can be regarded as consisting respectively of $2J_2 + 1$ and $2J_1 + 1$ close sublevels, where J_1 and J_2 are the total angular momentum.

The vector potential **A** of the linearly-polarized light pulse propagating along the Z axis is represented in the form

$$\mathbf{A} = \mathbf{a} \exp[i(kz - \omega t)], \quad \mathbf{a} = \mathbf{l}a, \quad \mathbf{l}^2 = 1, \quad k = \omega / c = 1/\lambda\!\!\!^-, \tag{1}$$

where c is the velocity of light in vacuum and a is a real slowly-varying amplitude:

$$\frac{\partial a_\alpha}{\partial z} \ll \frac{a_\alpha}{\lambda\!\!\!^-}, \quad \frac{\partial a_\alpha}{\partial t} \ll \omega a_\alpha.$$

The polarization current induced by the field (1) is equal to the integral, with respect to v, of the trace of the matrix

$$I_\alpha = -i\omega d_{m\mu}{}^\alpha R_{\mu m'}, \tag{2}$$

where $R_{\mu m}$ is the density matrix of the group of atoms (molecules) moving with velocity v, and $d^\alpha_{m\mu}$ is the dipole moment of the transition between the upper and lower levels. If we separate from the current (2) the slowly-varying amplitude

$$I_\alpha = j_\alpha \exp[i(kz - \omega t)],$$

then the fundamental equations are written in the form:

$$\left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial z}\right) a_\alpha = i \cdot 2\pi\lambda\!\!\!^- \int d\mathbf{v}\, \mathrm{Sp}\, j_\alpha, \tag{3}$$

$$\left(\frac{\partial}{\partial t} + ikv\right) j_\alpha + i\varepsilon_2 Q_{\alpha\sigma} j_\sigma - i\frac{\varepsilon_1}{\hbar} j_\alpha(\hat{\mathbf{J}}_1\mathbf{n}) + i\cdot\frac{3}{4}(2J_2+1)\gamma c\lambda\!\!\!^- (\rho_2{}^{\alpha\beta} - T_{\alpha\beta}\rho_1) a_\beta = 0, \tag{4}$$

$$\frac{\partial}{\partial t}\rho_2^{\alpha\beta}+i\varepsilon_2(Q_{\alpha\sigma}\rho_2^{\sigma\beta}-\rho_2^{\alpha\sigma}Q_{\sigma\beta})+i\frac{1}{\hbar c}(j_\alpha T_{\sigma\beta}-T_{\alpha\sigma}j_\beta{}^+)a_\sigma=0, \quad (5)$$

$$\frac{\partial}{\partial t}\rho_1+i\frac{\varepsilon_1}{\hbar}[(\mathbf{n}\hat{\mathbf{J}}_1)\rho_1-\rho_1(\mathbf{n}\hat{\mathbf{J}}_1)]-i\frac{1}{\hbar c}(j_\sigma-j_\sigma{}^+)a_\sigma=0, \quad (6)$$

where we use the following notation:

$$Q_{\alpha\beta}=(2J_2+1)d^\alpha_{m\mu}(\mathbf{n}\hat{\mathbf{J}}_2)_{\mu\mu'}d^\beta_{\mu'm'}/\hbar|d^{J_2}_{J_1}|^2,$$

$$T_{\alpha\beta}=d^\alpha_{m\mu}d^\beta_{\mu m'}/|d^{J_2}_{J_1}|^2,$$

$$\rho_2^{\alpha\beta}=d^\alpha_{m\mu}\rho_{\mu\mu'}d^\beta_{\mu'm'}/|d^{J_2}_{J_1}|^2,$$

$$\rho_1=\rho_{mm'},\quad \mathbf{n}=\mathbf{H}/H,$$

$$\varepsilon_1=\mu_0 g_1 H/\hbar,\quad \varepsilon_2=\mu_0 g_2 H/\hbar,$$

$$\gamma=4|d_{J_1}{}^{J_2}|^2/3(2J_2+1)\hbar\lambda^3.$$

Here $\rho_{\mu\mu'}$ and $\rho_{mm'}$ are the density matrices at the upper and lower levels, $d^{J_2}_{J_1}$ is the reduced dipole moment of the transition, γ is the probability of spontaneous emission, μ_0 is the Bohr magneton, and $\hat{J}_1$ and g_1 are respectively the operator of the total angular momentum and the g-factor of the lower level, while $\hat{J}_2$ and g_2 are analogous quantities for the upper level. The frequency ω of the pulse (1) coincides with the frequency of the resonant atomic transition. For convenience, Eqs. (3)–(6) are written in terms of the matrix indices of the operator of the total angular momentum of the lower level.

Prior to the passage of the light pulse, the distribution of the atoms over the velocities and the energy levels was described by the Maxwell and the Boltzmann distributions:

$$j_\alpha(\mathbf{r},-\infty)=0,$$

$$\rho_1(\mathbf{r},-\infty)=n_1 f/(2J_1+1),$$

$$\rho_2{}^{\alpha\beta}(\mathbf{r},-\infty)=n_2 f T_{\alpha\beta}/(2J_2+1),$$

where n_1 and n_2 are the densities of the atoms at the lower and upper levels respectively, at the initial instant $t=-\infty$ at the point r. The unit matrix is written out in explicit form. The atom-velocity distribution function f is

$$f=(1/\pi^{1/2}u)\exp(-v^2/u^2),$$

where u is the thermal velocity of the atom.

We consider first an electromagnetic pulse (1) in resonance with the atomic transition with change of total momentum $J_2=1\rightarrow J_1=0$. Then

$$Q_{\alpha\beta}=-ie_{\alpha\beta\gamma}n_\gamma,\quad T_{\alpha\beta}=\delta_{\alpha\beta}/3. \quad (7)$$

We make in Eqs. (3)–(6) the substitution

$$j_\alpha=l_\alpha j,\quad 3\rho_2{}^{\alpha\beta}l_\alpha l_\beta-\rho_1=N,$$

assuming that unit vector l satisfies the equation*

$$\partial\mathbf{l}/\partial t=\varepsilon_2[\mathbf{nl}]$$

and is a function of the argument $t - z/t$. Then Eqs. (3)–(6) with allowance for (7) reduce to the already investigated[2] system of equations:

$$(\partial/\partial t+c\partial/\partial z)a=i\cdot 2\pi\lambda\int dv\,j, \quad (8)$$

$$(\partial/\partial t+ikv)j+i\cdot 3\gamma c\lambda aN/4=0, \quad (9)$$

$$\partial N/\partial t+i\cdot 2a(j-j^*)/\hbar c=0. \quad (10)$$

*$[\mathbf{nl}]\equiv\mathbf{n}\times\mathbf{l}$.

A stationary solution of Eqs. (8)–(10) is the 2π-pulse

$$a=a_0/\mathrm{ch}\,[(t-z/v_0)/T]. \quad (11)$$

The duration T and the velocity v_0 for the 2π-pulse is determined by the relations

$$T=1/a_0(3\gamma\lambda/4\hbar)^{1/2},$$

$$\frac{c}{v_0}-1=-3\sqrt{\pi}N_0\lambda^2\gamma cT_0^2\int_0^\infty\frac{e^{-\xi^2}}{\xi^2+(T_0/T)^2}d\xi,$$

where $T_0 = 1/ku$ is the time of the Doppler relaxation; $N_0 = n_2/3 - n_1$ is the initial excess population of the levels prior to the passage of the 2π-pulse, and the quantity a_0 is a free parameter, which assumes different values depending on the form of the light pulse transmitted into the medium. However, the area of the 2π-pulse (11) has a fully defined value

$$\int_{-\infty}^{\infty}a\,dt=\pi a_0T=\sqrt{3}\pi\lambda\hbar/|d_{J_1}{}^{J_2}|. \quad (12)$$

If the area of the light pulse transmitted into the medium satisfies the condition (12), then the pulse assumes the form (11) after the establishment of the stationary regime. If the area of the transmitted pulse is a multiple of the quantity πa_0T, then other stationary regimes are possible[2,5].

Thus, in the case of an atomic transition with a change of the total angular momentum $J_2 = 1 \rightarrow J_1 = 0$, the amplitude a for a 2π-pulse $\mathbf{a} = \mathbf{l}a$ in an external magnetic field H is given by formula (11), and the components of the polarization vector are

$$l_1=\cos\varepsilon_2(t-z/c),\quad l_2=\sin\varepsilon_2(t-z/c),\quad l_3=0. \quad (13)$$

Since the velocity v_0 of the 2π-pulse differs from the velocity c of the light in vacuum, its polarization, according to (13), rotates around H. Indeed, at the point $z = v_0t$ of the maximum intensity of the 2π-pulse, the angular velocity of the rotation of the polarization vector l is $\epsilon_2(1 - v_0/c)$. Therefore the angle of rotation of the polarization of the 2π-pulse on the unit path $d\theta/dz$ is

$$d\theta/dz=\varepsilon_2(1-v_0/c)/v_0=-3\sqrt{\pi}N_0\lambda^2\varepsilon_2\gamma T_0^2\int_0^\infty\frac{e^{-\xi^2}}{\xi^2+(T_0/T)^2}d\xi. \quad (14)$$

The direction of the rotation (14) coincides with the direction of rotation of the polarization of the photon echo in a gas in the presence of a magnetic field[6]. In its nature, the rotation of the polarization of the 2π-pulse (14) is a Faraday rotation, although it differs in magnitude and direction from the ordinary Faraday rotation in a monochromatic wave passing through the same resonant beam. For comparison, we write out the formula for the Faraday rotation of a nonstationary short pulse in the linear approximation in the field:

$$d\theta/dz=3\pi N_0\lambda^2\gamma T_0e^{-\eta^2}\int_0^\eta e^{t^2}dt, \quad (15)$$

where $\eta = \epsilon_2T_0$, and where we also use the fact that the collision width of the level is small compared with the Doppler width. We see that the rotation (14) of the polarization of a 2π-pulse is opposite to the Faraday rotation (15). The Faraday rotation in an inert gas was investigated experimentally in[7].

If the frequency of a light pulse (1) is in resonance with the frequency of the atomic transition with change of total angular momentum $J_2 = 0 \to J_1 = 1$, then it is necessary to make in (14) the substitution

$$\varepsilon_2 \to \varepsilon_1, \quad \gamma \to \gamma/3, \quad N_0 \to n_2 - n_1/3.$$

An analogous result is obtained in the analysis of the atomic transition with change of the total angular momentum $J_2 = 1/2 \to J_1 = 1/2$. By virtue of the specific properties of the Pauli matrices σ_α, the operators $T_{\alpha\beta}$ and $Q_{\alpha\beta}$ assume the simple form

$$T_{\alpha\beta} = \sigma_\alpha\sigma_\beta/6, \quad Q_{\alpha\beta} = \sigma_\alpha(\mathbf{n}\sigma)\sigma_\beta/6.$$

If we make in (3)—(6) the substitution

$$j_\alpha = \sigma_\alpha(\mathbf{l}\sigma)j,$$
$$\rho_2^{\alpha\beta} = \rho\sigma_\alpha\sigma_\beta/6, \quad \rho - \rho_1 = N,$$

then we again arrive at Eqs. (8)—(10), with the substitutions $\lambda \to 2\lambda$ and $\gamma \to \gamma/6$. Now, however, the polarization vector $\mathbf{l}$ is defined by expressions (13), in which ϵ_2 is replaced by $(\epsilon_1 + \epsilon_2)/2$, and in formula (14) it is necessary to make the substitutions

$$\lambda \to 2\lambda, \quad \gamma \to \gamma/6,$$
$$N_0 \to (n_2 - n_1)/2, \quad \varepsilon_2 \to (\varepsilon_1 + \varepsilon_2)/2.$$

The rotation angle obtained in this manner for the polarization of the 2π-pulse differs from the corresponding result of[8] in the substitution $g \to (q_1 + q_2)/2$ and by a numerical factor, this being due to the fact that the degeneracy of the levels was neglected in[8] in the consideration of the atomic transition $J_2 = 1/2 \to J_1 = 1/2$ in ruby.

In the case of other atomic transitions, the rotation of the polarization and the change of the profile of the light pulse depends in a very complicated manner on the g-factors and the total angular momenta of the levels, and the problem cannot be solved in simple analytic form. An exception is the atomic transition with change of the total angular momentum $J_2 = 1 \to J_1 = 1$ and identical g-factors of the upper and lower degenerate levels, $g_1 = g_2$. In this case substitution in Eqs. (3)—(6)

$$j_\alpha = \hat{J}_\alpha(\mathbf{l}\hat{\mathbf{J}})j, \quad \rho_1 = q_1(\mathbf{l}\hat{\mathbf{J}})^2,$$
$$\rho_2^{\alpha\beta} = q_2\hat{J}_\alpha(\mathbf{l}\hat{\mathbf{J}})^2\hat{J}^\beta, \quad 6q_2 - q_1 = N,$$

also leads to Eqs. (8)—(10) with the substitution in them $\lambda \to 2\lambda$ and $\gamma \to \gamma/4$. However, formation of a 2π-pulse calls for excessively specialized initial conditions for the density matrices:

$$\rho_{\mu\mu'}(\mathbf{r}, -\infty) = C_1(\mathbf{l}_0\hat{\mathbf{J}})^2_{\mu\mu'}, \quad \rho_{mm'}(\mathbf{r}, -\infty) = C_2(\mathbf{l}_0\hat{\mathbf{J}})^2_{mm'},$$

where J is the operator of the total angular momentum, equal to unity; C_1 and C_2 are certain constants, and $\mathbf{l}_0$ is an arbitrary vector perpendicular to the wave vector $\mathbf{k}$.

An investigation of Eqs. (3)—(6) in the absence of a magnetic field (H = 0) shows that in the case of an inhomogeneously broadened line the 2π-pulse is produced only on atomic transitions with change of the total angular momentum $1 \rightleftarrows 0$, $1 \to 1$, $1/2 \to 1/2$, and $1/2 \rightleftarrows 3/2$. For other atomic transitions, stationary pulses may be produced with several maxima, as indicated in[4]. The external magnetic field noticeably reduces the number of atomic transitions on which formation of 2π-pulses in the absence of a magnetic field is possible.

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43