THIN SUPERCONDUCTING FILMS IN A UHF FIELD

I. DESTRUCTION OF SUPERCONDUCTIVITY BY A UHF CURRENT

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Dynamic destruction of the superconductivity of a thin film by a UHF current is considered within the framework of the phenomenological theory. Excitation of Cooper pairs through the energy gap by a UHF current of frequency $\omega \ll h^{-1}\Delta$ is discussed. The possibilities of studying the gap and the relaxation rate in the condensed state, which arise in this case, are also pointed out.

1. It has been possible to obtain a satisfactory understanding of the phenomenon of dc superconductivity only on the basis of microscopic theory.[1] In spite of this fact, it is difficult to overestimate the value of the phenomenological theory[2] in the study of superconductivity. Furthermore, it has been possible to justify the phenomenological approach (by means of the microscopic theory[3–5]) also for the study of ac superconductivity, where the picture of the phenomenon is much more complicated. For this to be possible, it is only necessary that there be a large difference between the frequency ω of the current and the relaxation rate T_S^{-1} of the ordering parameter ($\omega \ll T_S^{-1}$ or $\omega \gg T_S^{-1}$). We shall consider below a film of thickness $d \ll \xi_0$ (ξ_0 is the coherence length in the bulk specimen) with UHF current under the assumption that one of these conditions is satisfied. The inequality $d < \xi_0$ in the absence of external magnetic fields parallel to the film is a sufficient condition for the applicability of the phenomenological theory to a thin film.[6] At the same time, this is also the condition of constancy of the current density over the thickness. To simplify the problem, we shall assume the current density to be constant also over the surface of the film. The non-uniformity of the current distribution over the width of the film is compensated at UHF by the inductance of the superconducting component. Therefore, it suffices, to satisfy the latter condition, to have the width and length of the film much less than the wavelength of the radiation ($2\pi c\omega^{-1}$, where c is the velocity of light).

We assume that the free energy density, reckoned from the energy of the normal state, can be written for the film in the form

$$F_s = -f_1(x) + f_2(x, v_s^2), \tag{1}$$

where f_1 is the free energy density without a current, and

$$f_2 = \frac{mv_s^2}{2}\rho_t x, \tag{2a}$$

$x = \rho_S/\rho_t$; ρ_S is the density of superconducting electrons; ρ_t its value as the peak value of the free energy, $F_{S\,max}$, tends to zero (all quantities pertaining to this case will be called threshold quantities and will be denoted by the index t); $v_S = v\cos\omega t$ is the instantaneous velocity of the superconducting electrons, v its amplitude value.

Near the critical temperature,

$$-f_1 = \alpha\rho_s + {}^1/{}_2\beta\rho_s^2, \tag{2b}$$

where α and β do not depend on ρ_S. In order not to introduce any limitations on the temperature, we can assume that f_1 has some bell-shaped form. The admissibility of the representation of F_S in the form (1) for strong currents is not evident but is usually assumed (see, for example, [7]).

For $F_{S\,max} < 0$, the equilibrium value of ρ_S is determined in our case by the mean square value of the free energy (F_{Sa}). However, for $F_{S\,max} > 0$, this ceases to be valid since the relaxation rate of the ordering parameter cannot remain the same as before in that part of the period of the UHF when the instantaneous value of the free energy (F_S) is larger than zero. Here, for relaxation, the condensate need not obtain additional energy; therefore, its rate increases up to the rate of relaxation of the normal component τ^{-1}. In a thin or "dirty" conductor, it is not difficult to obtain $\tau^{-1} \gg \omega$ and thus, in spite of the sharp change in the conditions, to preserve the applicability of the phenomenological theory even for $F_S > 0$. According to estimates,[8] the relaxation time of the ordering parameter T_S is of the order of 10^{-8} sec, while for sufficiently thin films it is not difficult to obtain τ less than 10^{-12} sec. Although the upper bound of the relaxation rate is difficult to observe directly, the fact that $\tau \ll T_S$, even for sufficiently thin films, is evident from direct experiments on the study of the rate of destruction of superconductivity by a current.[9] The possibility of assuring a wide range of frequencies satisfying the required conditions $T^{-1} \ll \omega \ll \tau^{-1}$ is then evident.

2. As long as $F_{S\,max} < 0$, the equilibrium value of ρ_S can be determined from the equation

$$\frac{\partial F_{sa}}{\partial \rho_s} = -\frac{\partial f_1}{\partial \rho_s} + \frac{mv^2}{4} = 0. \tag{3}$$

By adding it to the equality

$$F_{s\,max} = -f_1 + \frac{mv^2}{2}\rho_s = 0, \tag{4}$$

it is possible to find the threshold velocity v_t and the threshold density ρ_t of the superconducting component. If (2b) is valid, then we can obtain

$$k = \rho_t / \rho_0 = {}^2/{}_3$$

(ρ_0 is the density of the superconducting component without current). Allowance for the succeeding terms of the expansion of f_1 in powers of ρ_0 does not lead to a significant change in k; it becomes close to unity.

In the framework of the two-fluid model, the total current density can be written in the form

$$j=-i\frac{e^2\rho_t}{m\omega}E\left[x+\frac{(\omega\tau)^2}{1+(\omega\tau)^2}(a-x)+i\frac{\omega\tau}{1+(\omega\tau)^2}(a-x)\right], \quad (5)$$

where $a = \rho/\rho_t$, ρ is the density of conduction electrons, E the electric field intensity, and e and m the charge and mass of the electron. For thin films ($d \ll \xi_0$) $\omega\tau \ll 1$ and $a \gg x$;[10] therefore

$$j\approx -i\frac{e^2\rho_t}{m\omega}E(x+ia\omega\tau). \quad (5a)$$

Hence

$$v_s=\frac{eE}{im\omega}=j\frac{1}{e\rho_t(x+ia\omega\tau)}. \quad (6)$$

If the current in the UHF circuit does not depend on the state of the superconductor (i.e., the internal resistance of the current source is much greater than the resistance of the superconductor), then j in (6) is also independent of x and therefore

$$v=v_t/|x+i\omega\tau|. \quad (6a)$$

For $F_S \geq 0$, the destruction of superconductivity takes place as for constant current. Since $x = e^{-t/\tau}$, it follows that

$$v\to v_t/a\omega\tau \gg v_t,$$

and F_S increases correspondingly. Thus, the conditions are created for an avalanche-type destruction of the superconductivity.

In the time interval in which $v_S(\omega t)$ falls to zero, condensation of pairs takes place at the rate T_S^{-1}, but in the next period they are destroyed again at the rate $\tau^{-1} \gg T_S^{-1}$. Therefore, practically complete destruction of the superconductivity takes place at the threshold. For its establishment, it is necessary to lower v to values smaller than v_t, which requires a reduction of the current in the UHF circuit by not less than the factor $(a\omega\tau)^{-1}$. (The current and the energy density of its field, at which establishment of superconductivity begins, will be called the critical current.) In other words, upon satisfaction of the conditions noted here, in a state almost unperturbed by the UHF current, instantaneous destruction of the superconductivity takes place at the threshold, owing to the excitation of the Cooper pairs through the energy gap by imparting to them of the kinetic energy necessary for this purpose. The phenomenon has a sharply expressed current threshold and a hysteretic character. The threshold character makes the effect an ideal indicator of the change in the resistance parameters (condensate density, energy gap, free energy, and so on).

3. If the internal resistance of the current source is of the order or smaller than the resistance of the superconductor in the normal state, the current in the superconductor will depend on the quantity x. The character of the destruction of superconductivity in this case changes, but in principle, the new situation arises only when the complete destruction of superconductivity leads to a falloff of the current density to a value J_{min} less than the critical J_{cr}. Such a situation can be created by cancelling the reactive part of the impedance of the superconductor, i.e., by tuning the UHF circuit to resonance with the current. In this case, the impedance will be determined by the losses of UHF energy in the superconductor. The density of these losses, according to (5a), is

$$P_s=\mathrm{Re}(jE)=\frac{e^2E^2}{m\omega}\rho_t a\omega\tau,$$

on the other hand, it is equal to

$$P_s=j^2\sigma^{-1},$$

where σ is the equivalent conductivity of the resonator. Equating the right-hand sides, we obtain

$$\sigma=\sigma_t\frac{x^2+(a\omega\tau)^2}{1+(a\omega\tau)^2}. \quad (7)$$

It is then seen that at x = 0 the conductivity, and consequently, the current in the resonator, is smaller by a factor $(a\omega\tau)^{-2}$ than the threshold value and consequently, is $(a\omega\tau)^{-1}$ times smaller than critical. In the general case, there are losses in the resonator which do not depend on x (damping in nonsuperconducting elements δ_0, by coupling with the external load δ_{ext}). Therefore, the falloff of current in the general case will be less. But even in this case, the actual reserve allows us to obtain $J_{min} < J_{cr}$. Then, after destruction of the superconductivity, conditions for its re-establishment automatically arise. In the superconductor, establishment of the initial density takes place with a time constant T_S, the damping decreases in the resonator, and the original current begins to be re-established. After achievement of the original current (or earlier), the process repeats itself. Thus, relaxation oscillations are established in the superconductor + resonator system.

If the UHF power (P) supplied to the resonator is equal to the threshold power (P_t), then the growth to the original threshold current takes an infinitely long time; consequently, the threshold powers correspond to zero frequency (Ω) of the relaxation oscillations. Increase of the power above threshold will increase Ω so long as J_{min} does not exceed J_c and collapse of the oscillations does not take place.

4. We now consider the more detailed picture of collapse of the relaxation oscillations. For a resonator impedance that is time-independent, the rate of change of current in it is proportional to

$$T_r^{-1}=\omega\delta,$$

where $\delta = \delta_{ext} + \delta_0 + \delta_S$, and δ_S is the damping in the superconductor.

In the general case,

$$\delta_s=\delta_{st}\frac{1+(a\omega\tau)^2}{x^2+(a\omega\tau)^2}. \quad (8)$$

Upon destruction of superconductivity, δ_S increases by a factor of $(a\omega\tau)^{-2}$ and T_r^{-1} i.e., the rate of attainment of a new stationary current J_{min}, also increases correspondingly. The collapse of the oscillations takes place for $J_{min} = J_c$. Consequently, one can assume that here the condensation begins at a current of constant amplitude and hence the problem of estimating the currents reduces to the calculation of a resonator with variable damping

$$\delta=\delta_t\left[1+C\frac{1-x^2}{(a\omega\tau)^2+x^2}\right], \tag{9}$$

where

$$C=\frac{\delta_{st}}{\delta_{ext}+\delta_0+\delta_{st}}$$

and determines the fraction of the losses in the superconductor relative to the total losses of the resonator. In this case, one can write for the current amplitude

$$\frac{dJ}{dt}+\frac{J}{T_{rt}}\left[1+C\frac{1-x^2}{(a\omega\tau)^2+x^2}\right]=\frac{MJ_t}{T_{rt}}, \tag{10}$$

$M^2 = P/P_t$ is the dimensionless UHF power fed to the resonator.

Since in this case

$$\frac{dx}{dt}=\frac{1-x}{T_s},$$

Eq. (10) can be rewritten differently:

$$(1-x)\frac{dJ}{dx}+J\frac{T_s}{T_{rt}}\left[1+C\frac{1-x^2}{(a\omega\tau)^2+x^2}\right]=MJ_t\frac{T_s}{T_{rt}}. \tag{10a}$$

At $t = 0$ $(x = 0)$ and $J = J_{min}$, it is not difficult to establish the fact that

$$J_{min}=\frac{MJ_t}{1+C(a\omega\tau)^{-2}}.$$

Then it is seen from (10a) that $dJ/dx = 0$ at $x = 0$. On the other hand,

$$v^2=\frac{J_s^2}{S^2e^2\rho_t^2x^2}=\frac{J^2}{S^2e^2\rho_t^2}\frac{1}{(a\omega\tau)^2+x^2}$$

(S is the cross section area of the film), therefore

$$\frac{df_2}{dx}=\frac{f_2}{x}\left[1-\frac{2x^2}{(a\omega\tau)^2}\right].$$

For $x \ll 1$, (2b) is valid; therefore,

$$\frac{df_1}{dx}=\frac{f_1}{x}\left[1-\frac{x}{3}\right].$$

Since the critical point correspond to $f_2 = f_1$, we have

$$\frac{dF_{s\,max}}{dx}=f_2\left[\frac{1}{3}-\frac{2x}{(a\omega\tau)^2}\right].$$

In our case,

$$dF_{s\,max}/dx<0 \quad \text{if} \quad x>(a\omega\tau)^2/6$$

and consequently, the repeated switching on of the mechanism of pair disruption cannot take place when $x > (a\omega\tau)^2/6$.[1)] For this reason, before the collapse, the period of the relaxation oscillations and their frequency have finite values.

The complete period of oscillations is composed of two parts: the time of the drop of the current to J_{cr}

$$T_1\approx T_{rt}(a\omega\tau)^2\ln\frac{CJ_{min}}{(a\omega\tau)^2(J_{min}-J_{cr})},$$

and the time of its growth

$$T_2\sim J_{min}^{-1}.$$

It is then seen that before the collapse ($J_{min} \to J_{cr}$) the time T_1 increases much more rapidly than T_2 decreases, and therefore the collapse ought to precede the change of sign in $d\Omega/dP$. Thus the increase in power behind the threshold increases the frequency of the relaxation oscillations from zero to a certain maximum value of the order of T_s^{-1}, after which the oscillations collapse. The connection of Ω_{max} with T_s allows us to use the effect for the study of relaxation characteristics of superconductors. This question will be discussed in more detail in a later paper.

5. Equation (10) is valid even for $J_{min} < J_c$ (only the initial conditions differ, $J \neq J_{min}$ at $x = 0$). We make use of them for the analysis of the process in the region $J \approx J_t$ for $M = 1$. Here $x = 1 - \epsilon$, $\epsilon \to 0$; therefore (10) can be rewritten in the form

$$\frac{T_{rt}}{T_s}\varepsilon\frac{dJ}{d\varepsilon}\approx-(J_t-J). \tag{10b}$$

Hence

$$J=J_t(1-q\varepsilon^{T_s/T_{rt}}), \tag{11}$$

where q is a constant of the order of unity.

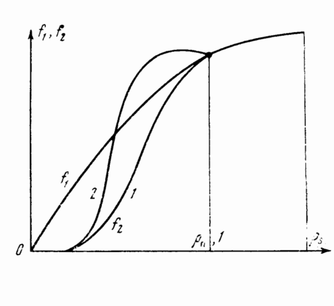
Using (11) and (2a), we get

$$f_2=f_{2t}(1-\varepsilon(2q\varepsilon^{T_s/T_{rt}-1}-1)),$$

whence it is evident that f_2 approaches f_1 monotonically only if $T_s < T_{pt}$. This case corresponds to the curve 1 in the drawing. If $T_s > T_{rt}$, the free energy has a maximum for $x < 1$ (curve 2) and therefore it is impossible to obtain zero frequency of the relaxation oscillations even at threshold. In this case, some hysteresis is possible even in the oscillatory regime.

6. In conclusion, we consider the physical picture of the effect and the possibilities of its use.

As a consequence of the strong non-adiabaticity, the superconductor behaves at low current densities as in direct current equal to the mean square value of the alternating current, but the instantaneous energy of the condensed state here changes in proportion to the square of the instantaneous value of the current. Upon increase in the current, this energy can reach as a maximum the energy of the normal state, while the density of Cooper pairs, which is determined by the mean square value of the current, is nevertheless large. The equality of the energies of the superconducting and normal states inserts a new, significantly more rapid relaxation mechanism, as a consequence of which the pair density falls to zero. In other words, the pairs are excited here because they acquire, within



[1)]In order that the disruption mechanism not be switched on repeatedly at smaller x, the UHF power (P) must be less than the power of collapse P_{coll}, while $(P_{coll}-P)/P_{coll} \leq 10^{-3}$. In other words, the collapse of the oscillations takes place in the range of powers lying beyond the limits of possibility of the ordinary methods of regulation of UHF power.

a very short time (much less than T_S), a kinetic energy exceeding the energy of the pair condensation.

Under certain conditions, as a consequence of the destruction of superconductivity, conditions for its restoration arise automatically. Therefore, relaxation oscillations of current appear in the UHF circuit, and oscillations of the condensate density take place in the superconductor.

The observation of the described effects makes possible still another method of investigation of such parameters of the superconductor as F_S, Δ, ρ_S and also a suitable method for the study of its relaxation characteristics.

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