THEORY OF SPIN WAVES IN ANTIFERROMAGNETICS WITH DOMAIN BOUNDARY TYPE MAGNETIC INHOMOGENEITIES

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We apply the second quantization method to give a theory of spin waves in antiferromagnetics with a simple domain structure. We evaluate the elementary excitation spectrum for orthorhombic antiferromagnetics. We show that two kinds of spin motion are possible in antiferromagnetics, to each of which there correspond three kinds of elementary excitations with different energy spectra. The results obtained are used to study the (thermal and resonance) properties of antiferromagnetics.

INTRODUCTION

It is well known that different defects (including magnetic ones) in crystals can affect strongly a number of their physical properties. Magnetic defects and inhomogeneities connected with the breakdown of the magnetic order in a crystal may be point defects, planar defects, or extended defects. Recently a number of papers have appeared which studied the properties of magnetic substances both with local[1] and planar[2] defects, as well as with extended defects,[3-8] amongst which belong domain boundaries in ferro- and antiferromagnetics. Localized vibrations are excited near point defects. Point defects change the density of states[1] and appreciably affect a number of properties of solids. Separate planar[2] and extended[4-6] defects cause the appearance of specific elementary excitations, which are damped fast when we get away from the defect. However, if there are in the sample extended defects that are connected with one another, specific elementary excitations may occur in the whole sample. Thus, in ferromagnetics with a periodic domain structure[9] there exist in the sample two additional kinds of elementary excitations. In this connection it is particularly important to develop methods of studying different kinds of elementary excitations in ferro- and antiferromagnetics containing extended defects that are connected with one another. The simplest case of such defects in ferro- and antiferromagnetics is their domain structure. The cause of the formation of domain structure in antiferromagnetics was considered in [10].

In [7] we studied elementary excitations in antiferromagnetics with a periodic domain structure using the classical equations of motion. For a consideration of several kinetic and relaxation phenomena in antiferromagnetics from a single point of view it is expedient to apply the second quantization method. In the present paper we give a general scheme of finding the elementary excitations in antiferromagnetics with a periodic domain structure.

1. STATEMENT OF THE PROBLEM. HAMILTONIAN

To simplify the calculations we restrict ourselves in our considerations to $CuCl_2 2H_2O$ type orthorhombic antiferromagnetics for which we can write the phenomenological Hamiltonian in the form

$$\mathcal{H}(\mathbf{r}) = \frac{1}{2}(B-A)\,m^2 + \frac{1}{2}A_\alpha\left(\frac{\partial l_{\alpha'}}{\partial x_\alpha}\right)^2 + \frac{1}{2}B_\alpha\left(\frac{\partial m_{\alpha'}}{\partial x_\alpha}\right)^2 + {}^1/_2 a_1 l_z^2 + {}^1/_2 a_2 l_y^2 + {}^1/_2 b_1 m_z^2 + {}^1/_2 b_2 m^2 - M_0 \mathbf{m}\mathbf{h}, \quad (1)$$

where $\alpha, \alpha' = 1, 2, 3$; $\{x_\alpha\} = x, y, z$; everywhere we have assumed summation to have taken place over twice repeated indices, $\mathbf{l}$ is the antiferromagnetism vector, and $\mathbf{m}$ the magnetization in units $2M_0$:

$$\mathbf{l} = {}^1/_2(\mathbf{m}_1 - \mathbf{m}_2), \quad \mathbf{m} = {}^1/_2(\mathbf{m}_1 + \mathbf{m}_2), \quad (2)$$

$\mathbf{m}_1 = \mathbf{M}_1/M_0$; $\mathbf{m}_2 = \mathbf{M}_2/M_0$; $\mathbf{M}_1$, $\mathbf{M}_2$ are the sublattice magnetization vectors, $i = 1, 2$; M_0 is the saturation magnitude of the sublattice magnetization; $B - A$ is the exchange interaction parameter, $B - A > 0$; a_1, a_2, b_1, b_2 are the anisotropy constants; $\mathbf{h}$ is the field connected with the non-uniformity of the vector $\mathbf{m}$:

$$\operatorname{rot}\mathbf{h} = 0, \quad \operatorname{div}\mathbf{h} = -8\pi M_0 \operatorname{div}\mathbf{m}. \quad (3)$$

The vectors $\mathbf{l}$ and $\mathbf{m}$ are connected through the relations

$$\mathbf{l}^2 + \mathbf{m}^2 = 1, \quad \mathbf{l}\mathbf{m} = 0. \quad (4)$$

The terms in (1) containing the coefficients A_α and B_α represent that part of the exchange energy which is connected with the non-uniformity of the vectors $\mathbf{l}$ and $\mathbf{m}$. The quantities

$$A_{\alpha\alpha} = B_\alpha + A_\alpha, \quad B_{\alpha\alpha} = B_\alpha - A_\alpha \quad (5)$$

represent the increase in the exchange energy connected with the non-uniformity $\mathbf{m}_j$ ($j = 1, 2$) inside and between the sublattices. When $a_1 > a_2$, $a_1 > 0$ the spins are parallel to the x (y) axis and in the sample 180 degree domains, parallel to the xy-plane are possible. We restrict our considerations to the simplest domain structure (Fig. 1) which consists of plane-parallel do-

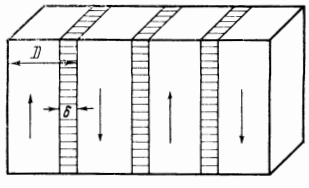

FIG. 1. Simplest Shirobokov type domain structure. The direction of the vector l in the domains is indicated by the arrows.

mains which are separated by transitional layers (in Shirobokov's model). In that case the antiferromagnetism vector changes in the whole sample according to the relations[10]

$$\cos\varphi = -\mathrm{sn}\, p_1 z, \quad \sin\varphi = \mathrm{cn}\, p_1 z, \quad \sqrt{1-k^2\cos^2\varphi} = \mathrm{dn}\, p_1 z, \tag{6}$$

where $\varphi = \varphi(z)$ is the azimuthal angle of the vector $\mathbf{l}$; in the transitional layer $\mathbf{m} = 0$

$$p = (a_2/A_3)^{1/2}, \quad p = kp_1, \quad k = 1-\varepsilon; \quad \varepsilon = 8e^{-pD}; \tag{7}$$

$\delta = 1/p$ is the effective thickness of the transitional layer, k the modulus of the elliptic functions. In antiferromagnetics $D/\delta \sim 10^2$ to 10^3; as $D/\delta \to \infty$, the quantity $\cos\varphi = -\tanh pz$.

According to the general scheme of [11] we change from the classical quantities $\mathbf{m}_j$ (j = 1, 2) to the corresponding quantal operators $\hat{\mathbf{m}}_j$, the components of which satisfy the commutator relations:[12]

$$[\hat{m}_{\alpha j}(\mathbf{r}), \hat{m}_{\alpha' j'}(\mathbf{r}')] = -i(\mu/M_0)\, e_{\alpha\alpha'\alpha''} m_{\alpha''}\delta(\mathbf{r}-\mathbf{r}'). \tag{8}$$

Here $e_{\alpha\alpha'\alpha''}$ is the antisymmetric third rank tensor, $\alpha, \alpha', \alpha''$ take on the values 1, 2, 3 cyclically, $\mu = \gamma\hbar$, $\gamma = g\mu_B$ is the spectroscopic splitting factor. By virtue of (2) the quantities $\mathbf{l}$ and $\mathbf{m}$ in the Hamiltonian (1) are also replaced by the corresponding operators $\hat{\mathbf{l}}$ and $\hat{\mathbf{m}}$.

In the original xyz system of coordinates the quantization axis is not fixed for an inhomogeneous system. It is in that case convenient to change to a local system of coordinates XYZ such that the X-axis is along the equilibrium vector $\mathbf{l}_0$ while the z-axis remains unchanged (Fig. 2). The components of the operators $\hat{\mathbf{l}}$ and $\hat{\mathbf{m}}$ then transforms as follows:

$$\begin{aligned} \hat{m}_x &= \hat{m}_X\cos\varphi - \hat{m}_Y\sin\varphi, \\ \hat{m}_y &= \hat{m}_X\sin\varphi + \hat{m}_Y\cos\varphi, \\ \hat{l}_x &= \hat{l}_X\cos\varphi - \hat{l}_Y\sin\varphi, \\ \hat{l}_y &= l_X\sin\varphi + \hat{l}_Y\cos\varphi. \end{aligned} \tag{9}$$

The commutation relations (8) remain invariant under such transformations. Further calculations turn out to be simplest if we assume $\mathbf{l}$ and $\mathbf{m}$ to be functions of the old coordinates (x, y, z).

Since there are no demagnetization fields in antiferromagnetics, the domain boundaries are more mobile than in ferromagnetics and can shift over several lattice constants. The rigidity of the domain boundaries caused by anisotropy and magneto-elastic forces will be taken into account by adding to the Hamiltonian a term of the form

$$R(l_Y^2 + m_Y^2), \tag{10}$$

where R is the "rigidity" parameter of the domain boundaries. The dipole energy plays in antiferromagnetics a secondary role and we shall take it into account in Winter's approximation in the form $8\pi M_0^2 m_Z^2$. Taking the term (10) into account, the Hamiltonian (1) can in the new set of coordinates be written in the form:

$$\begin{aligned} \mathcal{H}(\mathbf{r}) = {} & \frac{1}{2}(B-A)\,\hat{m}^2 + \frac{1}{2}A_\alpha\left(\frac{\partial\hat{l}_{\alpha'}}{\partial x_\alpha}\right)^2 + \frac{1}{2}A_3\left[2\left(\hat{l}_X\frac{\partial\hat{l}_Y}{\partial z} - \hat{l}_Y\frac{\partial\hat{l}_X}{\partial z}\right)\varphi' \right. \\ & \left. + (\hat{l}_X^2 + \hat{l}_Y^2)\varphi'^2\right] + \frac{1}{2}B_\alpha\left(\frac{\partial\hat{m}_{\alpha'}}{\partial x_\alpha}\right)^2 + \frac{1}{2}B_3\left[2\left(\hat{m}_X\frac{\partial\hat{m}_Y}{\partial z} - \hat{m}_Y\frac{\partial\hat{m}_X}{\partial z}\right)\varphi' \right. \\ & \left. + (\hat{m}_X^2 + \hat{m}_Y^2)\varphi'^2\right] + \frac{1}{2}a_1\hat{l}_z^2 + \frac{1}{2}a_2(\hat{l}_X^2\sin^2\varphi + \hat{l}_Y^2\cos^2\varphi \\ & + 2\hat{l}_X\hat{l}_Y\sin\varphi\cos\varphi) + \frac{1}{2}b_2(\hat{m}_X^2\sin^2\varphi + m_Y^2\cos^2\varphi + 2\hat{m}_X\hat{m}_Y\sin\varphi\cos\varphi) \\ & + \frac{1}{2}(b_1 + 16\pi M_0^2)\,\hat{m}_z^2 + R(\hat{l}_Y^2 + \hat{m}_Y^2). \end{aligned} \tag{11}$$

The sublattice magnetization operators $\hat{\mathbf{m}}_1$ and $\hat{\mathbf{m}}_2$ (by virtue of (2) also the operators $\hat{\mathbf{l}}$ and $\hat{\mathbf{m}}$) can be expressed in terms of the second quantization operators[11] $b_{\mathbf{r}}^j = b^j(\mathbf{r})$, $b_{\mathbf{r}}^{+j} = b^{+j}(\mathbf{r})$ (j = 1, 2) which satisfy the commutator relations

$$[b^j(\mathbf{r}), b^{+j'}(\mathbf{r}')] = \delta_{jj'}\delta(\mathbf{r}-\mathbf{r}'). \tag{12}$$

We can then express the Hamiltonian of the system as a power series in the operators $b_{\mathbf{r}}^j$ and $b_{\mathbf{r}}^{+j}$:

$$\mathcal{H}(\mathbf{r}) = \mathcal{H}_0(\mathbf{r}) + \mathcal{H}_2(\mathbf{r}) + \mathcal{H}_3(\mathbf{r}) + \mathcal{H}_4(\mathbf{r}) + \ldots, \tag{13}$$

where

$$\mathcal{H}_0(\mathbf{r}) = a_2k^{-2}(1-k^2\cos^2\varphi) + a_2\sin^2\varphi, \tag{14}$$

$$\begin{aligned} \mathcal{H}_2(\mathbf{r}) = {} & \frac{\mu}{4M_0}\left\{\alpha_{jj'}b_{\mathbf{r}}^{+j}b_{\mathbf{r}}^{j'} + \frac{1}{2}\beta_{jj'}(b_{\mathbf{r}}^j b_{\mathbf{r}}^{j'} + b_{\mathbf{r}}^{+j}b_{\mathbf{r}}^{+j'}) \right. \\ & \left. + A_{\alpha\alpha}\frac{\partial b_{\mathbf{r}}^{+j}}{\partial x_\alpha}\frac{\partial b_{\mathbf{r}}^j}{\partial x_\alpha} + \frac{1}{2}B_{\alpha\alpha}^{jj'}\left(\frac{\partial b_{\mathbf{r}}^j}{\partial x_\alpha}\frac{\partial b_{\mathbf{r}}^{j'}}{\partial x_\alpha} + \frac{\partial b_{\mathbf{r}}^{+j}}{\partial x_\alpha}\frac{\partial b_{\mathbf{r}}^{+j'}}{\partial x_\alpha}\right)\right\}. \end{aligned} \tag{15}$$

Here

$$\begin{aligned} & \alpha = 1, 2, 3; \quad \{x_\alpha\} = (x, y, z), \quad j, j' = 1, 2; \\ & \alpha_{11} = \alpha_{22} = B - A + 4a_2\cos^2\varphi + {}^1/_2(a_1 + \tilde{b}_1) - \\ & \quad - {}^1/_2(4 - k^{-2})a_2 + {}^1/_2 b_2 k^{-2} + 2R, \\ & \alpha_{12} = \alpha_{21} = {}^1/_2[(a_2 - b_2)k^{-2} + \tilde{b}_1 - a_1], \\ & \beta_{11} = \beta_{22} = {}^1/_2[(a_2 + b_2)k^{-2} - (a_1 + \tilde{b}_1)] + 2R, \\ & \beta_{12} = \beta_{21} = -(B - A) + {}^1/_2(a_1 - \tilde{b}_1) + {}^1/_2(a_2 - b_2)k^{-2}, \\ & B_{\alpha\alpha}^{jj'} = B_{\alpha\alpha}^{j'j} = B_{\alpha\alpha}, \quad B_{\alpha\alpha}^{jj} = 0, \quad \tilde{b}_1 = b_1 + 8\pi M_0^2. \end{aligned} \tag{16}$$

As in the homogeneous case, in the expansion (13) $\mathcal{H}_2 = \int\mathcal{H}_2(\mathbf{r})d\mathbf{V}$ determines after having been brought to diagonal form the sum of the energy of the elementary excitations. The higher-order terms $\mathcal{H}_3(\mathbf{r})$ and $\mathcal{H}_4(\mathbf{r})$ which we shall not consider here describe collision processes between different types of elementary excitations. They can be used to describe kinetic and relaxation effects in the inhomogeneous case.

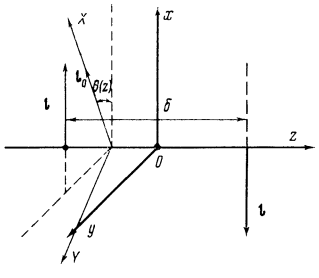

FIG. 2. System of coordinates. The X-axis is along the vector $\mathbf{l}_0$.

2. DIAGONALIZING THE QUADRATIC FORM $\mathcal{H}_2$

The coefficients in the quadratic form $\mathcal{H}_2(\mathbf{r})$ depend for an inhomogeneous antiferromagnetic on z and the diagonalization scheme is not the same as the scheme for a homogeneous antiferromagnetic.

Because of the symmetry of the crystal we can expand the operators $b^j(\mathbf{r})$ and $b^{+j}(\mathbf{r})$ in a two-dimensional Fourier series:

$$b^j(\mathbf{r}) = \frac{1}{\sqrt{S_0}}\sum_{\varkappa} b_\varkappa^j(z)\, e^{i\varkappa_\perp\rho}, \tag{17}$$

where S_0 is the area of a cross-section of the sample, $\varkappa_\perp = \varkappa_1\mathbf{e}_1 + \varkappa_2\mathbf{e}_2$, $\rho = x\mathbf{e}_1 + y\mathbf{e}_2$, $\mathbf{e}_1$, $\mathbf{e}_2$ are appropriate

unit vectors. In (17) the operators $b_\kappa^j(z)$ satisfy by virtue of (12) the commutator relation

$$b_\kappa{}^j(z)b_{\kappa'}{}^{+j'}(z') - b_{\kappa'}{}^{+j'}(z')b_\kappa{}^j(z) = \delta_{\kappa\kappa'}\delta_{jj'}(z-z'). \tag{18}$$

The quadratic form $\mathcal{H}_2$ is then transformed into

$$\mathcal{H}_2 = \frac{\mu}{4M_0}\sum_\kappa \int \Big\{ \alpha_{jj'}(\kappa) b_\kappa{}^{+j} b_\kappa{}^{j'} + \frac{1}{2}\beta_{jj'}(\kappa)\left(b_\kappa{}^j b_{-\kappa}{}^{j'} + b_\kappa{}^{+j} b_{-\kappa}{}^{+j'}\right) \tag{19}$$
$$+ A_{33}\frac{\partial b_\kappa{}^{+j}}{\partial z}\frac{\partial b_\kappa{}^j}{\partial z} + \frac{1}{2}B_{33}{}^{jj'}\left(\frac{\partial b_\kappa{}^j}{\partial z}\frac{\partial b_{-\kappa}{}^{j'}}{\partial z} + \frac{\partial b_\kappa{}^{+j}}{\partial z}\frac{\partial b_{-\kappa}{}^{+j'}}{\partial z}\right)\Big\} dz,$$

where

$$\alpha_{jj'}(\kappa) = \alpha_{jj'} + A_{11}\kappa_1^2 + A_{22}\kappa_2^2,$$
$$\beta_{jj'}(\kappa) = \beta_{jj'} + B_{11}\kappa_1^2 + B_{22}\kappa_2^2. \tag{20}$$

The operators $b_\kappa^j = b_\kappa^j(z)$, $b_\kappa^{+j} = b_\kappa^{+j}(z)$ are determined by the equations of motion

$$i\hbar\dot{b}_\kappa{}^j = (b_\kappa{}^j\mathcal{H}_2 - \mathcal{H}_2 b_\kappa{}^j). \tag{21}$$

Using (18) and (19) we get from (21)

$$i\hbar\dot{b}_\kappa{}^j = \frac{\mu}{4M_0}\left\{\alpha_{jj'}(\kappa)b_\kappa{}^{j'} + \beta_{jj'}(\kappa)b_{-\kappa}{}^{+j'} - A_{33}\frac{d^2b_\kappa{}^j}{dz^2} - B_{33}{}^{jj'}\frac{d^2b_{-\kappa}{}^{j'}}{dz^2}\right\} \tag{22}$$

$(j, j' = 1, 2)$.

Using the canonical transformation

$$b_\kappa{}^j = u_\kappa{}^{jj'}c_\kappa{}^{j'} + v_\kappa^{+jj'}c_{-\kappa}^{+j'}, \tag{23}$$

we can diagonalize the quadratic form (19); here $u_\kappa^{jj'} = u_\kappa^{jj'}(z)$, ... are unknown functions, c_κ^j new boson operators, $i\hbar\dot{c}_\kappa^j = \epsilon_\kappa c_\kappa^j$, and the quantity ϵ_κ is the energy of the elementary excitations, and

$$c_\kappa{}^j c_{\kappa'}^{+j'} - c_{\kappa'}^{+j'}c_\kappa{}^j = \delta_{\kappa\kappa'}\delta_{jj'}. \tag{24}$$

Substituting (23) into (22) we get a set of equations to determine $u_\kappa^{jj'}$, $v_{-\kappa}^{jj'}$, ϵ_κ:

$$B_{33}^{jj'}\frac{d^2u_\kappa^{j'j''}}{dz^2} - A_{33}\frac{d^2v_{-\kappa}^{j'j''}}{dz^2} + \beta_{jj'}(\kappa)u_\kappa^{j'j''} + (\alpha_{jj'}(\kappa) + \lambda_\kappa\delta_{jj'})v_{-\kappa}^{j'j''} = 0,$$
$$B_{33}^{jj'}\frac{d^2v_{-\kappa}^{j'j''}}{dz^2} - A_{33}\frac{d^2u_\kappa^{jj''}}{dz^2} + \beta_{jj'}(\kappa)v_{-\kappa}^{j'j''} + (\alpha_{jj'}(\kappa) - \lambda_\kappa\delta_{jj'})u_\kappa^{j'j''} = 0. \tag{25}$$

Here $\lambda_K = (4M_0/\mu)\epsilon_K$; j, j', j'' independently take on the values 1, 2. These equations must be solved with well-defined normalization conditions which we can obtain using Eqs. (18), (23), (24) and also the equations of motion (25):

$$u_\kappa^{jj'}(z)\,u_{\kappa'}^{+jj'}(z') - v_{-\kappa}^{jj'}(z')\,v_{\kappa'}^{jj'}(z) = \delta_{jj'}\delta(z-z'),$$
$$v_{-\kappa}^{jj'}(z)\,u_{\kappa'}^{jj'}(z) - v_{-\kappa}^{jj'}(z)\,u_{\kappa'}^{jj'}(z') = 0. \tag{26}$$

After diagonalization the quadratic form $\mathcal{H}_2$ has the form

$$\mathcal{H}_2 = \sum_{\kappa,\,j} c_\kappa{}^{+j}c_\kappa{}^j\varepsilon_\kappa{}^j + \Delta\mathcal{H}_0, \tag{27}$$

where

$$\Delta\mathcal{H}_0 = -\sum_\kappa |v_{-\kappa}^{jj'}|^2\varepsilon_\kappa{}^j. \tag{28}$$

Here j denotes the number of the kinds of motion. The number of spectral branches depends on the number of independent solutions of the set (25).

3. SOLUTION OF THE SET (25). FINDING THE ELEMENTARY EXCITATIONS AND THEIR ENERGY SPECTRUM

By virtue of the symmetry of the coefficients of (19) the set (25) can be reduced to an equivalent set of four second order differential equations with four unknown functions. All these equations have a similar form and the functions u_K^{11}, u_K^{21}, v_{-K}^{11}, and v_{-K}^{21} can differ only by a constant factor:

$$u_\kappa{}^{11} = C_{11}\psi(z),\quad u_\kappa{}^{21} = C_{12}\psi(z),\quad v_{-\kappa}{}^{11} = C_{21}\psi(z),\quad v_{-\kappa}{}^{21} = C_{22}\psi(z), \tag{29}$$

where the C_{ik} (i, k = 1, 2) are arbitrary constants, $\psi(z)$ is an unknown function. The sets of equations obtained have solutions which satisfy the normalization conditions (26), respectively, for the following cases:

$$C_{11} = -C_{12},\quad C_{21} = -C_{22}, \tag{30}$$

$$C_{11} = C_{12},\quad C_{21} = C_{22}. \tag{31}$$

In an antiferromagnetic without domains (in the uniform case) $d^2\psi/dz^2 = \kappa_3^2\psi$, and we get two different spectral branches.[11] The first branch (30) corresponds to the motion of the vector **l** within the ab-plane and to oscillations of the vector **m** at right angles to that plane. On the other hand, the second branch (31) corresponds to the motion of the vector **l** outside the ab-plane and a motion of the vector **m** in that plane.[7]

In antiferromagnetics the exchange interaction between sublattices is stronger than inside the sublattices. We may thus assume that

$$|\varepsilon_1| = |A_{33}/B_{33}| \ll 1. \tag{32}$$

The set of four equations can then be appreciably simplified and reduces to two independent equations:

$$\frac{d^2\psi}{d\xi^2} - (2k^2\,\mathrm{sn}^2\,\xi + A^{(j)})\psi = k^2\varepsilon_{11}(\mathrm{sn}^2\,\xi)\psi, \tag{33}$$

$$\frac{d^2\psi}{d\xi^2} - (2k^2\,\mathrm{sn}^2\,\xi + A^{(j)})\psi = k^2\varepsilon_{12}(\mathrm{sn}^2\,\xi)\psi, \tag{34}$$

where the $\psi(\xi)$ are the eigenfunctions of the problem and the A^j are the new eigenvalues which are connected with the old ones λ_K by the following equations:

$$(A_\kappa{}^j - \lambda_\kappa)C_{11} + B_\kappa{}^jC_{22} = 0,$$
$$B_\kappa{}^jC_{11} + (A_\kappa{}^j + \lambda_\kappa)C_{22} = 0; \tag{35}$$

j = 1, 2 correspond to cases (30) and (31); $p_1 z = \xi$;

$$A_\kappa^{(1)} = B - A + a_1 + b_2k^{-2} - 2a_2(1+k^{-2}) + 2R + A_{11}\kappa_1^2 + A_{22}\kappa_2^2 - A^{(1)}A_{33}p_1^2,$$
$$B_\kappa^{(1)} = -[B - A - a_1 + b_2k^{-2} + 2R + B_{11}\kappa_1^2 + B_{22}\kappa_2^2 - A^{(1)}B_{33}p_1^2],$$
$$A_\kappa^{(2)} = B - A + \tilde{b}_1 - (2+k^{-2})a_2 + 2R + A_{11}\kappa_1^2 + A_{22}\kappa_2^2 - A^{(2)}A_{33}p_1^2,$$
$$B_{\kappa_*}^{(2)} = -[B - A + \tilde{b}_1 - a_2k^{-2} - 2R + B_{11}\kappa_1^2 + B_{22}\kappa_2^2 - A^{(2)}B_{33}p_1^2],$$
$$\varepsilon_{11} = \varepsilon_1 + \varepsilon_2,\quad \varepsilon_{12} = \varepsilon_1 - \varepsilon_2,\quad C = C_{11}/C_{22} = 1 - \varepsilon_2,$$
$$\varepsilon_{2j} = \lambda_\kappa{}^j/A_\kappa{}^j \ll 1. \tag{36}$$

If the eigenvalues of Eqs. (33) or (34) are known, the energy spectrum of the elementary excitations is determined from (35):

$$\lambda_\kappa{}^j = \sqrt{(A_\kappa{}^j)^2 - |B_\kappa{}^j|^2}. \tag{37}$$

We cannot solve Eqs. (33) and (34) exactly. We can use the fact that the ϵ_{1j} are small to solve them by the method of successive approximations.

a) Zeroth Approximation

In zeroth approximation in ϵ_{1j} the two Eqs. (35) and (34) are the same:

$$\frac{d^2\psi}{d\xi^2} - (2k^2\,\mathrm{sn}^2\,\xi + A_0)\psi = 0. \tag{38}$$

This equation has three linearly independent solu-

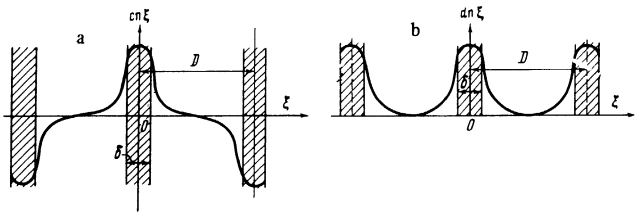

FIG. 3. The shape of the wave function: a) for the first b) for the second kind of excitation.

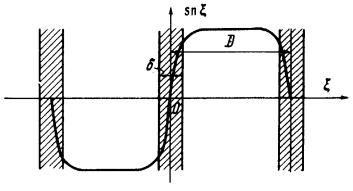

FIG. 4. The shape of the wave function for the third kind of excitation for the special case $\kappa_3 = 0$. In the general case it is represented by the superposition of two waves: $(\mathrm{sn}\,\xi \mp i\kappa_3 p_1^{-1})\exp(\mp i\kappa_3 p_1^{-1}\xi)$.

tions:[7,9]

$$A_{01}^{\,j} = -1, \quad \psi_1(\xi) = \mathrm{cn}\,\xi,$$
$$A_{02}^{\,j} = -k^2, \quad \psi_2(\xi) = \mathrm{dn}\,\xi, \tag{39}$$

$$A_{03}^{\,j} = -\left(1 + k^2 + \frac{\kappa_3^2}{p^2}\right), \quad \psi_3(\xi) = \left(\mathrm{sn}\,\xi \mp \frac{i\kappa_3}{p}\right)\exp\left\{\pm\frac{i\kappa_3}{p}\xi\right\}.$$

There are thus three kinds of elementary excitations (39) (see Figs. 3, 4) for each kind of motion. The solution of the set (25) corresponding to the two kinds of motion can be written in the form:

$$\text{I.}\quad u_{\varkappa i}^{(11)} = -u_{\varkappa i}^{(21)} = C_{11i}^{(1)}\psi_i, \quad v_{-\varkappa i}^{(11)} = -v_{-\varkappa i}^{(21)} = C_{21i}^{(1)}\psi_i, \tag{40}$$

$$\text{II.}\quad u_{\varkappa i}^{(11)} = u_{\varkappa i}^{(21)} = C_{11i}^{(2)}\psi_i, \quad v_{-\varkappa i}^{(11)} = v_{-\varkappa i}^{(21)} = C_{21i}^{(2)}\psi_i, \tag{41}$$

where the constants C_{ikl}^{j} (j = 1, 2; i = 1, 2, 3) are determined from (35) and a normalization condition such as (26):

$$C_{11i}^{j} = \frac{1}{2}\frac{\sqrt{A_{\varkappa i}^{\,j} + \lambda_{\varkappa i}^{\,j}}}{\sqrt{2\lambda_{\varkappa}^{\,j} D_i}}, \quad C_{22i}^{j} = -\frac{1}{2}\frac{B_{\varkappa i}^{\,j}}{|B_{\varkappa i}^{\,j}|}\frac{\sqrt{A_{\varkappa i}^{\,j} - \lambda_{\varkappa i}^{\,j}}}{\sqrt{2\lambda_{\varkappa}^{\,j} D_i}}, \tag{42}$$

$$\lambda_{\varkappa i}^{\,j} = \sqrt{(A_{\varkappa i}^{\,j})^2 - |B_{\varkappa i}^{\,j}|^2}. \tag{43}$$

$$D_1 = \frac{4N}{p_1 k^2}E, \quad D_2 = \frac{4N}{p_1}E, \quad D_3 = \frac{4N}{p_1}\left[\frac{1}{k^2}(K-E) + \frac{\kappa_3^2}{p_1^2}\right],$$
$$K = \frac{1}{2}pD(1+\varepsilon), \quad K' = \frac{\pi}{2}\left(1 + \frac{1}{2}\varepsilon\right) \quad E \approx 1, \quad N = \frac{L}{D}, \tag{44}$$

L is the size of the sample; the $A_{\kappa i}^{j}$, $B_{\kappa i}^{j}$ are determined from (36), using (43); the indices j = 1, 2 indicate the kinds of motion; i = 1, 2, 3 the kind of excitations (39).

The general expression for the $b_{\kappa}^{j}(z)$ has the form

$$b_{\varkappa}^{j}(z) = u_{\varkappa i}^{jj'} c_{\varkappa i}^{j'} + v_{\varkappa i}^{+jj'} c_{-\varkappa i}^{+j'}. \tag{45}$$

After diagonalizing the quadratic form we can finally write it as follows:

$$\mathcal{H}_2 = \Delta\mathcal{H}_0 + \sum_{\varkappa} n_{\varkappa i}^{\,j}\,\varepsilon_{\varkappa i}^{\,j}, \tag{46}$$

where

$$\Delta\mathcal{H}_0 = -\frac{1}{2}\left(\frac{\mu}{4M_0}\right)\sum_{i=1}^{3}\sum_{\varkappa,\,j}(A_{\varkappa i}^{\,j} - \sqrt{(A_{\varkappa i}^{\,j})^2 - |B_{\varkappa i}^{\,j}|^2}), \tag{47}$$

$$\varepsilon_{\varkappa i}^{\,j} = \frac{\mu}{4M_0}\lambda_{\varkappa i}^{\,j}, \tag{48}$$

$n_{\kappa i}^{j}$ indicates the average number of elementary excitations of kind i with a quasi-momentum κ for the j-th kind of motion.

b) First Approximation

In the first approximation in ϵ_{ij} the character of the solutions for the functions $u_{\kappa i}^{j}$ and $v_{-\kappa i}^{j}$ is not changed but the energy levels are shifted by a small amount. This shift is connected with the change in the eigenvalues of Eqs. (37), (38)

$$A_i^{\,j} = A_{0i}^{\,j} + A_{1i}^{\,j},$$

where

$$A_{1i}^{\,j} = -2\varepsilon_{ij}k^2\frac{\langle i|\mathrm{sn}^2\xi|i\rangle}{\langle i|i\rangle}, \tag{49}$$

$\langle i|\mathrm{sn}^2\xi|i\rangle$ are the diagonal matrix elements of $\mathrm{sn}^2\xi$ with respect to the functions ψ_i (i = 1, 2, 3), $\langle i|i\rangle$ the integral of the absolute square of ψ_i over ξ. Using (49) we can finally express the spin wave spectrum through the equations

$$\varepsilon_{\varkappa 1}^{(1)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + b_2 - a_2 + 2R + B_\perp\varkappa_\perp^2]$$
$$\times[a_1 - a_2 + A_\perp\varkappa_\perp^2 + a_2\varepsilon_{11}s_1]\}^{1/2},$$
$$\varepsilon_{\varkappa 2}^{(1)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + b_2 - a_2 + 2R + B_\perp\varkappa_\perp^2]$$
$$\times[a_1 - a_2 - 2a_2\varepsilon + A_\perp\varkappa_\perp^2 + a_2\varepsilon_{11}s_2]\}^{1/2},$$
$$\varepsilon_{\varkappa 3}^{(1)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + b_2 - a_2 + 2R + B_\perp\varkappa_\perp^2 + B_3\varkappa_3^2]$$
$$\times[a_1 + A_\perp\varkappa_\perp^2 + A_3\varkappa_3^2 + a_2\varepsilon_{11}s_3]\}^{1/2},$$
$$\varepsilon_{\varkappa 1}^{(2)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + \tilde{b}_1 - a_2 + B_\perp\varkappa_\perp^2]$$
$$\times[2R + 2a_2\varepsilon + A_\perp\varkappa_\perp^2 + a_2\varepsilon_{12}s_1]\}^{1/2},$$
$$\varepsilon_{\varkappa 2}^{(2)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + \tilde{b}_1 - a_2 + B_\perp\varkappa_\perp^2]$$
$$\times[2R + A_\perp\varkappa_\perp^2 + a_2\varepsilon_{12}s_2]\}^{1/2}, \tag{50, 51}$$
$$\varepsilon_{\varkappa 3}^{(2)} = \left(\frac{\mu}{2M_0}\right)\{[B - A + \tilde{b}_1 + B_\perp\varkappa_\perp^2 + B_3\varkappa_3^2]$$
$$\times[a_2 + 2R + A_\perp\varkappa_\perp^2 + A_3\varkappa_3^2 + a_3\varepsilon_{12}s_3]\}^{1/2};$$

$$s_1 = s_2 = \frac{1}{3}, \quad s_3 = \frac{(1 + \varkappa_3^2/p^2)(K-E)}{K - E + K\varkappa_3^2/p^2}$$

$$A_\perp\varkappa_\perp^2 = A_1\varkappa_1^2 + A_2\varkappa_2^2, \quad B_\perp\varkappa_\perp^2 = B_1\varkappa_1^2 + B_2\varkappa_2^2. \tag{52}$$

Equations (25) allow solutions in yet another case, when $\epsilon_2 = |B_{33}|/|A_{33}| \ll 1$. The corresponding solutions for ψ_i and ϵ_i^j can be expressed by Eqs. (39), (50), and (51) with the only difference that the value of the parameter ϵ_{ij} is replaced by ϵ_2. For arbitrary values of the parameters A_3 and B_3 we can look for a solution of the set (25) in the form of a linear combination of the functions ψ_i in (39), i.e., $\psi = a_i\psi_i$. To each type of motion there correspond then three kinds of elementary excitations; the form of the spectrum is essentially unchanged. Because of the complexity of the calculations this case shall not be considered separately. From this it follows that independent of the values of the parameters A_3 and B_3 the scheme assumed here describes correctly the basic characteristic properties of the elementary excitations in antiferromagnetics with a domain structure.

Of the three spectral branches for each type of mo-

tion the smallest gap occurs for two kinds of excitations (ϵ^j_{K1}, ϵ^j_{K2}) which differ little from one another. However, in some cases the quantities $2a_2\epsilon$ and $a_2\epsilon_{ij}s_j$ may be of the same order as 2R or $a_1 - a_2$. For thermal motion the first and second branches are first of all excited when $a_1 - a_2 < 2R$ for the first type of motion and when $a_1 - a_2 > 2R$ for the second type of motion. We shall consider separately the conditions for a parametric excitation of these types of spinwaves.

4. SOME APPLICATIONS OF THE ELEMENTARY EXCITATION SPECTRA

We can use the obtained elementary excitation spectrum to study different physical properties of antiferromagnetics with a domain structure (magnetic, thermal, resonance, kinetic, and relaxation properties). We shall restrict ourselves here to considering the two simplest applications.

1. In antiferromagnetics with a periodic domain structure there are up to three resonance frequencies $\omega^j_i = (\epsilon^j_{Ki}/\hbar)|_{K=0}$, for each kind of motion; two of those, ω^j_1 and ω^j_2 are connected with the presence of a domain structure in the sample:

$$\omega_1^{(1)} = \frac{\gamma}{2M_0}\{[B - A + b_2 - a_2][a_1 - a_2 + a_2\varepsilon_{11}s_1]\}^{1/2}, \qquad (53)$$

$$\omega_2^{(1)} = \frac{\gamma}{2M_0}\{[B - A + b_2 - a_2][a_1 - a_2 - 2a_2\varepsilon + a_2\varepsilon_{11}s_2]\}^{1/2}.$$

$$\omega_1^{(2)} = \frac{\gamma}{2M_0}\{[B - A + b_1 - a_2][2R + 2a_2\varepsilon + 2a_2\varepsilon_1 s_1]\}^{1/2}, \qquad (54)$$

$$\omega_2^{(2)} = \frac{\gamma}{2M_0}\{[B - A + b_1 - a_2][2R + a_2\varepsilon_1 s_2]\}^{1/2},$$

where $\gamma = \mu/\hbar$ is the spectroscopic splitting factor. The difference between the frequencies $\Delta\omega^j = \omega^j_1 - \omega^j_2$ depends on the quantity $2\epsilon a_2$. By measuring this difference we can estimate the domain size. Measuring $\omega_2^{(1)}$ and $\omega_2^{(2)}$ we can estimate the parameters $a_1 - a_2$ and 2R.

2. Using the elementary excitation spectrum we evaluate the spin part of the entropy and the specific heat in an antiferromagnetic sample with a periodic domain structure: $S = -\partial F/\partial T$, $C_S = -T\partial^2 F/\partial T^2$, where F is the free energy. We consider two special cases:

a) $\zeta^j_{0i} = \epsilon^j_{0i}/kT \ll 1$. Then we have

$$F_i^j = -G_i^j(kT)^3(1 - 3\zeta_{0i}^j)e^{-\zeta_{0i}j},$$

$$F_3^j = -G_3^j(kT)^3\left(\frac{\pi^4}{15} - \frac{\pi^2}{4}\zeta_{03}^j\right);$$

$$S_i^j = 3G_i^j k(kT)^2 e^{-\zeta_{0i}j}(1 - 9\zeta_{0i}^j),$$

$$S_3^j = G_3^j k(kT)^3\left[\frac{4\pi^2}{15} - \frac{\pi^2}{2}(\zeta_{03}^j)^2 - \frac{21}{2}(\zeta_{03}^j)^4\right];$$

$$C_{si}^j = 6G_i^j k(kT)^2 e^{-\zeta_{0i}(3)}(1 - 3\zeta_{0i}^{(3)}),$$

$$C_{s3}^j = 3G_3^j k(kT)^3\left\{\frac{4\pi^2}{15} - \frac{\pi^2}{6}(\zeta_{03}^j)^2 + \left(\frac{\pi}{2} - \zeta_{03}^j \ln\frac{\zeta_{03}^j}{kT}\right)\right.$$

$$\left.\times(\zeta_{03}^j)^3 + (1/2\,C - 35/2)(\zeta_{0i}^j)^4\right\}, \quad C \approx 0.58;$$

$$G_i^j = \frac{S_0}{C_i^j}\left(\frac{M_0}{\pi\mu}\right)^2 \frac{\pi}{\sqrt{A_1A_2}} \qquad G_3^j = \frac{1}{3}\frac{V}{(2\pi)^3}\left(\frac{2M_0}{\mu}\right)^3 \frac{1}{(C_3^j)^{3/2}}\frac{4\pi}{\sqrt{A_1A_2A_3}}$$

$$C_1^{(1)} = C_2^{(1)} = B - A + b_2 - a_2 + 2R, \quad C_1^{(2)} = C_2^{(2)} = B - A + b_1 + 16\pi M_0^2$$
$$C_3^{(1)} = B - A + b_2 + 2R, \qquad C_3^{(2)} = B - A + b_2 + 16\pi M_0^2;$$

b) $\zeta^j_{0i} = \epsilon^j_{0i}/kT \gg 1$. In that case

$$F_i^j = -G_i^j(kT)^3 e^{-\zeta_{0i}j}, \quad F_3^j = -3\sqrt{\pi/2}\,G_3^j(kT)^3(\zeta_{03}^j)^{-3/2}e^{-\zeta_{03}j};$$

$$S_i^j = G_i^j(kT)^2 k e^{-\zeta_{0i}j}(3 + 2\zeta_{0i}^j),$$

$$S_3^j = G_3^j \cdot 3\sqrt{\frac{\pi}{2}}\,k(\zeta_{03}^j)^{-3/2}e^{-\zeta_{03}j}(4 + \zeta_{03}^j)(kT)^3;$$

$$C_{si}^j = 2G_i^j(kT)^2 k e^{-\zeta_{0i}j}(2(\zeta_{0i}^j)^3 + 3\zeta_{0i}^j + 3),$$

$$C_{s3}^j = 3\sqrt{\pi/2}\,G_3^j k(kT)^3(\zeta_{03}^j)^{-3/2}e^{-\zeta_{03}j}\left((\zeta_{03}^j)^4 - 1/8\,(\zeta_{03}^j)^3 + 5\zeta_{03}^j + 12\right),$$

$$i = 1, 2, \quad j = 1, 2.$$

At low temperatures ($\zeta^j_{0i} \gg 1$) the specific heat for the first and second kinds of excitations is appreciably larger than for the third kind.

We shall consider the interaction between different kinds of excitations in another paper.

CONCLUSIONS

Two kinds of spin motion are possible in an antiferromagnetic sample with a periodic domain structure; and there are for each of them three kinds of elementary excitations with a different energy spectrum. For each type of motion the first two types of excitations are specific and are connected with the presence of the domain structure in the sample. These two kinds of excitations (Fig. 3) are plane waves propagating parallel to the domain boundary with a variable amplitude which attains a maximum in the middle of the transitional layer while vanishing in the middle of the domains. The third kind of excitations (Fig. 4) is a complicated spin wave which is similar to the homogeneous case but strongly deformed under the influence of the domain structure. For each kind of motion the energy gap in the first two kinds of excitations is appreciably smaller than for the third kind. In this connection the entropy, specific heat, resonance and relaxation properties of the sample change appreciably.

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