PLASMA INSTABILITY IN THE FIELD OF A WEAK TRANSVERSE WAVE

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Submitted November 3, 1969

Zh. Eksp. Teor. Fiz. 58, 1002—1006 (March, 1970)

Instability of a plasma located in the electromagnetic field of a transverse wave is studied with respect to buildup of non-potential high frequency oscillations possessing a frequency close to the frequency of the external wave. The threshold value of the external field strength is found and the maximum of the increment after the buildup threshold is determined.

IT is known[1] that a sufficient intense external HF electromagnetic field can lead to instability of a plasma against the buildup of both potential and non-potential oscillations. At not too high intensities E_0 of the external HF field, when the oscillation buildup increment is smaller than the frequency of the ion-acoustic oscillations, such instabilities can be subdivided into two types. One corresponds to the interaction of the external HF wave with the low-frequency ion-acoustic oscillations of the plasma, and is characterized by generation of ion-acoustic oscillations with frequency ω_s and of high-frequency oscillations with one or both combination frequencies $\omega_0 \pm \omega_s$ at an external field frequency equal to ω_0[2-4]. The second type of instability, which is not connected with the interaction of the external wave with the ion-acoustic oscillations, was investigated in[2-4] for the case of the buildup of longitudinal oscillations, when spatial oscillations are simultaneously excited with zero frequency, and also high-frequency oscillations with frequency equal to the frequency ω_0 of the external field.

In this paper we investigate the instability of the second type, which leads, in contrast to[2-4], to excitation of high-frequency nonpotential oscillations. In addition, we take into account the dependence of the field of the external transverse wave on the coordinates, which makes possible a nonzero frequency of excited low-frequency oscillations and a corresponding deviation of the frequency of the excited non-potential oscillations from ω_0.

1. Let us consider non-potential oscillations in a plasma situated in the field of a transverse electromagnetic wave

$$\mathbf{E}(\mathbf{r},t)=\mathbf{E}_0\sin(\omega_0 t-\mathbf{k}_0\mathbf{r}),$$
$$\mathbf{k}_0\mathbf{E}_0=0,\quad \omega_0^2=\omega_p^2+c^2k_0^2. \tag{1}$$

Here ω_p^2 is equal to the sum of the squares of the Langmuir frequencies $\omega_{L\alpha} \pm (4\pi e_\alpha^2 n_\alpha/m_\alpha)^{1/2}$ of the electrons and of the ions (α = e, i). We shall henceforth assume that the thermal velocity of the electrons $v_{Te} = (T_e/m_e)^{1/2}$ and the velocity of their oscillations in the field (1) of the external wave $v_E = eE_0/m_e\omega_0$ are much smaller than the velocity of light c.

Being interested in the dispersion properties of the plasma in the field of the wave (1), we represent the small perturbations of the electric field $\delta\mathbf{E}(\mathbf{r}, t)$ in the form of an expansion in the harmonics of the external field:

$$\delta\mathbf{E}(\mathbf{r},t)=\sum_{n=-\infty}^{\infty}\delta\mathbf{E}_n\exp\left[i(\mathbf{k}+n\mathbf{k}_0)\mathbf{r}-i(\omega+n\omega_0)t\right]. \tag{2}$$

Linearizing in the usual manner (see[1]) the kinetic equation[5] in terms of the deviations $\delta f_{e,i}$ of the electron and ion distribution functions from the ground state, and substituting the expansion (2) in the expressions for $\delta f_{e,i}$ in Maxwell's equations, we obtain a system of coupled equations for the amplitudes δE_n. We assume that the oscillations with frequencies $\omega \pm \omega_0$ and with wave numbers $\mathbf{k} \pm \mathbf{k}_0$ satisfy approximately the usual dispersion equation of the transverse oscillations, and the frequency ω and the increment γ do not exceed the frequency of the external wave ω_0. Under these conditions only the amplitudes δE_n at n = 0 and ±1 are not small in the expansion (2), and from the condition for the solvability of the system of equations for the amplitudes we obtain the dispersion equation for the non-potential high-frequency oscillations (see[3]):

$$\frac{1}{\delta\varepsilon_e(\omega+i\gamma,\mathbf{k})}+\frac{1}{1+\delta\varepsilon_i(\omega+i\gamma,\mathbf{k})}+\frac{1}{4}k^2\left\{\frac{[\mathbf{r}_E,\mathbf{k}-\mathbf{k}_0]^2}{(\mathbf{k}-\mathbf{k}_0)^2}\times\frac{(\omega+i\gamma-\omega_0)^2}{(\omega+i\gamma-\omega_0)^2\varepsilon^{tr}(\omega+i\gamma-\omega_0,\mathbf{k}-\mathbf{k}_0)-c^2(\mathbf{k}-\mathbf{k}_0)^2}\right.$$
$$\left.+\frac{[\mathbf{r}_E,\mathbf{k}+\mathbf{k}_0]^2}{(\mathbf{k}+\mathbf{k}_0)^2}\frac{(\omega+i\gamma+\omega_0)^2}{(\omega+i\gamma+\omega_0)^2\varepsilon^{tr}(\omega+i\gamma+\omega_0,\mathbf{k}+\mathbf{k}_0)-c^2(\mathbf{k}+\mathbf{k}_0)^2}\right\}=0. \tag{3}$$

Here $\mathbf{r}_E = e\mathbf{E}_0/m_e\omega_0^2$ is the amplitude of the oscillations of the electrons in the external field (1),

$$\varepsilon^{tr}(\omega,\mathbf{k})=1-\frac{\omega_p^2}{\omega^2}\left(1-i\frac{\nu_{\text{eff}}}{\omega}\right)$$

is the usual linear transverse dielectric constant, and ν_{eff} is the effective frequency of collisions between the electrons and the ions in the external HF field[6]:

$$\nu_{\text{eff}}=\frac{4}{3}\frac{\sqrt{2\pi}\,e^2e_i^2n_i}{m_e^{1/2}T_e^{3/2}}\ln\frac{v_{Te}}{\omega_0 r_{min}}. \tag{4}$$

For the partial contributions $\delta\epsilon_\alpha(\omega, \mathbf{k})$ to the linear longitudinal dielectric constant $\epsilon(\omega, \mathbf{k}) = 1 + \delta\epsilon_e + \delta\epsilon_i$ it is possible to use, in the cases of interest to us, the following expressions:

$$\delta\varepsilon_\alpha(\omega,\mathbf{k})=\frac{1}{k^2r_{D\alpha}^2}\left(1+i\sqrt{\frac{\pi}{2}}\frac{\omega}{kv_{T\alpha}}\right),\qquad|\omega|<kv_{T\alpha}; \tag{5}$$

$$\delta\varepsilon_\alpha(\omega+i\gamma,\mathbf{k})=-\frac{\omega_{L\alpha}^2}{(\omega+i\gamma)^2},\qquad|\omega+i\gamma|>kv_{T\alpha},\quad|\omega|<\gamma. \tag{6}$$

Here $r_{D\alpha} = (T_\alpha/4\pi e_\alpha^2 n_\alpha)^{1/2}$ is the Debye radius of the particles of type α.

Let us consider first the solutions of Eq. (3) at $|\omega + i\gamma| < kv_{Ti}$, when there are no natural low-frequency oscillations in the absence of an external field. Such a situation corresponds to screening of the field of the low-frequency wave $\delta E_0 \exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t + \gamma t)$ by the electrons and ions. It is easy to see from the dispersion equation (3) that the maximum value of the increment is reached at $\mathbf{k}\cdot\mathbf{E}_0 = 0$. Neglecting the contribution of the Cerenkov effect to the dielectric constants $\delta\epsilon_\alpha(\omega, \mathbf{k})$ (5), we obtain from (3) the following expressions for the frequency and for the increment

$$\omega = c^2\mathbf{k}\mathbf{k}_0 / \omega_0, \tag{7}$$

$$\gamma = -\tilde{\gamma} + \left[-\Delta^2 + \frac{1}{4}\omega_0\Delta\frac{r_E^2}{r_{De}^2 + r_{Di}^2}\right]^{1/2}, \quad |\omega + i\gamma| < kv_{Ti}. \tag{8}$$

Here the decrement is

$$\tilde{\gamma} = \frac{1}{2}\frac{\omega_p^2}{\omega_0^2}\nu_{\text{eff}},$$

and $\Delta = c^2k^2/2\omega_0$. We note that formulas (7) and (8) are actually applicable at $|\omega + i\gamma| < \omega_s$, where $\omega_s = \omega_{Li}r_{De}k$ is the usual frequency of the ion-acoustic oscillations. From (8) we obtain the following expression for the maximum increment and for the corresponding wave number, at which this value is reached:

$$\gamma_{max} = \frac{1}{8}\frac{\omega_p^4}{\omega_0^3}\frac{E_0^2 - E_{0,\text{thr}}^2}{4\pi(n_eT_e + n_iT_i)}, \tag{9}$$

$$c^2k_{max}^2 = \frac{1}{4}\frac{\omega_p^4}{\omega_0^2}\frac{E_0^2}{4\pi(n_eT_e + n_iT_i)}.$$

The threshold intensity of the external field $E_{0,\text{thr}}$ is determined by the relation

$$\frac{E_{0,\text{thr}}^2}{4\pi(n_eT_e + n_iT_i)} = 4\frac{\omega_0\nu_{\text{eff}}}{\omega_p^2}. \tag{10}$$

When $\omega_0 \approx \omega_p$, expression (10) coincides with the corresponding formula for $E_{0,\text{thr}}$ [2], at which high-frequency longitudinal oscillations with frequency equal to ω_0 are excited.

Unlike the case of the buildup of longitudinal oscillations, the excitation of non-potential high-frequency oscillations is possible also when $\omega_0 \gg \omega_p$ in a relatively weak external field, when $E_0^2/4\pi \ll n_eT_e + n_iT_i$. In addition, the excited potential oscillations can have frequencies $\omega_0 \pm \omega$ that differ from ω_0, since the frequency (7) may be different from zero as a result of allowance for the finite wavelength of the external field. This difference, however, is limited by the condition $|\omega| < kv_{Ti}$, which leads to the following inequality:

$$\left|\frac{\mathbf{k}\mathbf{k}_0}{kk_0}\right| < \frac{v_{Ti}}{c}\frac{\omega_0}{ck_0}.$$

It is seen from (9) that the requirement $\gamma < \omega_s$ leads to a limitation of the intensity of the external field

$$\frac{E_0^2}{4\pi(n_eT_e + n_iT_i)} < 16\frac{\omega_0^4\omega_{Li}^2}{\omega_p^6}\left(\frac{v_{Te}}{c}\right)^2. \tag{11}$$

We note, finally, that at not too strong a non-isothermy, when $e_i^2m_eT_e < e^2m_iT_i$, the imaginary parts of the partial dielectric constants $\delta\epsilon_\alpha$ (5) can be neglected in the case of low collision frequencies ν_{eff}[1]:

$$\frac{\nu_{\text{eff}}}{\omega_p} < \frac{\omega_0^3\omega_{Li}^2}{\omega_p^5}\left|\frac{e_i}{e}\right|\frac{T_e}{T_i}\left(\frac{v_{Te}}{c}\right)^2. \tag{12}$$

In the case of sufficiently large values of the increment, when $kv_{Te} > \gamma > \omega, \omega_s$, we obtain from the dispersion equation (3) the following expressions for the frequency and for the increment:

$$\omega = \frac{c^2\mathbf{k}\mathbf{k}_0}{\omega_0}\frac{\gamma^2}{\gamma^2 + \Delta^2}, \tag{13}$$

$$\gamma = -\tilde{\gamma} + \left\{\tilde{\gamma}^2 - \frac{1}{2}(\Delta^2 + \omega_s^2 + \tilde{\gamma}^2) + \frac{1}{2}\left[(\Delta^2 + \tilde{\gamma}^2 - \omega_s^2)^2 + 2\omega_0^2\Delta^2\frac{\omega_{Li}^2r_E^2}{c^2}\right]^{1/2}\right\}^{1/2}, \tag{14}$$

which are suitable at sufficiently small angles between the vectors $\mathbf{k}$ and $\mathbf{k}_0$, when $|\mathbf{k}\cdot\mathbf{k}_0| < k^2$. If at the same time the inequality

$$\frac{\nu_{\text{eff}}}{\omega_p} < \frac{\omega_0^3\omega_{Li}^2}{\omega_p^5}\left(\frac{v_{Te}}{c}\right)^2,$$

which is stronger than (12), is satisfied, then the decrement γ in (14) can be neglected; as a result we obtain from (13) and (14) for the frequency, the maximum increment, and the corresponding wave number,[2]

$$\omega(k_{max}) = \frac{c^2\mathbf{k}_{max}\mathbf{k}_0}{\omega_0}\frac{\gamma_{max}^2\cdot 4\omega_0^2}{c^4k_{max}^4}, \quad \frac{|\mathbf{k}_{max}\mathbf{k}_0|}{k_{max}^2} < 1,$$

$$\gamma_{max} = \frac{\omega_{Li}}{\sqrt{2}}\frac{v_E}{c}, \tag{15}$$

$$k_{max}^2c^2 = 2^{1/3}\left(\omega_{Li}\,\omega_p\,\omega_0\frac{v_E^2}{cv_{Te}}\right)^{2/3}.$$

When $\gamma > kv_{Te}$, the expression for the increment differs from (14) in the absence of the frequency ω_s. Therefore, if the quantities determined by formulas (15) do not satisfy the inequality $\gamma_{max} < k_{max}v_{Te}$, then the maximum of the increment is determined by formula (15), and the corresponding wave number is $k_{max} \approx \gamma_{max}/v_{Te}$. It should be noted that the assumption that the spectrum of the excited high-frequency oscillations is close to the spectrum of the transverse plasma waves, which is satisfied in this case when $\Delta \ll \omega_0$, leads to a limitation on the external-field intensity:

$$\frac{v_E}{v_{Te}} \ll \frac{\omega_0}{\omega_{Li}}, \quad \frac{\omega_0}{\omega_{Li}}\left(\frac{\omega_{Li}c}{\omega_p v_{Te}}\right)^{1/2}.$$

CONCLUSIONS

We have investigated above the instability of a plasma in the field of a transverse wave and have demonstrated the possibility of buildup of potential oscillations with frequencies $\omega_0 \pm \omega$ close to the frequency ω_0 of the external field. At the same time (and with the same increment), there is a buildup of low-frequency oscillations with frequency ω, the deviation of which from zero is connected with allowance for the finite wavelength of the external field ($k_0 \neq 0$). Such low-frequency oscillations, unlike in the case considered in [3], do not correspond to natural low-frequency (ion-acoustic) oscillations of the plasma.

Let us compare the value of E_0, determined in (10),

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[1]If the inequality (12) is not satisfied, then it is necessary to take into account the contribution of the Cerenkov effect to $\delta\epsilon_\alpha(\omega, \mathbf{k})$. This allowance, however, is essential only in a narrow region of values E_0, near the threshold, in which we are not interested at present.

[2]Expression (15) for the increment coincides with the corresponding result of [7], obtained in the limit of external-field frequencies that are much larger than those limited by the inequality inverse to (11), and for which $v_E/v_{Te} \gg \omega_0/\omega_{Li}$.

with the threshold value of E_0 for stimulated Mandel'shtam-Brillouin scattering (SMBS)[3], which corresponds to the instability of the plasma relative to the decay of a transverse external wave into a transverse wave and an ion-acoustic wave. The quantity $E^2_{0,thr}$ for SMBS is smaller by approximately γ_s/ω_s (where γ_s is the linear damping decrement of the ion-acoustic oscillations) than the value given in (10). In an isothermal plasma, when $T_e \approx T_i$, the decrement is $\gamma_s \sim \omega_s$, and the process described in[3] is impossible (at any rate, in the weak-coupling approximation). The nonlinear interaction of the external wave with the plasma can then be determined by the development of the instability considered above.

We note finally that the foregoing neglect of the inhomogeneity of the plasma is permissible for excited-oscillation wavelengths smaller than the characteristic dimension l of the plasma inhomogeneity. Therefore, if it turns out that the characteristic wave number is $k_{max} < 1/l$, then the maximum value of the increment and the threshold intensity of the external field) are determined in order of magnitude by the values of $k \sim 1/l$.

In conclusion I am grateful to V. P. Silin for valuable advice and for directing the work, and also to Yu. M. Aliev, N. E. Andreev, L. M. Gorbunov, R. R. Ramazashvili, and A. A. Rukhadze for stimulating discussions.

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Translated by J. G. Adashko
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