# WEAK TURBULENCE SPECTRUM AND SECOND SOUND IN A PLASMA

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Langmuir-turbulence spectra and the possibility of propagation of low-frequency oscillations of the second-sound type in a turbulent plasma are considered. If collisions between plasmons and ions are not taken into account, there should be two stationary nonequilibrium distributions that correspond to stationary fluxes of the kinetic energy and of the number of plasma waves. Two branches of second sound should correspondingly exist. An exact power-law solution of the form $k^{-7}$ is found for that region in phase space in which, along with four-plasmon interaction, nonlinear scattering of plasmons by ions is also important. It is proved that this distribution is stable with respect to low frequency disturbances. The stability is of a diffusion nature.

## 1. INTRODUCTION

THERE have been many recent investigations of the weak plasma turbulence due to nonlinear interaction of the waves resulting from various fragments of decay, coalescence, and scattering of the waves by one another. A characteristic feature of a plasma turbulence of this type is that it proceeds without direct energy exchange between the waves and the plasma particles. Such a turbulence is analogous in many respects to hydrodynamic turbulence. Zakharov[1] has shown that in an isothermal plasma without a magnetic field there is a region in the wave-number space, in which the distribution function (the turbulence spectrum) of the Langmuir plasmons has a universal power-law character. The existence of a stationary distribution of the plasma waves is due to the energy and conservation laws that hold when they are scattered or decay. This spectrum corresponds to a stationary flux of the plasmon energy from the generation region towards the region of larger wave numbers, where dissipation of the plasma-oscillation energy takes place. The character of the turbulent spectrum of the plasmons is analogous to the well known Kolmogorov-Obukhov law for turbulent motion of an incompressible liquid. Unlike hydrodynamic turbulence, in the theory of weak plasma turbulence it is possible to obtain a kinetic equation for the wave distribution function. The stationary distribution causes the integral of collisions between plasmons to vanish.

The presence of a region of universal equilibrium in phase space of the waves should lead to the existence of low-frequency oscillations of the type of second sound in helium[2,3] or solids[4]. In this respect, plasma oscillations are similar to phonons in condensed media. The mutual scattering of the plasma waves plays the role of normal collisions in which their total energy and total momentum do not change. It therefore becomes possible to describe hydrodynamically the vibrational motion of plasmons in the frequency region below the characteristic frequency of nonlinear interaction. The second-sound spectrum is linear, and the velocity is determined by the turbulence spectrum and by the magnitude of the phase volume in which a stationary wave distribution exists. The damping of these low-frequency oscillations is determined both by the interaction of the plasmons with one another, and by their scattering by electrons or ions, as a result of which a change takes place in the average energy, momentum, or number of plasma waves. Similar problems were discussed by Vedenov and Rudakov[5]. Second sound was considered within the framework of the quasilinear theory by Liperovskiĭ and Tsytovich[6] and also by Ishimaru[7] for ion-acoustic plasma turbulence.

The stationary distribution of plasma waves can exist also in the case when it is necessary to take into account the nonlinear scattering of the plasmons by the plasma particles, particularly by ions. The turbulence spectrum is established as the result of competition between the four-plasmon interaction and the scattering by ions. The question of the stationary turbulence spectra under these conditions was considered in[8] and in[9].

In this paper we investigate the weak-turbulence spectrum both in the case of four-plasmon interaction and with allowance for scattering by ions. An investigation of the stationary spectra for stability against low-frequency perturbations shows that the initial perturbations attenuate in time. The stability has a vibrational character (of the second-sound type) for the four-plasmon interaction, and is aperiodic when account is taken of plasmon scattering by ions. The absence of oscillations of the second-sound type in the latter case is the consequence of non-conservation of the energy and momentum of the plasmons in collisions with ions.

## 2. LANGMUIR SPECTRUM

We consider an isotropic weakly-turbulent plasma without a magnetic field. We assume first that the ions are infinitely heavy. We shall subsequently indicate the limitations imposed by allowance for the thermal motion of the ions. In this case the principal process of nonlinear interaction of the waves is the scattering of two plasmons by each other. Obviously, three-plasmon processes are forbidden by the energy conservation law. We write down the kinetic equation for the distribution function of the plasma oscillations $N_k(r, t)$ with momentum $\mathbf{k}$:

$$\frac{\partial N_{\mathbf{k}}}{\partial t}+\frac{\partial \omega_{\mathbf{k}}}{\partial \mathbf{k}}\frac{\partial N_{\mathbf{k}}}{\partial \mathbf{r}}+\nu_e N_{\mathbf{k}}+\gamma_{\mathbf{k}} N_{\mathbf{k}}+I_4\{N_{\mathbf{k}}\}=0. \quad (1)$$

Here

$$\omega_k = \omega_0[1 + {}^3/_2(kr_d)^2] \quad (2)$$

is the plasmon frequency, $\omega_0 = (4\pi e^2 n/m)^{1/2}$ is the plasma frequency, $r_d = (T_e/4\pi e^2 n)^{1/2}$ is the Debye radius, $T_e$ is the electron temperature, n is the electron density,

$$\nu_e \approx \omega_0/N_d \quad (3)$$

is the frequency of the Coulomb collisions of the electrons, $N_d \approx nr_d^3$ is the number of electrons in the Debye sphere. The quantity $\gamma_k$ is the nonlinear damping decrement of the plasmons on the electrons. According to[10], the order of magnitude of the decrement $\gamma_k$ is

$$\gamma_k \approx \frac{3\omega_0}{32\pi}(kr_d)^3\frac{\mathscr{E}}{nT_e}, \quad (4)$$

where

$$\mathscr{E} \approx \omega_0 \int N_k \, d^3k \quad (5)$$

is the energy density of the plasma waves. As is customary in the theory of weak turbulence, the ratio $\mathscr{E}/nT_e$ is assumed to be small compared with unity. It is then possible to neglect the change of the natural frequency of the plasmon as a result of nonlinear interaction of the waves, compared with the kinetic energy

$$\Omega_k = {}^3/_2\omega_0(kr_d)^2. \quad (6)$$

Finally, $I_4\{N_k\}$ is the collision integral due to the four-plasmon interaction[1],

$$I_4\{N_k\} = \int |V_{kk_1k_2k_3}|^2 \delta(k + k_1 - k_2 - k_3)\,\delta(\omega_k + \omega_{k_1} - \omega_{k_2} - \omega_{k_3}) \cdot \{N_k N_{k_2} N_{k_3} + N_{k_1} N_{k_2} N_{k_3} - N_k N_{k_1} N_{k_2} - N_k N_{k_1} N_{k_3}\}\, d^3k_1\, d^3k_2\, d^3k_3. \quad (7)$$

The matrix element of the four-plasmon interaction $|V_{kk_1k_2k_3}|^2$ is a fourth-order function of its variables, and in accordance with[11]

$$|V|^2 \sim k^4/m^2n^2 \quad (8)$$

(m is the electron mass). The explicit expression for the matrix element (8) is quite cumbersome and can be found in[1]. We note only that the function $V_{kk_1k_2k_3}$ is symmetrical relative to permutations within the first and second pairs of indices, and also with respect to mutual permutations of pairs of indices.

The four-plasmon collision integral can be estimated in the following manner:

$$I_4\{N_k\} \sim \gamma_4 N_k, \quad (9)$$

where

$$\gamma_4 \sim \omega_0(kr_d)^2(\mathscr{E}/nT_e)^2. \quad (10)$$

The collision integral (9) predominates when the conditions $\gamma_4 \gg \gamma_k$, $\nu_e$ are satisfied, i.e., in the wave-number interval

$$N_d^{-1/2}\, nT_e/\mathscr{E} < kr_d < \mathscr{E}/nT_e. \quad (11)$$

The left-hand inequality corresponds to the condition $\gamma_4 > \nu_e$, and the right one to $\gamma_4 > \gamma_k$. In order for the region (11) to be sufficiently broad, it is necessary to have

$$(\mathscr{E}/nT_e)^2 \gg N_d^{-1/2}. \quad (12)$$

This condition is more stringent than the inequality $\mathscr{E}/nT_e \gg 1/N_d$ in the quasilinear theory[11].

The integral for the collisions between waves satisfies three conservation laws: of the total number of plasmons $N = \int N_k d^3k$, of the total momentum $P = \int kN_k d^3k$, and of the total kinetic energy $T = \int \Omega_k N_k d^3k$. The conservation laws together with the symmetry properties and the homogeneity of the matrix element of the interaction define uniquely the stationary spectrum of the turbulence. The stationary distribution function $N_k$ must be obtained from the condition that the collision integral vanish

$$I_4(N_k) = 0. \quad (13)$$

Obviously, this distribution can exist only within a region (11) in which four-plasmon interaction predominates.

Following[1], let us consider the temporal evolution of a narrow wave packet of plasmons, with wave numbers $k \sim k_0$, situated inside an interval (11). The wave-number region $k \sim k_0$ will be called the energy-containing region. As will be shown below, it is precisely in this region that the kinetic energy of the plasmons has a maximum. Owing to the scattering of the waves by one another, plasmon fluxes are produced from the energy-containing region in the direction of larger and smaller values of k. The flux to the short-wave region of the spectrum is accompanied by an increase of the plasmon energy. Owing to the conservation of the kinetic energy, the packet as a whole diffuses in the region of small k in such a way that the total kinetic energy is conserved. On the short-wave boundary of the interval (11), when

$$k \approx k_s = \mathscr{E}/nT_e r_d, \quad (14)$$

energy dissipation takes place, and at small values

$$k \approx k_1 = nT_e/\mathscr{E} N_d^{*} \, r_d \quad (15)$$

there is plasmon absorption.

It must be emphasized that the energy absorption in the region (14) proceeds with conservation of the total number of plasmons, since the plasmon-electron collisions conserve the total number of plasma waves[10]: $\int \gamma_k N_k d^3k = 0$. Therefore in the short-wave region of the spectrum (between $k_0$ and $k_s$) there is established a stationary plasmon distribution corresponding to a flux of kinetic energy independent of k. Such a distribution was obtained by Zakharov[1] and is given by

$$j_N = \int d^3k I_4\{N_k\} \sim N_{k_0}{}^3 k_0{}^{11}. \quad (16)$$

This distribution is an exact solution of Eq. (13).

On the other hand, in the region between $k_1$ and $k_0$ there should be established a stationary flux of the number of plasmons. In order to determine in this region the dependence of $N_k$ on k, corresponding to the stationary plasmon flux, it suffices to express the change of the plasmon number in terms of the plasmon distribution function and wave number. In the region $k \sim k_0$ we have

$$N_k \sim k^{-11/3}.$$

The flux of plasmons $j_N$ does not depend on k if

$$N_k \sim k^{-13/3}. \quad (17)$$

The arbitrary constants in (16) and (17) can be obtained from the condition for the "joining together" of the distribution function $N_k$ at $k \approx k_0$ with (16) and (17).

It is easy to verify that this distribution is also one of the exact solutions of Eq. (13). In fact, this equation was transformed in[1] into (see Eq. (22) of[1])

$$\int_0^{\omega} d\omega' \int_{\omega-\omega'}^{\omega} d\omega'' \frac{W_{\omega,\omega'+\omega''-\omega,\omega',\omega''}}{\omega'^s\omega''^s(\omega'+\omega''-\omega)^s}\left[1+\left(\frac{\omega}{\omega'+\omega''-\omega}\right)^s-\left(\frac{\omega}{\omega'}\right)^s-\left(\frac{\omega}{\omega''}\right)^s\right]\left[1+\left(\frac{\omega}{\omega'+\omega''-\omega}\right)^{11/2+3s}-\left(\frac{\omega}{\omega'}\right)^{11/2+3s}-\left(\frac{\omega}{\omega''}\right)^{11/2+3s}\right]=0. \quad (18)$$

The solution is sought in the form of a power-law function $N_k \sim k^{2s}$, the variables chosen to be $\omega = k^2$, and $W_{\omega\omega_1\omega_2\omega_3}$ is a homogeneous function of its variables of the order of 5/2, with the same symmetry properties as $|V_{kk_1k_2k_3}|$.

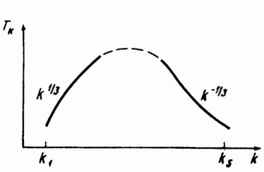
It is obvious that this equation is the solution not only when $11/2 + 3s = -1$, but also when $11/2 + 3s = 0$, i.e.,[1)]

$$2s = -{}^{11}/_3. \quad (19)$$

Another pair of power-law solutions with $s = -1$ (the Rayleigh-Jeans distribution) and $s = 0$ does not agree with the stationary conditions for any physical quantity in the wave-number region under consideration.

The structure of the four-plasmon collision integral is such that the turbulence is local and isotropic. A proof of this statement, as well as the convergence of the integrals in Eq. (13) when the obtained solution is substituted in it, can be obtained by the same method as in[1].

We thus arrive at the following picture of the turbulence of Langmuir plasmons. The waves of the initial packet, localized at the initial instant in the region $k \sim k_0$, are spread over the phase volume, both in the direction of $k > k_0$ and in the region $k < k_0$. A stationary flux of kinetic energy towards larger k sets in ($N_k \sim k^{-13/3}$), as well as a stationary flux of waves in the long-wave part of the spectrum ($N_k \sim k^{-11/3}$). Both regions of universal equilibrium exist only when the condition (12) is satisfied.




The figure shows schematically the spectral distribution of the density of the kinetic energy of the plasmons $T_k = (k^2/2\pi^2)\Omega_k N_k$ in the interval between k and k + dk. In the region $k < k_0$, where the spectrum (17) is realized, the dependence of $T_k$ on k obeys the asymptotic law

$$T_k \sim k^{1/3}, \quad (20)$$

and in the region $k_0 < k < k_s$,

$$T_k \sim k^{-1/3}. \quad (21)$$

If there is no generation of plasma waves when $t > 0$, then the distribution $N_k$ is non-stationary. The characteristic time of this non-stationarity is of the order of the time of nonlinear interaction

$$\tau \sim \frac{1}{\gamma_4} \sim \left[\omega_0 (k_0 r_d)^2 \left(\frac{\mathscr{E}}{nT}\right)^2\right]^{-1}.$$

The dissipation of the kinetic energy occurs in the region of wave numbers $k \sim k_s$, and wave absorption takes place at $k \sim k_1$. With decreasing energy and plasma-wave number in the energy-containing region, the quantity $k_1$ increases and $k_s$ decreases, i.e., a narrowing of the phase-space region in which the distributions (16) and (17) exist takes place. A quantitative consideration of the kinetics of such a relaxation is a complicated problem and calls for the solution of the nonlinear integral equation (1) in the non-stationary case. This is the situation when there is no source of plasma waves at $k \sim k_0$. The turbulence spectra (16) and (17) with stationary fluxes of the kinetic energy and of the wave number are established only in the presence of a source of plasma oscillations.

Let us discuss now the question of the influence of the thermal motion of the ions. In a non-isothermal plasma, the situation is complicated by the existence of ion sound and decays of Langmuir plasmons with participation of ion-acoustic oscillations. We shall therefore consider an isothermal plasma. For simplicity we confine ourselves to the wave-number region

$$(kr_d)^2 \gg m / 9M, \quad (22)$$

in which plasmon scattering is almost elastic, i.e., the relative change of the plasmon energy in collision with the ion is small:

$$\frac{|\Omega_k - \Omega_{k'}|}{\Omega_k} \approx \frac{|k-k'|}{k} \approx \left(\frac{m}{9M}\right)^{1/2} \frac{1}{kr_d} \ll 1.$$

Inasmuch as the process of energy transfer in collisions is slow, the corresponding term in the kinetic equation (1) describes nonlinear diffusion of plasmons in k-space. According to[8], this additional term is of the form

$$\hat{\nu}_i\{N_k\} = \frac{1}{2^5 3^3 \pi} \frac{m}{MnT_e} \left(\frac{\omega_0}{r_d}\right)^2 N_k \frac{\partial}{\partial k}(k^2 N_k). \quad (23)$$

In order of magnitude, the collision frequency of the plasmons with ions is

$$\nu_i = \frac{1}{N_k}\hat{\nu}_i\{N_k\} \sim 3\cdot 10^{-4} \frac{m}{M} \frac{\omega_0^2 N_k k}{nT_e r_d^2} \sim 10^{-4} \frac{m}{M} \frac{\mathscr{E}}{nT_e k^2 r_d^2}. \quad (24)$$

The results obtained above remain valid also when account is taken of the thermal motion of the ions under the condition that $\nu_e \gtrsim \nu_i$, i.e., $(kr_d)^2 \geq 10^{-4}(m/M)(\mathscr{E}/nT_e)N_d$. If this inequality is violated, then the lower limit of the region of existence of stationary spectra (16) and (17) will be determined not by the paired collisions of the electrons, but by the induced scattering from the ions. The total damping on the electrons and ions has a minimum at

$$kr_d \approx (m / 300\, M)^{1/5}. \quad (25)$$

Therefore the stationary distributions (16) and (17) can be realized in a relatively narrow region, where $(m/300M)^{1/5} < \mathscr{E}/nT_e < 1$ near the minimum of the total damping. Thus, allowance for the thermal motion of the ions leads to a decrease of the phase-space region in which the stationary spectra (16) and (17) are established.

It is interesting to note that a similar Langmuir turbulence can be realized in principle also in an electron solid-state plasma, for example in semiconductors. Intense plasma oscillations can be produced by electron beams or by optical pumping. Under these conditions,

[1)]V. E. Zakharov has advised us that he obtained a similar turbulence spectrum for waves on the surface of a liquid [12].

the limitations due to stimulated scattering of electrons by ions do not play an important role, since the effective mass of the ion $M_{eff} = \rho/n = MN_i/n$ ($N_i \gg n$) is larger by many orders of magnitude than the true mass (here $\rho$ is the mass density of the crystal, $N_i$ is the atom concentration). Then the lower limit of the turbulence spectra $k_1$ is determined by the collisions of the electrons with the impurities, phonons, and other scatterers.

A stationary distribution of plasmons exists also in that region of k space where besides four-plasmon interaction it is necessary to take into account also scattering by ions (23). In this case Eq. (1) for the stationary distribution

$$\nu_i\{N_k\} + \gamma_4\{N_k\} = 0 \tag{26}$$

is an expression of the condition that the change of the plasmon density $N_k$ due to their mutual scattering is offset by the stimulated scattering by the ions. It is easy to obtain an exact power-law solution for this equation, by using the properties of homogeneity and symmetry of the four-plasmon interaction operator. We put $N_k = Ck^{-\nu}$. We then get from (26)

$$\text{const}\cdot C^2k^{1-2\nu} = C^3k^{-3\nu+8}, \tag{27}$$

whence $\nu = 7$, and the constant C coincides with const, which contains the k-independent factors of the collision integrals (7) and (23). Consequently, the spectrum of the stationary turbulence is

$$N_k \sim k^{-7}. \tag{28}$$

It should be mentioned that the possible existence of a stationary spectrum in this case was already pointed out in[8], where the non-stationary width of a plasmon packet, due to competition between four-plasmon interaction and nonlinear scattering by ions, was determined. Just as in the cases (16) and (17), turbulence with a spectrum (28) is local and isotropic. This spectrum is also non-stationary, strictly speaking, if there is no source of plasma waves. The characteristic non-stationarity time is of the order of $(\nu_i + \gamma_4)^{-1}$. It should be noted that the exact solutions obtained above for the weak-turbulence spectra differ from those obtained in[8,9] by approximate calculations.

## 3. STABILITY OF DISTRIBUTIONS AND THE POSSIBILITY OF SECOND SOUND IN A TURBULENT PLASMA

In this section we investigate the stability of the turbulence spectra against low-frequency perturbations. We assume that the perturbation frequency is small compared with the characteristic frequency of the nonlinear interaction that determines the stationary distribution of the plasmons. In the case when the thermal motion of the ions is not taken into account ($M \to \infty$), the stability of the spectra (16) and (17) has an oscillatory character. These oscillations constitute second sound in a turbulent plasma. When account is taken of the ion motion, the stability of the distribution (28) has an aperiodic (diffusion) character, because of the nonconservation of the energy and momentum of the plasmons when they are scattered by the ions.

In order to determine the dispersion law and the velocity of the second sound, we write out the equations for the total number of plasma oscillations, for the summary momentum, and for the total kinetic energy. We neglect here the stimulated scattering of the plasma waves by plasma particles and collisions between electrons, i.e., we put $\nu_e = \gamma_k = 0$ in (1). Allowance for these collisions leads to damping of the second sound. We multiply (1) by unity, by the plasmon wave vector **k**, and by the kinetic energy $\Omega_k$, and integrate over the phase volume in which the plasmons have a stationary distribution. Owing to the conservation laws, the terms containing the averaged collision integral drop out. As a result we obtain the system of three equations

$$\frac{\partial N}{\partial t} + 3\omega_0 r_d^2 \operatorname{div}\mathbf{P} = 0, \tag{29}$$

$$\frac{\partial P_i}{\partial t} + \frac{\partial \Pi_{ij}}{\partial x_j} = 0, \tag{30}$$

$$\frac{\partial T}{\partial t} + \operatorname{div}\mathbf{Q} = 0, \tag{31}$$

where

$$\Pi_{ij} = 3\omega_0 r_d^2 \int k_i k_j N_k d^3\mathbf{k}$$

is the momentum flux density tensor,

$$\mathbf{Q} = 3\omega_0 r_d^2 \int \Omega_k \mathbf{k} N_k d^3\mathbf{k}$$

is the kinetic-energy flux density, and $\mathbf{P} = \int \mathbf{k} N_k d^3\mathbf{k}$ is the plasmon momentum density.

Second sound can exist in the frequency region

$$\omega \ll \gamma_4. \tag{32}$$

Therefore at each instant of time there exists a quasi-stationary distribution

$$N_k(\mathbf{r}, t) = N_k^0 - (\mathbf{kV} + \mu + \Omega_k\Theta)\,\partial N_k^0/\partial\Omega_k, \tag{33}$$

which causes the integral of the plasmon-plasmon collisions to vanish. The non-equilibrium addition to the distribution function (33), due to delay effects in these collisions, leads to an additional damping of the second sound, on the order of $\omega/\gamma_4$. The parameters **V**, $\mu$, and $\Theta$ are small quantities and depend on the coordinates and on the time. $\mathbf{V}(\mathbf{r}, t)$ is the average velocity of the plasmon gas as a whole, $\mu(\mathbf{r}, t)$ is their chemical potential, and $\Theta(\mathbf{r}, t)$ is the dimensionless average energy. $N_k^0$ represents the stationary turbulence spectrum (16) or (17). Small additions to N, **P**, and T, and also to $\Pi_{ij}$ and **Q**, can be expressed in terms of the distribution parameters (33) as follows:

$$\delta N = A\mu + B\Theta, \quad \delta P_i = \frac{2}{9\omega_0 r_d^2} BV_i, \quad \delta T = \mu B + \Theta C, \tag{34}$$
$$\Pi_{ij} = {}^2/_3\delta_{ij}(B\mu + C\Theta), \quad Q_i = {}^2/_3 V_i C,$$

where the constants A, B, and C are given by

$$A = -\int \frac{\partial N_k}{\partial\Omega_k} d^3\mathbf{k},\ B = -\int \Omega_k \frac{\partial N_k}{\partial\Omega_k} d^3\mathbf{k},\ C = -\int \Omega_k^2 \frac{\partial N_k}{\partial\Omega_k} d^3\mathbf{k}. \tag{35}$$

Substituting (34) in (29)—(31), we can easily verify that in the case of second-sound oscillations the chemical potential remains unchanged, i.e., $\mu(\mathbf{r}, t) = 0$. Therefore the equations for the determination of the second-sound spectrum are of the form

$$\frac{\partial \mathbf{V}}{\partial t} + 3\omega_0 r_d^2 \frac{C}{B}\nabla\Theta = 0, \quad \frac{\partial\Theta}{\partial t} + \frac{2}{3}\operatorname{div}\mathbf{V} = 0.$$

Eliminating the velocity **V** from these two equations, we obtain the wave equation

$$\frac{\partial^2\Theta}{\partial t^2} - 2\omega_0 r_d^2 \frac{C}{B}\Delta\Theta = 0.$$

It follows therefore that the spectrum of the second-sound oscillations is linear:

$$\omega(q) = qs, \quad (36)$$

and the velocity of second sound is determined by the formula

$$s = (2\omega_0 r_d^2 C / B)^{1/2}. \quad (37)$$

The presence of two stationary distributions of plasmons leads to the existence of two branches of second sound—fast and slow. The fast second sound is realized in region (16) with a constant plasmon kinetic energy flux. Its velocity can be readily obtained with the aid of formulas (37) and (35). The main contribution to B is made by the energy-containing region ($k \sim k_0$), and in C the principal role is played by large $k \sim k_s$. As a result of the calculations we get

$$s_1 \approx 2v_e(k_0 r_d)^{2/3}(k_s r_d)^{1/3}, \quad v_e = (3T_e/2m)^{1/2}.$$

The velocity of the slow second sound is determined by the distribution (17), for which the plasmon flux is constant. In this case the quantity C is determined by the energy-containing region, and in B the main contribution is made by small $k \sim k_1$. The expression for the velocity is given by

$$s_2 \approx v_e(k_0 r_d)^{2/3}(k_1 r_d)^{1/3}. \quad (38)$$

The velocity of the second sound is approximately $(k_s/k_1)^{1/3}$ larger than the velocity of the slow sound.

The damping of the second sound is determined both by dissipative mechanisms and by the fact that the plasmon distribution "lags" the quasistationary distribution (33). The relative damping decrement of the fast sound is of the order of

$$\omega/\gamma_4 + \gamma_k/\omega \quad (39)$$

and is small in the frequency interval $\gamma_k < \omega < \gamma_4$. For slow sound, the relative damping is given by the same formula (39), in which, $\gamma_k$ must be replaced by $\nu_e$. By the same token, it can be stated that the distributions (16) and (17) are stable against similar low-frequency perturbations.

It follows from the foregoing analysis that second sound constitutes oscillations of the average number and of the energy of the plasmons. These oscillations can be excited by periodic variation of the external parameters, for example, by changing the number or energy of the plasma oscillations in the region of their generation.

Let us consider now the evolution of low-frequency perturbations in the case when the turbulence spectrum (28) is determined by the competition between the mutual plasmon scattering and their scattering by ions. Owing to the non-conservation of the energy and momentum of the plasma waves there appear in Eqs. (30) and (31) additional terms $\int \mathbf{k}\hat{\nu}_i\{N_k\}d^3k$ and $\int \Omega_k\hat{\nu}_i\{N_k\}d^3k$, describing the transfer of momentum and energy to the ions. Now the derivative with respect to time in (30), as well as the derivatives with respect to time and the coordinates in (31) must be neglected compared with these additional dissipative terms. Relation (29) remains unchanged, since the total number of plasmons is conserved upon collision with the ions [Eq. (23)]. As a result we obtain the following system of equations:

$$A\frac{\partial\mu}{\partial t} + B\frac{\partial\Theta}{\partial t} + \frac{2}{3}B\,\mathrm{div}\,\mathbf{V} = 0, \quad (40)$$

$$B\nabla\mu + C\nabla\Theta + {}^3/_2\int \mathbf{k}\hat{\nu}_i\{N_k\}d^3\mathbf{k} = 0, \quad (41)$$

$$\int \Omega_k\hat{\nu}_i\{N_k\}d^3\mathbf{k} = 0. \quad (42)$$

Linearized expressions for the change of the summary momentum and energy of the plasmons can be represented in the form

$$\int \mathbf{k}\hat{\nu}_i\{N_k\}d^3\mathbf{k} = {}^1/_3\mathbf{V}a,\ a \approx \langle\nu_i k^2\rangle,$$

$$\int \Omega_k\hat{\nu}_i\{N_k\}d^3\mathbf{k} = b\mu + c\Theta,\ b \approx \langle\nu_i\Omega_k\rangle,\ c = \langle\nu_i\Omega_k^2\rangle. \quad (43)$$

Here $\nu_i$ is given by (24), the angle brackets denote averaging over the turbulence spectrum (28) with the function $-\partial N_k^0/\partial\Omega_k$. Changing over to Fourier components, we can easily obtain the solution of the dispersion equations of the system (40)—(42) in the form

$$\omega = -i\frac{4}{3}q^2\frac{B}{a}\frac{Bc - bC}{Ac - bB} \approx -iq^2\frac{B^2}{aA}. \quad (44)$$

It follows therefore that the stability of the spectrum (28) has an aperiodic character, and the initial low-frequency perturbation spreads out diffusely with a diffusion coefficient $D \approx B^2/aA$.

In conclusion, we take the opportunity to thank Ya. B. Faĭnberg and the participants of the seminar under his direction for a discussion of the results of the work.

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