INVESTIGATION OF THE RADIATION SPECTRUM OF A LASER EMPLOYED AS A DETECTOR OF A DOPPLER-SHIFTED SIGNAL

I. M. BELOUSOVA, O. B. DANILOV, A. F. ZAPRYAGAEV, and N. N. ROZANOV

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The radiation spectrum of a helium-neon laser subjected to the action of a signal reflected from a moving body is investigated. Stimulated generation produced following cessation of the action of the external signal on the laser is observed.

1. MODULATION OF LASER RADIATION BY A REFLECTED SIGNAL WITH FREQUENCY SHIFT

In the experiments[1] on the use of the Doppler effect for the measurement of the velocity of objects, they used a Michelson interferometer scheme. A much better method is to receive the signal directly in the resonator of the laser[2,3] whose beam was used to probe the object. An advantage of the latter method is that the received signal is amplified by the active medium of the laser, and that the receiver is highly compact, thereby greatly simplifying the problem of mechanical stabilization of the receiver parts.

This paper is devoted to an investigation of the emission spectrum of a laser acted upon by a signal reflected from a moving object.

EXPERIMENTAL SETUP

A diagram of the experimental setup is shown in Fig. 1. The radiation of a neon-helium laser (1), generating simultaneously at the wavelengths $\lambda_1 = 0.63$ and $\lambda_2 = 3.39\ \mu$, was aimed on the object (2). The object was either a mirror or a reflected with a scattering surface. The object could be moved on a track over a length $t = 400$ mm at speeds V from 1 to 60 mm/sec. The laser operated in TEM_{00q} modes. The laser resonator was made up of spherical mirrors Z_1, Z_2, and a flat mirror Z_3. The reflection coefficients were 99.5, 98, and 16%, respectively. Mirror Z_3 served as the longitudinal-mode selector. To monitor the number of generated longitudinal modes, part of the laser radiation was deflected by a plate (3) and directed to a Fabry-Perot etalon (5) through a lens (4). The interference pattern was observed with the aid of a collimator (6). The laser radiation passing through the mirror was incident on a photomultiplier (7). The photomultiplier signal was fed to an SKCh-3 spectrum analyzer (8) or to an oscilloscope (9). Thus, we observed the spectrum of the beats of the frequencies present in the laser radiation, as well as the depth of the modulation of the radiation. The depth of modulation was measured using a disc modulator (10).

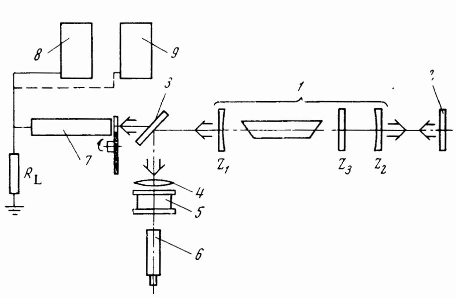


FIG. 1. Diagram of experimental setup.

RESULTS OF EXPERIMENT

Single-frequency Laser Regime

1. The spectrum of the low-frequency beats, obtained when the laser signals at the wavelengths $\lambda_1 = 0.63$ and $\lambda_2 = 3.39\ \mu$ is reflected from a moving mirror, is shown in Fig. 2. As seen from the figure, the beats occurred at two frequencies. Both frequencies depend linearly on the mirror velocity. The beat frequencies were calculated at a known mirror velocity, using the formula $\Delta\nu = 2V\nu/c$, where $\Delta\nu$ is the Doppler shift, ν the laser generation frequency, V the velocity of the object, and c the velocity of light. Calculation has shown that a larger beat frequency corresponds to a wavelength $\lambda_1 = 0.63\ \mu$ and the smaller one to $\lambda_2 = 3.39\ \mu$. The photomultiplier is not sensitive to the 3.39-μ radiation. The appearance of beats at a frequency corresponding to this wavelength can be explained in the following manner. It is known that in neon transitions with wavelengths $\lambda_1 = 0.63$ and $\lambda_2 = 3.39\ \mu$ have a common upper level $3s_2$Ne. When modulation takes place at the wavelength $\lambda_2 = 3.39\ \mu$, the population of

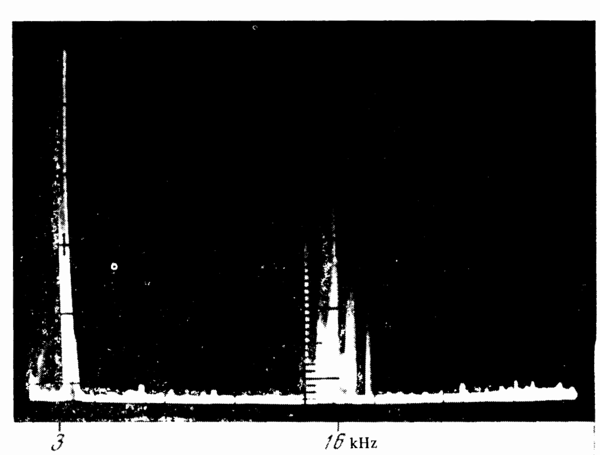


FIG. 2. Beats at the frequencies $\Delta\nu_{3.39} = 3$ kHz and $\Delta\nu_{0.63} = 16$ kHz.

FIG. 3. Noise at a radiation modulation depth 100%.

the level will pulsate at the beat frequency, causing modulation of the 0.63-μ radiation.

2. Under the given experimental conditions, we measured the depth of modulation of the laser radiation. The result is shown in Fig. 3. The radiation with $\lambda = 3.39\,\mu$ was suppressed by a K8 glass plate, installed at the Brewster angle in the laser resonator. As seen from the figure the depth of modulation is close to 100%. This indicates that the intensities of the main signal in the resonator and of the signal with shifted frequency are approximately equal. Knowing the geometry of the setup, the divergence angle of the radiation, and the transmission coefficient of the resonator mirror, it is easy to show that the fraction of the radiation energy returning to the laser is 10^{-4} of the output energy. Thus, a weak signal from the laser resulted in excitation of oscillations at the frequency of this signal. The depth of the radiation modulation in the single-frequency regime is independent of the distance between the moving mirror and the laser resonator.

Multifrequency Regime

3. The general picture of the spectra of the beats produced when a signal reflected from a moving object (mirror) is incident on a laser is shown in Fig. 4. The laser generated on five longitudinal modes. It is seen from the figure that the spectrum contains beats at wavelengths λ_1 and λ_2 not only of the fundamental frequency of the Doppler signal but also of its harmonics.

We investigated also the behavior of the beat spectrum and the depth of modulation as a function of the

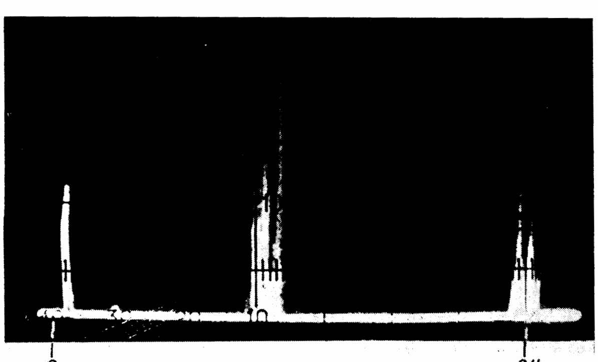


FIG. 4. Beat signals at frequencies $\Delta\nu_{3.39} = 2.2$ kHz, $2\Delta_{3.39} = 4.4$ kHz, $\Delta\nu_{0.62} = 12$ kHz, and $2\Delta\nu_{0.63} = 24$ kHz.

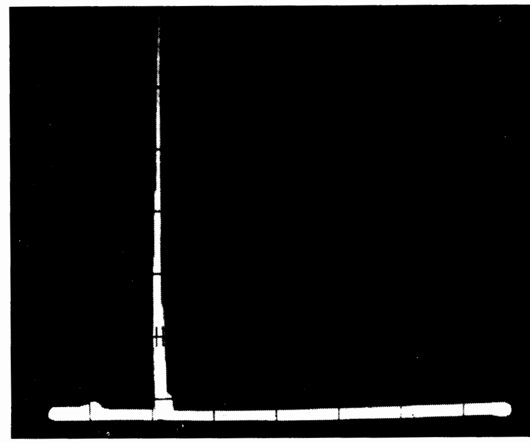


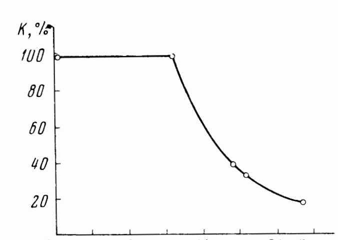
FIG. 5. Beat signal when the distance between the laser mirror and the object is an integer multiple of the resonator length.

distance between the object and the laser mirror. The results are shown in Fig. 5.(the generation on $\lambda_2 = 3.39\,\mu$ is suppressed here). As seen from the figure, when the distance between the laser mirror and the object is an integer multiple of the resonator length, the depth of the laser radiation modulation is close to 100%, and the beat spectrum contains one frequency equal to the fundamental frequency of the Doppler shift. As the object moves away from this point, the depth of modulation decreases, and at the same time higher harmonics appear in the beat spectrum.

4. In the case of multifrequency laser operation, when the distance from the object was an integer multiple of the resonator length, neutral light filters were placed between the laser and the object in order to attenuate, in final analysis, the radiation acting on the laser. To prevent the signal reflected from the neutral filters from entering into the laser, the filters were placed at an angle. At the same time, the depth of modulation was fixed. The result is shown in Fig. 6. The ordinates represent the depth of modulation K, and the abscissas the attenuation ξ in decibels. As seen from the figure, the depth of modulation is close to 100% at an attenuation up to ~13 dB, which is determined by the divergence of the laser beam. With further increase of the attenuation, the depth of modulation decreases gradually, reaching 20% at 27 dB, i.e., at a signal attenuation by approximately 500 times. (We disregard here the attenuation of the signal by the laser mirror, which amounts to ~20 dB.)

5. Under the experimental conditions described above (the distance to the object is an integer multiple of the resonator length), a lens was placed between the laser and the object, making it possible to feed into the

FIG. 6. Depth of modulation K of the laser radiation against the attenuation of the intensity of the back signal ξ.



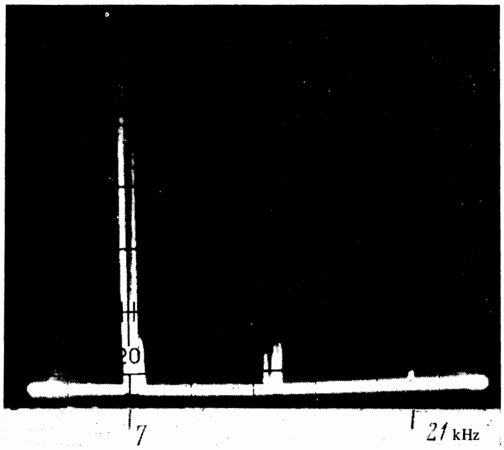


FIG. 7. Signals at the fundamental frequency $\Delta\nu = 7$ kHz and at higher harmonics.

laser an energy approximately five times larger than in the preceding experiments. In this case the depth of modulation, naturally, was close to 100%. The beat spectrum is shown in Fig. 7. As seen from the figure, the beat spectrum now contains harmonics which were previously missing (Fig. 5).

6. An experiment was performed with an object having a scattering surface. The results of this experiment turned out to be similar to those considered above.

DISCUSSION OF EXPERIMENTAL RESULTS

The problem of the modulation of laser radiation by a signal reflected from a moving object reduces to a solution of a partial differential equation (wave equation) with allowance for the boundary conditions on the moving and stationary mirrors and with nonlinear polarization of the medium.

In such a rigorous formulation, the problem is practically insoluble. Nonetheless, certain qualitative results pertaining to the single-frequency laser regime can be obtained by considering a problem that is analogous to some degree, namely that of synchronization of a large number of longitudinal modes in the case of single-band resonance modulation. An analytic solution was obtained for the case of a laser with a homogeneously broadened line. We can use these results, since the fundamental and the Doppler-shifted frequencies certainly lie inside a single dip in the amplification contour in our case.

Let us consider the case when the laser generates in the initial state at a single frequency. We liken the oscillations separated by the Doppler frequency to longitudinal modes. The frequency dependence of the saturated gain of the "modes" is equivalent to the corresponding dependence of the Q of the harmonics. We shall assume that the frequency of one of the "modes" (with index n = 0) coincides with the atomic frequency. Then the distribution of the amplitudes of the forced oscillations is given by the formula

$$E_n = 0 \text{ if } n < 0, \qquad (1)$$

$$E_n/E_0 = (n_{max}^n/n!)^2, \quad n_{max} = \sqrt{\varepsilon}(\gamma/\nu_m). \qquad (2)$$

Here γ is the resonator bandwidth, ν_m is the frequency of radiation modulation (the frequency of the Doppler beats), and ϵ is the fraction of the radiation energy returned to the resonator.

At $n_{max} = 1$ we have $E_0 = E_1 = 4E_2 = 36E_3 = \ldots = (n!)^2E_n$. In this case the amplitudes of the oscillations having the natural generation frequency and those having the Doppler-shifted frequency are equal. The intensities of the remaining force oscillations are relatively small and amount to $\leq 1/16$ of the intensity of the natural oscillations.

Thus, at $n_{max} \approx 1$ in the single-frequency regime, we have approximately 100% depth of modulation. The higher harmonics of the Doppler signal will be small. This conclusion agrees with the experimental data described in Subsections 2 and 4 of the preceding section. It is easy to estimate the value of ϵ in this regime. At $n_{max} \approx 1$ we have $\epsilon = (\nu_m/\gamma)^2$. Putting $\nu_m \approx 10^4$ Hz and $\gamma \approx 2 \times 10^6$ Hz, we get $\epsilon \sim 5 \times 10^{-5}$, which is in satisfactory agreement with experiment.

With further increase of ϵ, additional frequencies appear in the laser radiation spectrum, besides the fundamental frequency and the Doppler-signal frequency. The distances between these frequencies are all equal to ν_m. Thus, harmonics appear in the beat spectrum. This conclusion agrees with the experimental results given in Subsection 5 of the preceding section. It should be recalled here that expression (2) has been obtained neglecting the influence of the combination tones. Therefore the foregoing can be regarded only as a qualitative description of the modulation of the laser radiation by the moving mirror.

According to the data of [6], for our geometry and laser operating conditions, the dips in the spontaneous emission line are overlapped by the wings. However, the spatially longitudinal modes are weakly overlapped. If the distance from the object to the resonator is an integer multiple of the resonator length, the response signals corresponding to the longitudinal modes strike the laser mirror at the same phase, equal to the phase of the longitudinal mode itself. Therefore the fields of the response signals and of the longitudinal modes overlap little. In this case the laser behaves just as in the single-frequency regime.

If the distance to the object is not an integer multiple of the resonator length, the fields of the response signals corresponding to a single longitudinal mode turn out to overlap spatially the fields of other longitudinal modes. The interaction of these fields leads to the occurrence of combination tones.[5] The frequencies of the latter correspond to the frequencies of the signals with double the Doppler shift (i.e., harmonics appear in the beat spectrum).

Thus, to obtain harmonics at relatively smaller response signals it is important that three longitudinal modes be present. The amplitude of the harmonics should decrease strongly in the presence of two longitudinal modes. The latter effect was observed experimentally on going over from multifrequency to two-frequency laser operation.

If the moving mirror is replaced by scattering objects, then the incident radiation will be reflected into the laser with a worse directivity pattern. Since the scattering object acts on the laser radiation like a complicated diffraction grating,[6] the response signal consists of individual maxima ("lobes"). The object, as a rule, is located at a considerable distance from the laser, and only one of such lobes will be incident on the laser. Therefore the difference between the actions exerted on the laser by signals reflected from a mirror and those reflected from a scattering object is purely a matter of energy, as is indeed confirmed by experiment.

2. STIMULATED LASER GENERATION

It was shown above that when a response signal of $\sim 10^{-4}$ of the output power acts on the laser, beats appear in the laser radiation; their frequency is determined by the Doppler law, and the depth of modulation is close to 100%, i.e., the laser is excited at an additional new frequency under the influence of the external field. It is of interest to investigate the duration of the generation of the induced oscillations in the absence of a driving signal.

Results of Experiment

The experiment was performed in the following manner (Fig. 1). Mirror 2 was moved and the Doppler beat signal was observed. The depth of modulation of this beat signal was close to 100%. Mirror 2 was then covered with a screen. It was observed that under certain conditions the beat signal did not vanish. Thus, the laser excited by the external signal continues to generate at the frequency of this signal, i.e., it "remembers" the frequency of the excitation signal. The results are shown in Fig. 8a. When the velocity of mirror 2 change changes (when the excitation-signal frequency changes), the frequency of the "remembered" signal coincides with the frequency of the exciting signal (Fig. 8b).

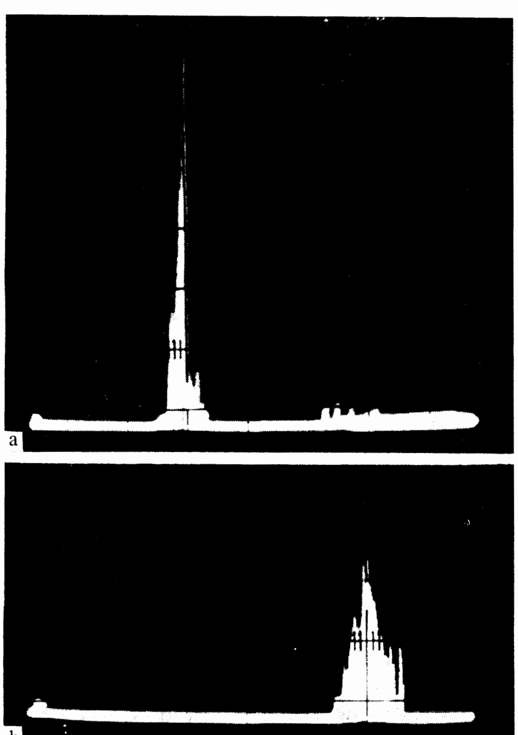


FIG. 8. Spectrograms of laser-emission modulation signals with the external reflector covered: a – Doppler beat frequency $\Delta\nu = 9$ kHz; b – Doppler beat frequency $\Delta\nu = 20$ kHz.

It turned out that the "memory" effect depends strongly on the exact longitudinal adjustment of the resonator. To register the effect described here, one of the resonator mirrors was mounted on a piezoceramic plunger, with the aid of which the resonator length is changed by an amount smaller than the wavelength. At certain mirror positions, we observed the stimulated generation of the laser (the "memory" effect).

It was observed in addition that the "memory" effect is directly connected with the presence in the laser harmonics of a beat frequency corresponding to the Doppler shift. In particular, the "memory" effect appears when the distance to the object mirror is not an integer multiple of the resonator length. In this case, as shown above, harmonics appear in the beat spectrum.

The "memory" effect was not observed in the absence of harmonics in the beat spectrum. The harmonics were eliminated by placing the object mirror at a distance that was an integer multiple of the resonator length (see above).

Discussion of Experimental Results

The most important question arising in the analysis of the phenomena described above is that of the stability of the "memory" regime. The distance between the fundamental and "remembered" frequencies is in this case ~ 10 kHz, and according to [7] lies near the capture region. It might seem therefore that the "memory" regime should terminate at the single-frequency regime. However, taking into account the experimental fact that the "memory" regime is observed only when harmonics are present in the beat spectrum, we can present the following interpretation of the stability of the "memory" phenomenon.

Assume that when the signal reflected from the moving mirror enters into the laser the laser emission spectrum consists of two frequencies: the natural frequency ν_0, the Doppler-shifted frequency ν_1, and the doubly-Doppler-shifted frequency ν_2 (i.e., the beat spectrum will contain the Doppler frequency ν_D and its harmonics $2\nu_D$). The fields of these three frequencies (ν_0, ν_1, and ν_2) act together on the laser active medium. As will be shown below, combination terms with frequencies $(2\nu_1 - \nu_0)$ and $(2\nu_1 - \nu_2)$ will appear in the expression for the polarization of the medium. The former frequency is equal to ν_2, and the latter to ν_0. Thus, each two neighboring frequencies stimulate the generation of a third. This may be the reason for the stability. We shall consider below the theory of this question and determine the region of stability of such a regime.

We assume that in the presence of an additional moving object mirror, a number of frequencies are excited within the resonator linewidth corresponding to each longitudinal mode. These frequencies differ by an amount equal to the Doppler shift. A rigorous determination of their amplitudes (and phases) should be based on the solution of a wave equation in the form

$$\left(-c^2\frac{\partial^2}{\partial z^2}+\frac{\nu}{Q}\frac{\partial}{\partial t}+\frac{\partial^2}{\partial t^2}\right)E^{(+)}=-\frac{1}{\varepsilon_0}\frac{\partial^2}{\partial t^2}p^{(+)}. \qquad (3)$$

Here $E^{(+)}$ and $p^{(+)}$ are the complex field and polarization of the medium, c is the velocity of light, ν is the generation frequency, Q is the quality factor describing the distributed losses in the resonator, and ϵ_0 is the dielectric constant.

Equation (3) must be supplemented by the boundary conditions on the mirrors and by the dependence of p on E. In the case of large mirror reflection coefficients we can attempt to replace the mirror losses by distributed losses[8] and seek a solution of Eq. (3) in the form of a sum over the modes and the harmonics, whose spatial distribution is close to the distribution of the field in a standing wave:

$$u_{n,m} = \sin(K_{n,m} z), \tag{4}$$

where n is the mode index, m the harmonic index, and $K_{n,m}$ is the wave vector, which is connected with the frequency by the relation $K_{n,m} = \nu_{n,m}/c$ ($\nu_{n,m} = \nu_n + m\nu_D$ are the frequencies in the absence of the active medium). The variation of the field amplitudes along the longitudinal coordinate z is neglected.

The field $E^{(+)}$ can now be written in the form

$$E^{(+)} = \frac{1}{2} \sum_{n,m} E_{n,m} \exp\left[-i(\nu_{n,m} t + \varphi_{n,m})\right] \sin K_{n,m} z. \tag{5}$$

In accordance with expression (5), the projection of the polarization of the medium on the n-th mode is represented in the form

$$p_n^{(+)} = \frac{1}{2} \sum_m p_{n,m} \exp[-i(\nu_{n,m} t + \varphi_{n,m})]. \tag{6}$$

Substituting (6) and (5) in (3), we obtain the following equations:

$$(\nu_{n,m} + \dot{\varphi}_{n,m} - cK_{n,m})E_{n,m} = -{}^1/{}_2(\nu/\varepsilon_0)\,\mathrm{Re}\, p_{n,m} + \varepsilon F_1, \tag{7}$$

$$\dot{E}_{n,m} + {}^1/{}_2(\nu/Q_{n,m})E_{n,m} = -{}^1/{}_2(\nu/\varepsilon_0)\,\mathrm{Im}\, p_{n,m} + \varepsilon F_2. \tag{8}$$

The terms $\epsilon F_{1,2}$, which have been introduced phenomenologically, describe the action of the radiation reflected from the object mirror:

$$\varepsilon F_1 = -\frac{1}{2}\varepsilon\frac{\nu}{Q} E_{n,m-1} \sin(\varphi_{n,m} - \varphi_{n,m-1} - \theta), \tag{9}$$

$$\varepsilon F_2 = \frac{1}{2}\varepsilon\frac{\nu}{Q} E_{n,m-1} \cos(\varphi_{n,m} - \varphi_{n,m-1} - \theta), \tag{10}$$

where θ is a certain constant phase and ϵ is the fraction of the radiation returned to the laser resonator ($\epsilon \ll 1$).

Equations (7) and (8) are a generalization of the ordinary Lamb equations.[8] They can be justified in the following manner. Let us consider the steady-state field produced by an incident wave with amplitude $E_{n,m-1}$ and frequency $\nu_{n,m}$ in an empty resonator (i.e., $p_{n,m} = 0$). The resonator quality factor is Q. Then Eq. (7) will be satisfied at $\varphi_{n,m} - \varphi_{n,n-1} - \theta = 0$, and (8) yields

$$E_{n,m} = \varepsilon(Q_{n,m}/Q)E_{n,m-1}.$$

From this we find that the introduced "Q of the harmonic" $Q_{n,m}$ should describe the decrease of the amplitude of the steady-state field with increasing difference between the field frequency and the resonator frequency. The phase shift is of no interest to us, since we confine ourselves to the case $\epsilon = 0$ (there is no moving mirror).

In our experiments, the modulation frequency (the Doppler shift) is much smaller than the laser transition linewidth γ. Therefore, the amplitude and phase of the total field of the frequencies excited in the laser change extremely little during the lifetime of the atom. We can thus use Lamb's[8] for the polarization by representing the field of each mode in the form of a sum of the harmonic fields. The difference between the wave vectors $K_{n,m}$ for different m (harmonics) has no effect on the linear polarization of the medium.

Single-mode Regime

We consider the regime in which one longitudinal mode is generated, with a frequency that coincides with the atomic frequency of the working transition. The terms describing the pulling and repulsion can be neglected.

We confine ourselves to the case when the significant intensities have only three frequencies separated by the Doppler shift, with indices m = 0 and ±1. We omit the index of the longitudinal mode. We introduce the notation

$$\Psi = 2\varphi_0 - \varphi_1 - \varphi_{-1}, \tag{11}$$

$$\alpha_m = \alpha_y - \frac{1}{2}\frac{\nu}{Q_m}, \tag{12}$$

α_y is the unsaturated gain, and β is the saturation parameter. Using (8) and (9), we get

$$\begin{gathered}
(\nu_0 + \dot{\varphi}_0 - \Omega_0)E = 2\beta E_0 E_1 E_{-1} \sin\Psi, \\
\dot{E}_0 = \alpha_0 E_0 - \beta E_0^3 - 2\beta E_0 E_{-1}^2 - 2\beta E_0 E_1^2 - 2\beta E_0 E_1 E_{-1}\cos\Psi, \\
(\nu_1 + \dot{\varphi}_1 - \Omega_1)E_1 = -\beta E_0^2 E_{-1}\sin\Psi, \\
\dot{E}_1 = \alpha_1 E_1 - \beta E_1^3 - 2\beta E_1 E_0^2 - 2\beta E_1 E_{-1}^2 - \beta E_0^2 E_{-1}\cos\Psi, \\
(\nu_{-1} + \dot{\varphi}_{-1} - \Omega_{-1})E_{-1} = -\beta E_0^2 E_1 \sin\Psi, \\
\dot{E}_{-1} = \alpha_{-1}E_{-1} - \beta E_{-1}^3 - 2\beta E_{-1}E_0^2 - 2\beta E_{-1}E_1^2 - \beta E_0^2 E_1\cos\Psi.
\end{gathered} \tag{13}$$

In the stationary regime we obtain from the frequency equations

$$\nu_i = \Omega_i, \ \Psi = \pi. \tag{14}$$

It is easy to see that the other solution ($\Psi = 0$) is unstable. Substituting the value $\Psi = \pi$ in the amplitude equations (13), we find that the last (combination) term in them leads to amplification of the corresponding frequency.

We now proceed to find the stationary solutions of the amplitude equations and to investigate their stability. First, arbitrarily specifying the stationary values of the amplitudes, we find the region of stable and unstable solutions. It is then already easy to change over from the stationary values to the parameters α_m/β and determine by the same token the parameters at which stable generation regimes of three harmonics are possible.

The stability is investigated in the usual manner (Lyapunov's first method). The result is shown in Fig. 9. The vertically-shaded region corresponds to values $k_1 = E_1/E_0$ and $k_2 = E_{-1}/E_0$, at which the stationary solution E_i of the system (13) is stable when (14) is taken into account.

It is now necessary to connect the stationary values of the intensities with the parameters of the laser and to determine the region of physically real parameters. From (13) with allowance for (14) we obtain

$$\alpha_0 / \beta E_0^2 = 1 + 2k_1^2 + 2k_2^2 - 2k_1k_2,$$
$$\alpha_1 / \beta E_0^2 = 2 + k_1^2 + 2k_2^2 - k_2 / k_1, \qquad (15)$$
$$\alpha_{-1} / \beta E_0^2 = 2 + k_2^2 + 2k_1^2 - k_1 / k_2.$$

From the first equation of (15) we see that $\alpha_0 / \beta E_0^2 \geq 1$. With the aid of (15), we can determine α_0, α_1, and α_{-1} for any pair of values k_1 and k_2 (which, naturally, should be chosen from the stability region) and for a specified intensity of the middle harmonic. It is also easy to solve the inverse problem, namely to find E_0^2, k_1, and k_2 from the known values of α_0, α_1, and α_{-1}.

According to (12), α_i is defined as the difference of the unsaturated gain and the loss coefficient. It is natural to assume that the linear gain does not depend on the number of the frequencies under consideration, and that α_i is determined only by the resonator circuit. We then arrive at the requirement

$$2\alpha_0 > \alpha_1 + \alpha_{-1}. \qquad (16)$$

The condition (16) separates two physically real regions of values of k_1 and k_2 in the stability region obtained above. These regions are shaded horizontally in Fig. 9.

We now consider the question of the influence of the exact longitudinal setting of the laser on the "memory" effect in the case of three frequencies. Let Δ_m be the difference between the m-th and atomic frequencies: $\Delta_m = \Delta_0 + m\omega_m$ (ω_m is the beat frequency). Since $\Delta_m \ll \nu/Q$, we can confine ourselves to a quadratic dependence of α_m on Δ_m, namely $\alpha_m = c_1 - c_2\Delta_m^2$, where c_1 and c_2 are certain constants. At $\Delta_0 = 0$ we have $k_1 = k_2$, corresponding in Fig. 9 to unphysical values of the gains at the frequencies under consideration. Thus, stable regimes with noticeable relative frequency amplitudes k_1 and k_2 are possible only at an appreciable value of the ratio Δ_0/ω_m, i.e., at a relatively large shift of the central frequency. On the other hand, however, the value of this shift should not exceed the value at which an additional intense frequency, determined by the Doppler shift, appears near the atomic frequency.

We see from the foregoing that longitudinal adjustment of the laser resonator has a very strong effect on the appearance of the "memory" regime, and determines this regime in practice, as is indeed observed in the experiments.

In addition to the investigated three-frequency regime, the system (13) admits of stable single-frequency regimes. It is easy to show that it is possible to find at least one single-frequency regime when the values of the the parameters α_0, α_1, and α_{-1} correspond to any pair of values k_1 and k_2 from the stability region of the three-frequency regime. Thus, when $k_1 > k_2$, the stable regime can be $E_0 = E_{-1} = 0$, $E_1^2 = \alpha_1/\beta$; when $k < k_2$, the stable regime can be $E_0 = E_1 = 0$, $E_{-1}^2 = \alpha_{-1}/\beta$.

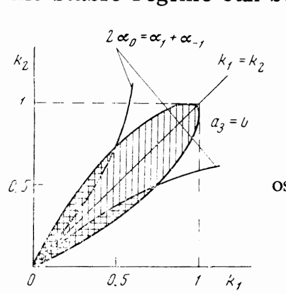


FIG. 9. Stability diagram of stimulated oscillations in a laser.

The regime that actually sets in (single-frequency or three-frequency) depends on the initial conditions (strong coupling). If the distribution of the amplitudes and their deviation from the atomic frequency are close to the parameters of stability of the three-frequency regime at the instant when the driving force is turned off, then this regime sets in. If one of the parameters deviates, an unstable regime can arise, which in turn can lead to "capture," i.e., to generation at a single frequency.

Multimode Regime

An analysis of the possible generation regimes that occur after the signal reflected from the moving object ceases to act on the laser is greatly complicated if several longitudinal modes are present. We shall therefore give here a rough qualitative picture of what occurs when many longitudinal modes are taken into account.

We assume, as before, that the frequency of the mode separated by us is close to the atomic frequency. Considering the fields of the remaining modes as specified, we arrive at the conclusion that the remaining modes exert a twofold action on the central mode.

1) Owing to the overlap of the dips of the longitudinal modes with the lines of the spontaneous emission, saturation causes the gain at the central modes to decrease. This decrease, however, is the same for all the Doppler-shifted frequencies. This effect is therefore irrelevant in our case.

2) If the distance to the object is not an integer multiple of the resonator length, then the overlap of the field of the response signal of one mode with the field of another mode (see above) gives rise to combination terms, whose frequencies coincide with the frequencies that are located, say, near the central mode at distances determined by the Doppler shift. Thus, the interaction of the latter increases even more.

This factor can be taken into account in the following simple manner, by supplementing the first parts of the amplitude equations with small terms in the form $\kappa_{ik}E_k$. It is easy to write out the explicit form of κ. We assume that none of the κ_{ik} are negative. This is equivalent to assuming that in the investigated regime, in accordance with the results of the preceding section, the parametric interactions should correspond to mutual amplification (and not attenuation) of the frequencies. As already mentioned, the additional terms are assumed to be small. Therefore the change of the value of Ψ in the amplitude equations would lead to the appearance of terms of second order of smallness. In addition, the smallness of the additional terms leads to a small change in the boundaries of the stability region (Fig. 9). Allowance for these terms is important only in the determination of the stationary solutions of the equations (i.e., in the determination of the physically real parameters).

For the system of equations altered in the indicated manner, we can easily repeat all the operations performed on the system (13). Without presenting these results, we indicate that, at reasonable values of κ_{ik}, stable generation regimes at the Doppler frequencies exist in a much wider range of parameters α_i than in the case of the system (13). In particular, regimes with a symmetrical distribution of the amplitudes $k_1 = k_2$ become possible.

Thus, in the multifrequency laser operation regime, the "memory" regime is easier to produce than in the single-frequency case.

CONCLUSION

We have investigated experimentally the emission spectrum of a laser acted upon by a signal reflected from a moving mirror or a moving scattering object. We have shown that the laser emission becomes modulated, with a depth close to 100%, at a significantly low intensity of the response signal.

Harmonics were observed in the spectrum of the low-frequency beats. It is assumed that they are due to two causes: 1) multiple passage of the signal between the laser and the moving object; 2) in the case of the multifrequency laser regime, it is due to nonlinear interaction between the field of the response signal of one mode with the field of another mode.

The stimulated laser generation regime that sets in after the cessation of the signal reflected from the moving object (the "memory" effect) was obtained experimentally. It is shown that the existence of the "memory" effect is connected with the presence of harmonics of the Doppler signal in the beat spectrum of the laser radiation. The region of stability of such a regime was determined theoretically for the case of three frequencies belonging to one longitudinal mode. The influence of many longitudinal modes is discussed.

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47