EXCITATION OF TRANSVERSE WAVES IN INDIUM ANTIMONIDE IN AN EXTERNAL MAGNETIC FIELD BY A STRONG CURRENT

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It is shown that a strong current flowing through indium antimonide located in a strong magnetic field excites Cerenkov radiation of helicon waves. The magnitudes of the currents and fields necessary for excitation to take place are found. The strength of the self-field of the current is found to be much less than that of the external magnetic field. For a cylindrical sample, with a radius much smaller than its length, the values of the frequency and critical current are close to those found experimentally.

1. CERENKOV EXCITATION OF HELICON WAVES

In a number of experiments (see the reviews [1-3]), microwave radiation (frequency range 10^9-10^{11} Hz) has been observed from samples of indium antimonide placed in a strong electric and, generally speaking, a magnetic field. It was found that this radiation occurs for a magnetic field H = 0 or for $\mathbf{H} \parallel \mathbf{E}$ or for $\mathbf{H} \perp \mathbf{E}$ (E is the electric field). However, if the magnetic field is so strong that $H > c/\mu_-$ ($\mu_\mp$ is the mobility of electrons or holes and c is the speed of light), then, first, a lower constant current density j_0 is necessary for excitation of transverse waves than at H = 0, and second, the amplitude of the oscillations is much greater. Therefore, a real situation is presented in which, for different values and orientations of the magnetic field relative to the direction of the current, different mechanisms of excitation of the instability of transverse waves are important.

In this paper we limit ourselves to the case $\mathbf{H} \parallel \mathbf{j}_0$ and to values of the magnetic field $c/\mu_+ > H > c^2/\mu_-\mu_+$ (the case $H > c/\mu_+$ does not occur in experiment for indium antimonide). We consider samples of cylindrical shape with radium R much less than the length L. In correspondence with the experimental situation, we shall assume that the electric field is so strong that shock ionization leads to the equality of concentrations of electrons and holes: $n_- = n_+$. We note that the theory of this phenomenon was considered in [4-7]. However, in these papers, no physical mechanism was given for the appearance of growing transverse oscillations. Furthermore, the boundary conditions on the lateral surfaces of the sample were not taken into account. Finally, the magnetic field of the current that is necessary for excitation of the oscillations is greater than the external field of the current and it is necessary to take it into account.

We shall consider one of the possible reasons for excitation of transverse waves. In the presence of a magnetic field, of the "magnetizing" electron and the "nonmagnetizing" hole, weakly attenuated helicon waves exist in the crystal with frequency $\omega = ck\mathbf{k}\cdot\mathbf{H}/4\pi ne$ (here e is the charge on the electron and **k** is the wave vector). If the drift velocity of the carriers v_{dr} becomes greater than the velocity of helicon waves $c\mathbf{k}\cdot\mathbf{H}/4\pi ne$, then "Cerenkov" radiation of helicon waves takes place.[1)] This radiation is analogous to the well-known emission of sound when the drift velocity of the carriers is greater than the sound velocity. The condition $v_{dr} > c\mathbf{k}\cdot\mathbf{H}/4\pi ne$ or $j_0 > ckH/4\pi$ and the expression for the frequency differs from the condition obtained below, first, by the fact that in place of a wave vector **k** with all three components there is only the single component of **k** along the cylindrical axis in the obtained expression and, second, by a numerical factor less than unity. These differences are connected with the boundary conditions on the lateral surfaces of the cylinder and with the relations between the kinetic coefficients. Here the magnetic field of the current (the "self-field") H_s is less than the external field, by a factor greater than R/L; therefore, its effect can be neglected.

In spite of the fact that the resultant frequencies and currents lie in the range of experimental values, there is no complete assurance that the observed "Cerenkov" mechanism of excitation of helicon waves is responsible for microwave emission in indium antimonide. For an unambiguous determination of the reason for microwave emission, it would be necessary, on the one hand, to study the wave emission in the excitation of "hole" sound, and on the other, to consider the excitation of helicon waves for $\mathbf{H}_0 \perp \mathbf{j}_0$ and H = 0, and also to construct a nonlinear theory which one could compare with experiment. However, this takes us beyond the framework of the present paper.

2. SYSTEM OF EQUATIONS AND THE BOUNDARY CONDITIONS

The set of equations describing the problem inside the sample has the form

$$\partial\mathbf{H} / \partial t = -c \operatorname{rot} \mathbf{E}, \tag{1}$$

$$4\pi\mathbf{j} / c = \operatorname{rot} \mathbf{H}, \tag{2}$$

[1)]We note that in a number of cases (see, for example, [6]) the problem of "Cerenkov" radiation of hole sound has been considered. However, the excited oscillations are longitudinal. In the dispersion equation, the equations for transverse and longitudinal waves are divided by a small parameter $(v/c)^2$. Therefore, it was not previously evident that excitation of hole sound could lead to the radiation of electromagnetic waves.

$$\mathbf{E} = \eta \mathbf{j} + \eta_1 [\mathbf{jH}] + \eta_2 \mathbf{H}(\mathbf{jH}). \quad (3)^*$$

The kinetic coefficients η, η_1, η_2 are given for example, in [8]. For $\mu_+ H/c < 1 < \mu_- \mu_+ H^2/c^2$, they are equal to $\eta \approx -\eta_2 H^2 \approx \mu_+ H^2/nec^2$; $\eta_1 H = H/nec$. As is seen, the kinetic coefficients are determined only by the hole mobility. The holes do not heat up at not too high a concentration ($n < 3 \times 10^{15}$ cm^{-3}). Therefore, we can disregard the dependence of the kinetic coefficients on the field. Also, we shall not take into account the inhomogeneity of the concentration along the radius of the cylinder, due to the pinch effect, since calculation shows that this inhomogeneity is negligible. Finally, neglect of the displacement current and of the temporal dispersion is valid up to frequencies of 10^{12} Hz, which is much higher than the frequencies obtained below. Outside the sample, (2) changes to $\partial \mathbf{E}'/\partial t = c \operatorname{curl} \mathbf{H}'$. On the surface of the cylinder it is necessary that all three components of the magnetic field be continuous. Finally, the fields should vanish as $r \to \infty$ (where r is the distance from the axis of the cylinder).

We linearize (1)—(3) and introduce a cylindrical set of coordinates (r, φ, z) and set $\mathbf{E}'$, $\mathbf{H}'$, $\mathbf{j}' \sim f(r) \times \exp\{ikz + im\varphi - i\omega t\}$. We shall see below that $H_S = 2\pi j_0 r/c \ll H_0$. Therefore, we shall neglect H_S in (3). Furthermore, we shall neglect the constant current $j_{0\varphi}$ originating in the sample, since $j_{0\varphi}/j_{0z} \approx H_S/H_0 \ll 1$. We note that for linearization, one must take into account the dependence of η on H^2. Substituting (1) and (3) in (2), we get

$$i(\omega + ck\mathbf{j}\eta_1)\mathbf{H}' = \left(\frac{ic^2 \mathbf{kH}}{4\pi}\eta_1 + \eta_2 c\mathbf{jH}\right) \operatorname{rot} \mathbf{H}'$$
$$- c\eta_2 [\mathbf{H}\nabla](\mathbf{jH}') - \frac{c^2}{4\pi}\eta_2 [\mathbf{H}\nabla](\mathbf{H} \operatorname{rot} \mathbf{H}') - \frac{2\eta}{H^2} c [\mathbf{j}\nabla](\mathbf{HH}'). \quad (4)$$

We take the curl of (4) and substitute in it the value of $\operatorname{curl}_z \mathbf{H}'$ found from (4). Projecting the resultant expression on the z axis, we find

$$(\Delta + \varkappa_1^2)(\Delta + \varkappa_2^2) H_z' = 0, \quad (5)$$

where $\kappa_{1,2}^2$ are the roots of the following expression:

$$\left(\frac{c^2}{4\pi}\right)^2 \eta\eta_0 \varkappa^4 - \left[i\omega \frac{c^2}{4\pi}\eta - \frac{c^4 (\mathbf{kH})^2}{(4\pi)^2}\eta_1^2 + \frac{ic^3 \mathbf{kj}}{2\pi}\eta_0\eta_1\right]\varkappa^2$$
$$- \left(\tilde{\omega}^2 + i\omega \frac{c^2 k^2}{4\pi}\eta + \frac{4ic^3 \mathbf{kj}}{4\pi} k^2 \eta_0 \eta_1\right) = 0, \quad (6)$$

here $\eta_0 = \eta + \eta_2 H^2$, $\tilde{\omega} = \omega + c\eta_1 \mathbf{k} \cdot \mathbf{j}$.

The solution of (4), which is finite as $r \to 0$, has the form

$$H_z' = C_1 J_m(\sqrt{\varkappa_1^2 - k^2}\, r) + C_2 J_m(\sqrt{\varkappa_2^2 - k^2}\, r). \quad (7)$$

Taking the projections of (4) on the r and φ axes and replacing $\operatorname{curl}_z \mathbf{H}'$, we get a system for the determination of H_r' and H_φ' in terms of H_z'. Its solution has the form (with account of $\operatorname{div} \mathbf{H}' = 0$ and (7)):

$$\left\{\begin{matrix} H_r' \\ H_\varphi' \end{matrix}\right\} = (A_1^2 - k^2 A_2)^{-1} \left[\frac{c^2}{4\pi}\eta_2 H^2 (\varkappa^2\eta + i\tilde{\omega}) + A_2 A_3\right] \quad (8)$$
$$\times \left[\left\{\begin{matrix} ik \\ -A_1/A_2 \end{matrix}\right\} \frac{\partial H_z'}{\partial r} - \frac{m}{r}\left\{\begin{matrix} iA_1/A_2 \\ -k \end{matrix}\right\} H_z'\right],$$
$$A_1 = i\tilde{\omega} + \frac{c^2 \varkappa^2 \eta}{4\pi}, \quad A_2 = \frac{ic^2 \eta_1}{4\pi}\mathbf{kH} + \eta_2 c\mathbf{jH},$$
$$A_3 = \frac{ic^2 \eta_1}{4\pi}\mathbf{kH} + \eta_0 \frac{cj}{H}.$$

*$[\mathbf{jH}] \equiv \mathbf{j} \times \mathbf{H}$, $(\mathbf{jH}) \equiv \mathbf{j} \cdot \mathbf{H}$.

Outside the sample, the solution of the system leads to

$$H_z' \sim C_3 N_m\left(\sqrt{\frac{\omega^2}{c^2} - k^2}\, r\right) + C_4 J_m\left(\sqrt{\frac{\omega^2}{c^2} - k^2}\, r\right).$$

As we shall see below, $\omega < ck$, and therefore $J_m(\sqrt{\omega^2/c^2 - k^2}\, r) \to \infty$ as $r \to \infty$, which leads to $C_4 = 0$. Using $\operatorname{div} \mathbf{H}' = 0$, we find, finally,

$$H_r' = C_3 \frac{\partial}{\partial r} N_m\left(\sqrt{\frac{\omega^2}{c^2} - k^2}\, r\right), \quad H_\varphi' = iC_3 \frac{m}{r} N_m\left(\sqrt{\frac{\omega^2}{c^2} - k^2}\, r\right),$$
$$H_z' = -\frac{iC_3}{k}\left(\frac{\omega^2}{c^2} - k^2\right) N_m\left(\sqrt{\frac{\omega^2}{c^2} - k^2}\, r\right). \quad (9)$$

3. FREQUENCY AND CONDITION FOR EXCITATION OF TRANSVERSE WAVES

The boundary conditions for $r = R$ lead to a homogeneous system for the determination of C_1, C_2, and C_3 with coefficients determined by (7)—(9). The condition of the non-zero solution of this system gives the dispersion equation. It has the form of a sum of products of three Bessel and Neumann functions of the arguments

$$(\sqrt{\varkappa_1^2 - k^2}\, R), \quad (\sqrt{\varkappa_2^2 - k^2}\, R), \quad (\sqrt{\omega^2/c^2 - k^2}\, R),$$

and we shall not write it down, because of its complexity. The exact solution is possible only numerically; we shall solve it approximately. We write, as usual, $k = \pi p/L$, where $p = 1, 2, \ldots$. Then we find $kR = \pi Rp/L < 1$ for not too large p. (In practice, $L/R \approx 10$—30, so that $p < 3$—10.) Further, we take it into account that one of the roots of (6) is much larger than the other. Let $\kappa_1^2 \ll \kappa_2^2$, then $\kappa_2 R \gg 1$. Using the approximate expressions of the Bessel and Neumann functions for small and large arguments, we obtain, for $m \neq 0$,

$$J_m(\sqrt{\varkappa_1^2 - k^2}\, R) = {}^1\!/_2 B(kR); \quad (9')$$

here B is a number much less than unity. We shall be interested in the smallest root of (9′), since it is easy to see that even for $\sqrt{\kappa_1^2 - k^2}R \approx 3.8$ (i.e., close to the first non-zero root of J_1), the critical current is such that $2\pi j_0 R/cH > 1$ and our consideration is invalid. If $\sqrt{\kappa_1^2 - k^2}R \ll 1$, then, expanding the Bessel function in a series and substituting κ_1^2 from (6), we find the condition at which the frequency is real:

$$j_0 = j_{0cr} = \frac{ckH}{4\pi}\left(1 + \frac{3 + 2B}{B}\frac{\eta_0}{\eta}\right)^{-1}\sqrt{1 + B} \quad (10)$$

and the frequency itself is

$$\omega = \frac{ck^2 H}{4\pi ne}\frac{\sqrt{1 + B}(3 + 2B)}{3 + 2B + \eta/\eta_0}. \quad (11)$$

As is seen from (10), $2\pi j_0 R/cH < 1$, i.e., our neglect of H_S and $j_{0\varphi}$ is justified. It is also seen that the expression for the frequency is close to the corresponding one for helicon waves (in the unbounded specimen). It is also seen from (10) that the carrier drift velocity at which $\operatorname{Im} \omega > 0$, should be greater than the velocity of the helicon wave, in agreement with the qualitative picture of Sec. 1.

We estimate the critical current and the frequency in indium antimonide for the concentration $n \approx 10^{14}$ cm^{-3}, $\mu_- \approx 10^8$ absolute units, $\mu_-/\mu_+ \approx 36$, $L = 1$ cm and magnetic field $H_0 \approx 3 \times 10^3$ Oe. Here $j_{0cr} = 3 \times 10^3$ A/cm^2, $\omega \approx 2 \times 10^9$ Hz, which is close to the experimental data. Here $\omega/ck < 1$.

4. ON THE POSSIBILITY OF EXCITATION OF HELICON WAVES UNDER OTHER CONDITIONS

The calculation set forth makes it possible to draw a number of conclusions. If $\mu_-\mu_+H^2c^{-2} < 1 < \mu_-H/c$, then the kinetic coefficients depend not only on the hole mobility but also on the electron mobility. In this case, it is necessary to take into account the heating of the electrons. However, if $1 + \partial \ln \eta/\partial \ln E^2$ and $1 + \partial \ln \eta_2/\partial \ln E^2$ are close to unity, which is the case in indium antimonide, the result of the calculation is almost unchanged; therefore, the expressions (10) and (11) are true in the interval of values of the magnetic field $c/\mu_- < H < c/\mu_+$, If the magnetic field is so strong that $H > c/\mu_+$, i.e., the holes are "magnetized," then at equal carrier concentrations there are no helicon waves and Alfven waves are generated, the velocity of which is much higher. Therefore, the current necessary for their radiation is much greater. Similarly, if the carrier concentrations are strongly different, then, as calculation shows, the critical current increases in the ratio $(\eta/\eta_2 H^2)$, which is greater than unity. Therefore excitation should be expected in certain materials with high ratios of mobilities. Thus, in addition to indium antimonide, there is bismuth, where for L = 1 cm, $H_0 = 10^2$ Oe and $T \approx 4°K$, $j_{0\,cr} = 10^2$ A/cm^2 and $\omega \approx 3 \times 10^4$ Hz.

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