ENERGY LOSSES BY A CHARGED PARTICLE IN AN ISOTROPIC PLASMA LOCATED IN AN EXTERNAL HIGH-FREQUENCY ELECTRIC FIELD

T. L. TAVDGIRIDZE and N. L. TSINTSADZE

Physics Institute, Georgian Academy of Sciences

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Polarization losses of energy by a charged particle in an isotropic plasma located in an external HF electric field is considered. It is shown that in weak HF electric fields the losses grow quadratically with respect to the HF field amplitude; in strong HF fields the losses are much smaller, and on this basis suppression of proper Langmuir oscillations can be assessed.

A charged particle passing with constant velocity through an isotropic plasma, loses energy to the excitation of plasma oscillations. This question has been subject to a large number of investigations[1)]. In this paper we consider the influence of an external HF electric field on the excitation of plasma waves emitted by a charge.

Starting from a detailed investigation of the energy loss of the charged particles passing through an isotropic plasma placed in an external HF electric field, we consider the problem of amplification and suppression of natural HF Langmuir oscillations excited by the charged particles. It is shown that an external homogeneous HF electric field, up to a definite value of the amplitude, contributes to intensification of the natural Langmuir oscillations, and the growth of the energy loss is quadratic in the amplitude of the HF field. Further growth of the amplitude of the HF field leads to a suppression of the natural Langmuir oscillations. Expressions for the energy lost by the charge are obtained for two cases, when the amplitude of the HF field is either parallel or transverse to the charge motion. A comparison of the expressions shows that the suppression of the natural oscillations in the latter case occurs at lower amplitudes of the HF field than in the former case.

The frequency of the Langmuir oscillations considered by us

$$\omega = \omega_{Le} + {}^3/_2 k^2 v_{Te}^2 / \omega_{Le}$$

is much lower than the frequency ω_0 of the external electric field. As was shown in[3], the dielectric constant ϵ_e for Langmuir oscillations in the HF field remains practically unchanged. Recognizing that the charged particle executes not only linear motion with velocity $\mathbf{v}_0$, but also vibrational motion in the HF electric field $\mathbf{E} = \mathbf{E}_0 \sin \omega_0 t$, we obtain for the polarization energy loss of the particles the expression

$$Q = \frac{e^2}{2\pi}\int \frac{d\mathbf{k}\, d\omega}{k^2}|\omega| \sum_n J_n^2\left(\frac{\mathbf{k}\mathbf{v}_\sim}{\omega_0}\right)\delta(\omega + n\omega_0 - \mathbf{k}\mathbf{v}_0)\,\delta(\varepsilon_e), \quad (1)$$

where $\mathbf{v}_\sim = e\mathbf{E}_0/m\omega_0$ and $J_n(x)$ is the Bessel function.

Expression (1) follows from the formula for the power of the radiation of longitudinal waves from an extraneous charge[4,5]

$$Q = \lim_{T\to\infty} \frac{(2\pi)^6}{T}\int \frac{|\omega|\, d\mathbf{k}\, d\omega}{k^2}|\rho(\mathbf{k}, \omega)|^2 \delta(\varepsilon_e), \quad (2)$$

where $\rho(\mathbf{k}, \omega)$ is the Fourier component of the charge density and is given by

$$\rho(\mathbf{k}, \omega) = \frac{e}{(2\pi)^4}\int dt \exp\{-i[\mathbf{k}\mathbf{r}(t) - \omega t]\}, \quad (3)$$

with $\mathbf{r}(t)$ the trajectory of motion of the charge with velocity $\mathbf{v}_0$ in a homogeneous HF field:

$$\mathbf{r}(t) = \mathbf{v}_0 t - (e\mathbf{E}_0/m\omega_0)\sin\omega_0 t. \quad (4)$$

Using the expression for the dielectric constant $\epsilon_e = 1 - \omega_{Le}^2/\omega^2$ and integrating with respect to ω with the aid of the δ-function, we obtain

$$Q = \frac{\omega_{Le}^2 e^2}{2\pi}\sum_n \int \frac{d\mathbf{k}}{k} J_n^2\left(\frac{\mathbf{k}\mathbf{v}_\sim}{\omega_0}\right)\delta(\omega_{Le} + n\omega_0 - \mathbf{k}\mathbf{v}_0). \quad (5)$$

It is obvious that only small n take part in the process, since $kr_0 \ll 1$ for Langmuir waves, and inasmuch as $k \sim n\omega_0/v_0$, it follows that

$$nv_{Te}/v_0 \ll \omega_{Le}/\omega_0, \quad v_0 \gg v_{Te},$$

where v_{Te} is the thermal velocity of the electrons and $r_0 = v_{Te}/\omega_{Le}$ is the Debye length.

We consider two cases.

1. The amplitude of the applied HF field is directed along the motion of the charge, $\mathbf{E}_0 \parallel \mathbf{v}_0$. Expression (5) then takes the form

$$Q = Q_0 J_0^2\left(\frac{v_\sim}{v_0}\frac{\omega_{Le}}{\omega_0}\right) + \sum_{n\neq 0} Q_n J_n^2\left(\frac{nv_\sim}{v_0}\right), \quad (6)$$

where

$$Q_0 = \frac{e^2\omega_{Le}^2}{v_0}\ln\frac{k_0 v_0}{\omega_{Le}} \quad (7)$$

is the known expression for the polarization loss in an isotropic plasma[6,7];

$$Q_n = \frac{e^2\omega_{Le}^2}{v_0}\ln\frac{k_0 v_0}{n\omega_0}.$$

From formula (6) we get for the energy loss at $v_\sim/v_0 \ll 1$:

$$Q = Q_0 + \frac{\omega_{Le}^2 e^2}{v_0}\left(\frac{v_\infty}{v_0}\right)^2 \ln\left[\frac{k_0 v_0}{\omega_0}\right]. \quad (8)$$

In this expression we discarded terms of the order

[1)]See the review articles by Bolotovskii [1] and Ginzburg [2], and also the literature cited in these articles.

of $(v_\sim/v_0)^4$ and higher. It is seen from (8) that when $v_\sim/v_0 \ll 1$ there is added a term quadratic in the amplitude of the HF field to the known expression for the energy loss Q_0 as determined in (7). When the amplitude of the HF field increases to a definite value, the energy loss increases and reaches a maximum, and further growth of the amplitude ($v_\sim/v_0 > 1$) leads to a strong suppression of the oscillations.

2. The amplitude of the applied HF field is directed perpendicular to the direction of motion of the charge ($\mathbf{E}_0 \perp \mathbf{v}_0$). Carrying out the corresponding integration with respect to φ and ϑ, we obtain the following expression for the energy loss:

$$Q = \frac{\omega_{Le}^2 e^2}{v_0}\left\{\int\limits_{\omega_{Le}/v_0}^{k_0} \frac{dk}{k} J_0^2\left[\frac{v_\sim}{\omega_0}\sqrt{k^2 - \left(\frac{\omega_{Le}}{v_0}\right)^2}\right] + \sum_{n\neq 0}\int\limits_{n\omega_0/v_0}^{k_0} \frac{dk}{k} J_n^2\left[\frac{v_\sim}{\omega_0}\sqrt{k^2 - \left(\frac{n\omega_0}{v_0}\right)^2}\right]\right\}. \quad (9)$$

We rewrite expression (9) in terms of new variables

$$z = \frac{v_\sim}{\omega_0}\sqrt{k^2 - \frac{\omega_{Le}^2}{v_0^2}}, \quad z' = \frac{v_\sim}{\omega_0}\sqrt{k^2 - \frac{n^2\omega_0^2}{v_0^2}},$$

where at the upper limit of integration

$$z_{max} = \frac{v_\sim}{\omega_0}\sqrt{k_0^2 - \frac{\omega_{Le}^2}{v_0^2}}, \quad z'_{max} = \frac{v_\sim}{\omega_0}\sqrt{k_0^2 - \frac{n^2\omega_0^2}{v_0^2}},$$

but since $k_0 \gg \omega_0/v_0$ and $k_0 \gg \omega_{Le}/v_0$, we can put at the upper limit

$$z_{max} = z'_{max} = a = k_0 v_\sim/\omega_0.$$

We have

$$Q = \frac{\omega_{Le}^2 e^2}{v_0}\left\{\int\limits_0^a \frac{J_0^2(z)\,z\,dz}{z^2 + z_0^2} + \sum_{n\neq 0}\int\limits_0^a \frac{J_n^2(z)\,z\,dz}{z^2 + z_n^2}\right\}, \quad (10)$$

where $z_0 = (v_\sim/v_0)\omega_{Le}/\omega_0$ and $z_n = v_\sim/v_0$.

We consider here two cases:

1) Weak HF fields, $a \ll 1$ ($v_\sim \ll \omega_0/k_0$). Since $z \gg 1$, we can expand the Bessel functions in terms of the small argument and confine ourselves to the first two terms of the integration, and then integrate at n = 0 and n = 1. We have

$$Q = \frac{\omega_{Le}^2 e^2}{v_0}\left\{\ln\frac{k_0 v_0}{\omega_{Le}} + \frac{1}{2}\left(\frac{k_0 v^\sim}{\omega_0}\right)^2\left[1 + \left(\frac{\omega_0}{k_0 v_0}\right)^2 \ln\left(\frac{\omega_0}{k_0 v_0}\right)^2\right]\right\}. \quad (11)$$

2) Strong HF fields, $a \gg 1$ ($v_\sim \gg \omega_0/k_0$), and then the integration in formula (9) can be extended to infinity and the following expression is obtained

$$Q = \frac{\omega_{Le}^2 e^2}{v_0}\left\{I_0\left(\frac{\omega_{Le}}{\omega_0}\frac{v_\sim}{v_0}\right)K_0\left(\frac{\omega_{Le}}{\omega_0}\frac{v_\sim}{v_0}\right) + \sum_{n\neq 0} I_n\left(n\frac{v_\sim}{v_0}\right)K_n\left(n\frac{v_\sim}{v_0}\right)\right\}. \quad (12)$$

Let us consider the case $(v_\sim/v_0)(\omega_{Le}/\omega_0) \ll 1$ and $v_\sim/v_0$ is arbitrary; then

$$Q = \frac{\omega_{Le}^2 e^2}{v_0}\left\{\ln\left(2\frac{v_0}{v_\sim}\frac{\omega_0}{\omega_{Le}}\right) + \sum_{n\neq 0} I_n\left(n\frac{v_\sim}{v_0}\right)K_n\left(n\frac{v_\sim}{v_0}\right)\right\}. \quad (13)$$

Comparison of expressions (12) and (13) for the energy loss with formula (11) shows that in the case of strong HF fields the energy loss turns out to be much smaller, i.e., the presence of an external HF electric field leads to the suppression of the natural Langmuir oscillations.

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