COOLING OF CARRIERS THAT SCATTER THEIR ENERGY ON OPTICAL-LATTICE VIBRATIONS

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If the mean free time for a current carrier with respect to optical phonon emission in a dielectric or high resistance semiconductor is much smaller than its energy relaxation time involved in quasielastic scattering mechanisms, then in electric fields of intermediate strength the isotropic component $f_0(\epsilon)$ of the quasi-isotropic distribution function becomes independent of the electric field, and the carrier mobility in the substance becomes ohmic. This "second" (in distinction to the equilibrium, or "first") ohmic section must necessarily precede the section of drift-velocity saturation in the low-temperature case ($\hbar\omega_0 \gg kT$), but it can exist also when there is no saturation. The form of function $f_0(\epsilon)$ in the "second" ohmic section is determined by the energy dependence of the mean free path $l(\epsilon)$ relative to elastic scattering. If the length is small at $\epsilon \approx 0$ and decreases sufficiently rapidly as $\epsilon \to 0$, then the current-carrier gas will be cooler in the low temperature case than in the equilibrium case.

1. INTRODUCTION

THE usual result of the theory of "hot" electrons in dielectrics and semiconductors is the increase of the total carrier energy in an electric field[1-3]. However, the increase of the total energy with increasing field is not obligatory, since all that is required to obtain a stationary distribution of the carriers is an increase of the energy flux transferred by the carriers to the lattice. In most cases this last condition leads indeed to a growth of the total energy. We shall show in this paper that in the case of predominant scattering of the carrier energy by generation of optical phonons, in the low temperature case, situations are possible in which the growth of the electric field leads to a lowering of the total energy of the carriers, and this lowering, in principle, can be "absolute," i.e., the carrier energy becomes smaller than the equilibrium value (3/2)kT. We note that this effect differs essentially from the effect considered in[4-8], where the electron temperature (or the "thermal" energy[7]) is reduced and the total carrier energy is simultaneously increased; in our case it is precisely the latter which is decreased.

Our problem differs in its formulation from most analogous problems (see[4-8]) in that we investigate the case of extremely low carrier densities, when their interaction can be neglected and an electron temperature cannot be introduced.

The main results obtained by us concern the case

$$e^{\eta_0} \gg 1 \tag{1}$$

($\eta_0 \equiv \epsilon_0/kT$ and $\epsilon_0 = \hbar\omega_0$ is the energy of the optical phonons, the dispersion of which is neglected), when the free path time relative to emission of an optical phonon τ_0 is much shorter than the free path time relative to absorption $\tau_0 \exp(\eta_0)$. In addition to these times, the problem is characterized by the free path time τ relative to elastic scattering, and the time of energy scattering by the quasi-elastic mechanisms $\tau_\epsilon = \tau\kappa$, where κ is the inelasticity factor ($\kappa \gg 1$). In the case when the time τ in the energy region $\epsilon < \epsilon_0$ (passive region in accordance with the terminology of[9]) is much shorter than the time $\tau_0 \exp(\eta_0)$, the carrier energy distribution is determined in this region by the isotropic component of the distribution function $F_0(\epsilon)$ defined by the equation

$$P(\varepsilon)\frac{dF_0(\varepsilon)}{d\varepsilon} + D(\varepsilon)F_0(\varepsilon) = -j_0(\varepsilon). \tag{2}$$

The left side of this equation yields (apart from the sign) the flux of carriers along the energy axis, which is connected with the field heating and with the quasi-elastic interaction with the lattice. Then,

$$P(\varepsilon) = {}^2/_3 e^2E^2\varepsilon v(\varepsilon)\tau(\varepsilon) + kTD(\varepsilon), \tag{3}$$

where $v(\epsilon) = \sqrt{2\epsilon/m}$, m is the effective carrier mass and is assumed to be constant and isotropic (τ is also assumed to be isotropic); the function $D(\epsilon)$ characterizes the quasi-elastic energy exchange with the lattice:

$$D(\varepsilon) = kTp(\varepsilon)\,/\,\tau_\varepsilon(\varepsilon) \tag{4}$$

(here $p(\epsilon) \equiv mv(\epsilon)$ is the carrier momentum).

The presence of optical scattering causes the flux indicated above to be different from zero:

$$j_0(\varepsilon) = \int_0^\varepsilon \Phi(\varepsilon')\left[(N_0+1)F_1(\varepsilon') - N_0F_0(\varepsilon')\right]d\varepsilon' \tag{5}$$

Here $F_1(\epsilon)$ is the isotropic component of the distribution function in the energy region $\epsilon_0 < \epsilon < 2\epsilon_0$ (active region in accordance with the terminology of[9]), $N_0 \equiv (\exp(\eta_0) - 1)^{-1}$ and

$$\Phi(\varepsilon) = p(\varepsilon)\,/\,\tau_0(\varepsilon), \tag{6}$$

where $1/\tau_0(\epsilon) \sim p(\epsilon + \epsilon_0)$ in the case of a deformation potential of carrier scattering by optical phonons, and

$$\frac{1}{\tau_0(\varepsilon)} \sim \frac{1}{p(\varepsilon)}\ln\frac{p(\varepsilon+\varepsilon_0)+p(\varepsilon)}{p(\varepsilon+\varepsilon_0)-p(\varepsilon)}$$

in the case of a polarization potential. In both cases, $\tau_0(\epsilon)$ tends to a finite value when $\epsilon \ll \epsilon_0$, so that $\Phi(\epsilon) \sim \sqrt{\epsilon}$.

2. "SECOND" OHMIC SECTION

The carrier flux $j_0(\epsilon)$, generally speaking, depends on the energy: the conserved quantity in the case of optical scattering in the generalized flux, equal to the sum of fluxes in all the energy intervals $(0, \epsilon_0)$, $(\epsilon_0, 2\epsilon_0)$, $(2\epsilon_0, 3\epsilon_0)$, etc. In the case when (1) is satisfied and the fields are sufficiently weak, the fluxes in all the intervals except the first two can be neglected because they are exponentially small; then

$$j_0(\varepsilon)+j_1(\varepsilon)=j\equiv \text{const.} \tag{7}$$

We shall henceforth be interested only in the situation in which

$$\tau_\varepsilon(\varepsilon_0)\gg\tau_0(0). \tag{8}$$

If the electric field is much weaker than a certain critical value, equal to

$$E_A=p(\varepsilon_0)\,/\,e\sqrt{\tau_0(0)\tau(\varepsilon_0)} \tag{9}$$

when $\tau_0(0) > \tau(\epsilon_0)$ and

$$E_B=p(\varepsilon_0)\,/\,e\tau_0(0) \tag{10}$$

when $\tau_0(0) < \tau(\epsilon_0)$, then the penetration of the carriers into the active region is limited to a narrow energy interval near ϵ_0, so that the flux $j_1(\epsilon)$ is comparable with j only near the bottom of the band.

In such fields, $F_0(\epsilon_0)$ is much smaller than $F_0(\epsilon)$ in almost the entire interval $(0, \epsilon_0)$, so that the Eq. (2) can be solved with the approximate boundary condition

$$F_0(\varepsilon_0)\approx 0. \tag{11}$$

This solution takes the form

$$F_0(\varepsilon)=\int_\varepsilon^{\varepsilon_0}\frac{d\varepsilon'\,j_0(\varepsilon')}{P(\varepsilon')}e^{I(\varepsilon')-I(\varepsilon)}, \tag{12}$$

where

$$I(\varepsilon)=\int_0^\varepsilon\frac{D(\varepsilon')}{P(\varepsilon')}d\varepsilon'. \tag{13}$$

Taking the foregoing into account, we can replace $j_0(\epsilon')$ by j almost everywhere in (12), and we can determine this constant from the normalization condition

$$\int_0^{\varepsilon_0}d\varepsilon\, g(\varepsilon)F_0(\varepsilon)=-\frac{2}{3}g_0\int_0^{\varepsilon_0}\varepsilon^{3/2}d\varepsilon\frac{dF_0}{d\varepsilon}=n_0, \tag{14}$$

where $g(\epsilon)\equiv g_0\sqrt{\epsilon}$ is the density of states along the ϵ axis.

Let us analyze expressions (12)—(14). When $E\to 0$ ($y(\epsilon)\equiv 2e^3E^2\epsilon v(\epsilon)\tau(\epsilon)/3kTD(\epsilon)\ll 1$ everywhere), the function $F_0(\epsilon)$, defined by formula (12), becomes Maxwellian almost everywhere (with the exception of the region $\sim kT$ near ϵ_0, owing to the inaccurate condition (11)). The average energy tends in this case to the equilibrium value $(3/2)kT$, and Ohm's law is satisfied. In sufficiently strong fields, such that

$$I(\varepsilon_0)\ll 1 \tag{15}$$

and

$$y(\varepsilon)\gg 1 \tag{16}$$

the function (12) for actual values of ϵ takes the form

$$F_0(\varepsilon)=C\int_\varepsilon^{\varepsilon_0}\frac{d\varepsilon'}{\varepsilon' v(\varepsilon')\tau(\varepsilon')}, \tag{17}$$

where the redefined (compared with (12)) normalization constant can be obtained from (14) and is independent of the field. For the same reason, the conductivity defined with the aid of the function (17) is likewise independent of the field. Thus, a "second" ohmic section should be observed on the current-voltage characteristics (the "first" section corresponds to $E\to 0$). The mobility in this section, if it is assumed that the change of $\tau(\epsilon)$ with energy is determined by the power-law function

$$\tau(\varepsilon)\sim\varepsilon^r, \tag{18}$$

can be obtained from

$$\mu=e\tau(\varepsilon_0)(1-r)\,/\,m, \tag{19}$$

and the average energy from the formula

$$\bar{\varepsilon}=3(1-r)\varepsilon_0\,/\,5(2-r). \tag{20}$$

We note that the mobility and the galvanomagnetic coefficients in the range of fields corresponding to the "second" ohmic section were calculated recently by Rabinovich[10]. He proposed that the indicated section should take place only in the case when

$$\tau\gg\tau_0 \tag{21}$$

and thus must precede the section of saturation of the drift velocity. In fact, as will be shown below, in order for it to appear, generally speaking, it is necessary to satisfy a much less stringent condition, so that this saturation need not necessarily occur in this section. The conditions for the existence and the length of this section can be obtained more accurately by comparing (15) and (16), which limit this section on the weak-field side, with the condition

$$E\ll E_A, \tag{22}$$

which limits it on the strong-field side at $\tau_0(0) > \tau(\epsilon_0)$, or else with the condition

$$E\ll p(\varepsilon_0)\,/\,e\tau(\varepsilon_0)(1-r) \tag{23}$$

at $\tau_0(0) < \tau(\epsilon_0)$. The field defined by the right side of the inequality (23) is the field at which saturation is reached (it can be calculated by taking into account the possible anisotropy of the distribution function in the kinetic equation, for example, by Baraff's method[11]).

Postponing to the next section the case $r \gtrsim 1$, when formulas (19), (20), and (23) are known not to be valid, let us consider the limits of the "second" ohmic section in the case when $\tau(\epsilon)$ and $\tau_\epsilon(\epsilon)$ are determined by the scattering from the deformation potential of the acoustic phonons: $r=-(1/2)$, $D(\epsilon)\sim\epsilon^2$, and $y(\epsilon)=y(\epsilon_0)\epsilon_0/\epsilon$. The transition to the "second" section from below is determined by the inequality (15) (since (16) is satisfied as soon as heating sets in, i.e., at $y(kT) > 1$). The inequality (15) yields

$$y(\varepsilon_0)\gg\eta_0\,/\,2$$

or

$$E^2\gg\frac{3m^2s^2\varepsilon_0}{e^2\tau^2(\varepsilon_0)kT}, \tag{24}$$

where s is the velocity of the longitudinal sound. Comparing (24) with the conditions (22) and (23), we can see that the "second" ohmic section takes place when

$$\tau(\varepsilon_0)\,/\,\tau_0(0)>3ms^2\,/\,2kT, \tag{25}$$

if $\tau_0(0) > \tau(\epsilon_0)$, and when

$$27ms^2/8kT < 1, \tag{26}$$

if $\tau_0(0) < \tau(\epsilon_0)$. Inasmuch as the right side of (25) is much smaller than unity at $T > 1°K$ for reasonable values of m, the criterion (25) is much less stringent than the criterion (21) used in[10].

We note also that for the existence of a "second" ohmic section the inequality (1) is not a limiting one. If a condition of the type (25) is satisfied, the "second" ohmic section exists for any relation between ϵ_0 and kT. If $kT \gg \epsilon_0$, then the mobilities on the "first" and "second" sections hardly differ from each other. But their difference can be quite appreciable already when $kT \approx \epsilon_0$.

3. "COOLING" OF CARRIERS

When $r > 1$ formulas (19), (20), and (21) become meaningless. We shall therefore consider this section in greater detail. Interest in this section is stimulated by the fact that in the scattering of carriers by a charged impurity, relation (18) for $\tau(\epsilon)$ with $r = 3/2$ holds approximately in a wide range of energies.

The analysis depends significantly on the relation between three numbers: $r + 3/2$, d, and 5/2, where d is the exponent in the relation $D(\epsilon) \sim \epsilon^d$. Since the case $d > 5/2$, which leads to interesting consequences, is so far only of academic interest (we do not know the corresponding mechanisms), and the case $r + 3/2 < 5/2$ was considered above, we shall focus our attention on the situation in which

$$r + {}^3/_2 > {}^5/_2 > d. \tag{27}$$

We assume that the fields are so strong that the condition (15) is satisfied, for which it is necessary to have

$$y(\varepsilon_0) \gg \eta_0 \frac{1-({}^3/_2 - d + r)\,[y(\varepsilon_0)]^{(d-r-{}^1/_2)/(d+r+{}^1/_2)}}{d-r-{}^1/_2}. \tag{28}$$

(When $d = r + 1/2$, this condition reduces to $y(\epsilon_0) \gg \eta_0 \ln y(\epsilon_0)$, and when $r + 1/2$ exceeds d by several units it reduces to $[y(\epsilon_0)]^{1/(d - r + 1/2)} \gg \eta_0$.) Satisfaction of (15) still does not imply satisfaction of (16), and therefore the distribution function is given by the formula

$$F_0(x) = C_1 \int_x^1 \frac{dx}{y(\varepsilon_0)\,x^{r+{}^3/_2} + x^d}, \tag{29}$$

where $x = \epsilon/\epsilon_0$, and the normalization constant has again been redesignated. (Formula (29) is obtained by neglecting the second term in the left side of (2); thus, carrier heating by the lattice is taken into account, but their cooling is omitted.)

Substituting (29) in the expression for the drift velocity, we have

$$\bar{v} = \frac{eE\tau(\varepsilon_0)}{m} \int_0^1 dx\, x^{r+{}^3/_2}\, dF_0/dx \Big/ \int_0^1 dx\, x^{{}^3/_2}\, dF_0/dx \tag{30}$$

where

$$\approx \frac{eE\tau(\varepsilon_0)\,(r+{}^3/_2-d)}{m\pi} \sin\left(\pi \frac{{}^5/_2-d}{r+{}^3/_2-d}\right) y^{[({}^5/_2-d)/(r+{}^3/_2-d)]-1} \sim y^\nu$$

$$\nu = \frac{{}^7/_2-(d+r)}{2(r+{}^3/_2-d)}, \tag{30'}$$

and y stands for $y(\epsilon_0)$. Since the denominator of ν, in accordance with (27), is positive, the sign of ν is governed by the numerator. In the case when $d + r = 7/2$ the drift velocity saturates in fields satisfying the condition (28). This case is realized approximately, in particular, energy scattering by the deformation potential of the acoustic phonons ($d = 2$) and in momentum scattering by the charged impurity ($r = 3/2$). In the latter case the saturated value of $\bar{v}$ is

$$\bar{v} = \frac{\tau(\varepsilon_0)}{m\pi} \left[\frac{3kTD(\varepsilon_0)}{2\varepsilon_0 v(\varepsilon_0)\tau(\varepsilon_0)}\right]^{1/2} = \frac{\sqrt{6}}{\pi} s, \tag{31}$$

i.e., it is practically independent of the temperature. (An estimate shows that the saturated value is reached after $\bar{v}$ passes through a maximum near $y(\epsilon_0) \sim \eta_0$.) If the piezopotential ($d = 1$) predominates in the energy scattering, and the ions predominate in the momentum scattering, as before, then $r + d = 5/2$ and the current increases in the investigated region in proportion to $\sqrt{E}$. Unfortunately, there are no simple examples for the case $d + r > 7/2$, when the current-voltage characteristic should be decreasing.

When (28) is satisfied, the average carrier energy is calculated differently, depending on whether $r > 2$ or $r < 2$. When $1 < r < 2$ (but is not too close to the limits of the interval) we have

$$\bar{\varepsilon} = \varepsilon_0 \frac{3(r+{}^3/_2-d)}{5\pi(2-r)} \sin\left(\pi \frac{{}^5/_2-d}{r+{}^3/_2-d}\right) y^{(1-r)/(r+{}^3/_2-d)}, \tag{32}$$

i.e., the average energy decreases with increasing field. When $r = 3/2$ and $d = 2$ we have

$$\bar{\varepsilon} = \frac{6}{5\pi} \frac{\varepsilon_0}{[y(\varepsilon_0)]^{1/2}} \sim E^{-1}; \tag{33}$$

and when $r = 3/2$ and $d = 1$

$$\bar{\varepsilon} = \frac{6\sqrt{2}}{5\pi} \frac{\varepsilon_0}{[y(\varepsilon_0)]^{1/4}} \sim E^{-1/2}.$$

Without stopping on the unrealistic situation $r > 2$, let us investigate in greater detail the average energy determined by formula (33). It exceeds $(3/2)kT$ so long as the field does not reach the value

$$E_1 \equiv \frac{4\sqrt{6}}{5\pi} \frac{ms}{e\tau(\varepsilon_0)} \eta_0. \tag{34}$$

(As above, it is assumed that the dependence $D \sim \epsilon^2$ is ensured by energy scattering in the interaction with the deformation potential of the acoustic phonons.) Since the field E_1 is much larger than the field with which satisfaction of condition (28) starts, we can conclude that $\epsilon = \epsilon(E)$ is a non-monotonic function. In weak fields ϵ increases with increasing field, and then goes through a maximum at $y(\epsilon_0) \approx \eta_0$, after which it decreases in accordance with (33). This decrease is limited in different manners, depending on the relation between $\tau_0(0)$ and $\tau(\epsilon_0)$, and also on the deviation of the real $\tau(\epsilon)$ dependence for scattering by a charged impurity from an idealized power-law. Ignoring the latter for the time being, let us consider the mechanisms of limitation in the idealized model. With increasing field, the region of "production" of the carriers near the bottom of the band, after emission of the optical phonons, broadens as the result of their deepening in the active region. This effect is taken into account by calculating the explicit dependence of $j_0(\epsilon)$ in the expression for $F_0(\epsilon)$ (see (12)). To calculate $j_0(\epsilon)$ it is necessary to find the function

$F_1(\epsilon)$. If (15) is satisfied, the latter is determined by the relation

$$\frac{d^2F_1}{du^2}-\frac{dF_1}{du}-\frac{3}{4}F_1=\frac{3}{4}\sqrt{\frac{\varepsilon_0}{\varepsilon}}\frac{\tau_0(0)}{\tau(\varepsilon_0)}F_1, \qquad (35)$$

where

$$u=\frac{1}{eE\varepsilon_0\tau(\varepsilon_0)}\int_0^\varepsilon d\varepsilon\, p(\varepsilon)=\frac{2p(\varepsilon_0)x^{3/2}}{3eE\tau(\varepsilon_0)}. \qquad (36)$$

This equation is obtained under the assumptions that 1) the energy scattering proceeds only via emission of an optical phonon, owing to satisfaction of (15); 2) the spilling of carriers from the passive region with absorption of optical phonons can be neglected; 3) besides the phonon emission, there is an appreciable elastic-scattering mechanism, given by $\tau \approx \tau(\epsilon_0)$; 4) the possible anisotropy of the distribution function can be stylized after Baraff[11]. When

$$x<(\tau_0(0)/\tau(\varepsilon_0))^2 \qquad (37)$$

the term in the right side of (35) is larger than the second and third terms on the left side. Since we are interested in values $x \lesssim \eta_0^{-2}$, the indicated two terms can be neglected if $\tau_0(0) > \tau(\epsilon_0)/\eta_0$. Retaining only that solution (35) (without the indicated terms) which decreases as $\epsilon \to \infty$, and discarding in expression (5) for $j_0(\epsilon)$ the term connected with the thermal spilling into the active region, we obtain in place of (29)

$$F_0(x)=C_1\int_x^1\frac{1-\alpha z^{-3/5}x^{3/4}K_{3/5}[4/5(x^{5/4}/z)]}{y(\varepsilon_0)x^{r+3/2}+x^d}dx, \qquad (38)$$

where

$$z=\left(\frac{4}{3}e^2E^2\frac{\tau(\varepsilon_0)\tau_0(0)}{p^2(\varepsilon_0)}\right)^{1/2},\quad \alpha=\frac{2}{\Gamma(3/5)}\left(\frac{2}{5}\right)^{3/5},$$

and $K_\nu(x)$ is the Macdonald function. Using the function (38) in the normalization condition (14), we can see that in weak fields the divergence of the normalization integral is cut off by the second term in the denominator, which expresses the heating of the carriers by the lattice, and in strong fields it is cut off by the presence of the subtrahend of the numerator, which expresses the nonlocality of the spilling.

Let us calculate the drift velocity and the average energy in the last case, neglecting the term x^d in the denominator. Integrating the normalization integral by parts, we get[12]

$$\bar v=\frac{2eE\tau(\varepsilon_0)(r-1)z^{4(r-1)/5}}{m\alpha(5/2)^{(7-4r)/5}\Gamma((5-2r)/5)\Gamma((7-2r)/5)}, \qquad (39)$$

with $1 < r < 5/2$ (when $r > 5/2$, the nonlocality of the spilling does not cut off the divergence of the normalization). When $r = 3/2$ we have $\bar v \sim E^{7/5}$, and the average energy is

$$\bar\varepsilon=\frac{3}{5}\varepsilon_0\frac{\Gamma(3/5)}{\Gamma(2/5)\Gamma(4/5)}\left(\frac{5}{2}z\right)^{2/5}\sim E^{2/5}. \qquad (40)$$

At a certain field E_2, the energy becomes equal to $(3/2)kT$. It is obvious that the condition for the existence of a region of "absolute" cooling ($\bar\epsilon < (3/2)kT$) is

$$E_2>E_1. \qquad (41)$$

From (41) there follows a stringent limitation on the temperature and on the parameters of the material

$$\frac{s}{v(\varepsilon_0)}\eta_0^{7/2}<20\sqrt{\frac{\tau(\varepsilon_0)}{\tau_0(0)}}. \qquad (42)$$

When $\tau_0(0) \ll \tau(\epsilon_0)/\eta_0$, the condition (42) must be replaced by the condition

$$\frac{s}{v(\varepsilon_0)}\eta_0^4<8\frac{\tau(\varepsilon_0)}{\tau_0(0)}, \qquad (42')$$

which is obtained if one neglects the right side of (35) (then the numerator under the integral sign in (38) is equal to $1 - e^{-u/2}$). Conditions (42) and (42′) are the results of the limitations imposed on the carrier cooling, because of their heating by the lattice (by absorption of acoustic phonons) and penetration into the active region. We shall consider below two other mechanisms that limit the cooling, of which the second is particularly important.

Needle-like Distribution in the Passive Region

When $\tau_0(0) < \tau(\epsilon_0)$, if the field E reaches a value $p(\epsilon_0)/e\tau(\epsilon_0)$, a needle-like distribution[13] becomes established in the passive region, but does not encompass the entire region, because of the strong scattering near $\epsilon = 0$. An exact calculation of the strong anisotropy of the distribution function is quite laborious; we therefore use the procedure of Baraff[11], who proposed to use in place of the usual method of cutting off the chain of Davydov equations

$$f_2=0 \qquad (43)$$

a different cutoff method

$$f_2={}^5/_3f_1. \qquad (44)$$

(Here f_1 and f_2 are the first and second spherical harmonics of the distribution function.) This method yields exact results in limiting cases of quasi-isotropic and needle-like distributions; in the intermediate case, a certain interpolation is obtained. In our case of large fields, when the condition (15) is satisfied and the cooling of the carriers by the quasi-elastic channels can be neglected, the equations determining the function $F_0(\epsilon)$ and the first spherical harmonic of the distribution function $\varphi_0(\epsilon)$ ($\varphi_0 \equiv f_1$ in the passive region) take the form

$$-\frac{2}{3}eE\varepsilon\varphi_0+kTD(\varepsilon)\frac{dF_0}{d\varepsilon}=-j_0(\varepsilon), \qquad (45)$$

$$eEv(\varepsilon)\tau(\varepsilon)\left[\frac{dF_0}{d\varepsilon}+\frac{2}{3\varepsilon^{3/2}}\frac{d}{d\varepsilon}(\varepsilon^{3/2}\varphi_0)\right]=-\varphi_0. \qquad (45')$$

The second term in the square brackets in (45′) is the consequence of the transition from the cutoff method (43) to Baraff's method (44). From (45) and (45′) we can obtain an equation determining $F_0(\epsilon)$:

$$P(\varepsilon)\frac{dF_0}{d\varepsilon}=-j_0\left(1+\frac{eEv(\varepsilon)\tau(\varepsilon)}{3\varepsilon}\right). \qquad (46)$$

(In the derivation of (46) we took into account the fact that the term with the second derivative $F_0(\epsilon)$ is small everywhere compared with the other terms of the equations, and we also took into account the insignificance of the second term in the round brackets in that region where the change of $j_0(\epsilon)$ with energy is appreciable. Cooling by the quasi-elastic mechanisms can then be taken into account by adding the term $D(\epsilon)F_0$ in the left side.)

Equation (46) must be solved with the boundary condition

$$F_0(\varepsilon_0) = {}^1/_3\varphi(\varepsilon_0), \quad (47)$$

where $\varphi_0(\epsilon_0)$ can be calculated from (45). The condition (47) goes over into the condition (11) when $\varphi_0(\epsilon_0) \ll F_0$, and can be obtained by comparing momentum-space carrier fluxes crossing the surface $\epsilon = \epsilon_0$. Solving (46) with condition (47) and calculating the drift velocity in analogy with (30), we get

$$\bar{v} = \frac{eE\tau(\varepsilon_0)}{my(\varepsilon_0)}\Big[\frac{3w(\varepsilon_0)}{y(\varepsilon_0)} + \pi\Big/\Big(r+\frac{3}{2}-d\Big)\sin\Big(\pi\frac{{}^5/_2-d}{r+{}^3/_2-d}\Big)[y(\varepsilon_0)]^{({}^5/_2-d)/(r+{}^3/_2-d)}\Big]^{-1} \quad (48)$$

where $w(\epsilon_0) \equiv 2eE\tau(\epsilon_0)/3p(\epsilon_0)$. For the average energy we obtain

$$\bar{\varepsilon} = \frac{3}{5}\frac{\varepsilon_0}{y(\varepsilon_0)K(\varepsilon_0)}\Big[\frac{1}{2-r} + \frac{5}{3}w(\varepsilon_0)\Big], \quad (49)$$

where $K(\epsilon_0)$ is the expression in the square brackets from (48). At $r = 3/2$ and $d = 2$, the drift velocity, calculated from formula (48), barely differs from that calculated from formula (30). On the other hand, the average energy in fields on the order of

$$E_3 = 9p(\varepsilon_0) / 5e\tau(\varepsilon_0) \quad (50)$$

saturates, and the saturated value, which equals

$$\varepsilon_s = \frac{2\sqrt{2}}{\pi\sqrt{3}}\varepsilon_0\frac{s}{v(\varepsilon_0)}, \quad (51)$$

is small compared with (3/2)kT practically at all times when the scattering of carriers with energy ϵ_0 by the acoustic phonons can be regarded as quasi-elastic. When $s/v(\epsilon_0) \approx 10^{-2}$ and $\epsilon_0 = 400°K$, the energy ϵ_s amounts to 2—3°K.

Thermal Spilling of Carriers Into the Active Region

When $\tau_0 \to 0$, the carrier distribution function in the active region $F_1(\epsilon)$ is determined entirely by the thermal spilling of the carriers from the passive region (with absorption of optical phonons), and is equal to

$$F_1(\varepsilon) = F_0(\varepsilon)e^{-\eta_0}. \quad (52)$$

If we neglect the quasi-elastic energy exchange with the lattice, then Eq. (7) with allowance for (52) yields

$$\frac{2}{3}e^2E^2v(\varepsilon_0)\tau(\varepsilon_0)(x^{r+{}^3/_2} + e^{-\eta_0})\frac{dF_0}{dx} = -j.$$

Determining from this $F_0(x)$ and calculating at $r = 3/2$ the average energy, we obtain

$$\bar{\varepsilon} = \frac{9}{5\pi}\varepsilon_0(e^{\eta_0/6} - 1)^{-1}. \quad (53)$$

This quantity is smaller than (3/2)kT at

$$\eta_0(e^{\eta_0/6} - 1)^{-1} < 2{,}5. \quad (53')$$

The criterion (53) has a meaning opposite to criterions (42) and (42′), and together with them establishes a narrow temperature region in which "absolute" cooling is possible via the proposed scattering mechanisms. According to (53) and (42), this region exists if

$$\frac{v(\varepsilon_0)}{s}\frac{\tau(\varepsilon_0)}{\tau_0(0)} > 800. \quad (54)$$

At typical values of the velocities ($s = 5 \times 10^5$ cm/sec and $v(\epsilon_0) = 3 \times 10^7$ cm/sec) it is necessary to have a very strong inequality of the relaxation time ($\tau_0(0) < \tau(\epsilon_0)/15$). The deviation of the "real" law of scattering by a charged impurity from a power-law (18) with $r = 3/2$ increases the right side of (54).

4. ALLOWANCE FOR THE "REAL" LAW OF CARRIER SCATTERING BY A CHARGED IMPURITY

As already mentioned in the introduction, the main results of the present article concern the case of small carrier concentrations, when it is impossible to introduce an electron temperature. An interaction of the carriers with one another, significant at least in the entire passive region, would lead to an additional flux of energy from this region into the active region, where it would be rapidly transferred to the lattice[9]. (At the same time, the interaction between the carriers, which is significant only at the bottom of the band in the energy region $\epsilon \ll \epsilon_0$, apparently does not affect qualitatively the conclusions of the work.)

On the other hand, to realize the cooling effect it is necessary that the impurity scattering predominate in the entire passive region up to $\epsilon = \epsilon_0$. These two requirements can be made compatible in compensated semiconductors (dielectrics), where the concentration of the ionized donors N_d is almost equal to the concentration of the ionized acceptors N_a and each of them greatly exceeds the concentration of the free carriers. The calculation of the exact $\tau(\epsilon)$ dependence for carrier scattering by the indicated donors and acceptors encounters serious difficulties (see the review[14]), and in approximate calculations taken from plasma theory[15,16], a concept wherein "multiple collisions" are regarded as pair collisions, predominates; this concept leads in the case of singly-charged centers to the formula (see[17])

$$\frac{1}{\tau(\varepsilon)} = \frac{\pi e^4 n_1}{m^{1/2}\varkappa^2(2\varepsilon)^{3/2}}\Big[\ln(1+b) - \frac{b}{1+b}\Big],$$
$$b = \frac{8\pi m\varkappa kT}{e^2h^2n_2}\varepsilon = \alpha x, \quad (55)$$

where κ is the dielectric constant, n_1 is the concentration of the scattering centers, and n_2 is the carrier concentration; in the case when carriers of one polarity predominate we have

$$n_1 = N_a + N_d, \quad n_2 = |N_a - N_d|. \quad (56)$$

According to (55), $\tau(\epsilon)$ decreases more slowly with increasing ϵ than in accordance with (18) at $r = 3/2$, and near $\alpha x \sim 1$ it reaches a minimum value. Both these singularities are typical of scattering by an impurity, regardless of the chosen model for the cutoff of the Coulomb potential, so that there appears one more limitation of the cooling level, connected with the real $\tau(\epsilon)$ dependence. In the case when (55) is valid, the minimum value is

$$\bar{\varepsilon} = \frac{3}{5}\varepsilon_0\frac{\ln(1+\alpha) - 3(1 - \alpha^{-1/2}\operatorname{arctg}\alpha^{1/2})}{\alpha^{1/2}\operatorname{arctg}\alpha^{1/2} - \ln(1+\alpha)}. \quad (57)$$

When $\epsilon_0/k = 400°K$, $T = 70°K$, $\kappa = 20$, and $m = 10^{-28}$ we get $\alpha \approx 2.5 \times 10^{18}/n_2$, and since we are interested in carrier densities $n_2 < 10^{14}$ cm^{-3}, $\bar{\epsilon}/k < 10°K$, so that the cooling-limitation mechanisms considered above are more effective. Inasmuch as to ensure predominance of impurity scattering it is necessary to have $n_i \gtrsim 10^{16}$ cm^{-3}, the Debye radius greatly exceeds the distance between impurities (so that the number of ions in the Debye sphere amounts to several thousand), and it can be assumed that the upper scattering radius in (55) is overestimated. The lower limit of this quantity was apparently used in the well known formula of Connwell-Weiskopf (see[14]), which leads to a formula for $\bar{\epsilon}$ which is quite similar to (57) at large α, but differs in having an essentially different value of α:

$$\alpha = \varepsilon_0 \varkappa / e^2 n_i^{1/3}; \qquad (58)$$

at $n_i = 10^{16}$ cm^{-3} and at the previous values of ϵ_0 and κ, formula (58) yields only $\alpha = 20$, so that the minimum value of $\bar{\epsilon}$ rises to 40–50°K and the given limitation mechanism becomes significant.

When the energy reaches its limiting value (53) or (57) (essentially, the larger of the two) with increasing field, the drift velocity increases linearly with the field (regardless of whether $\bar{\epsilon}$ is larger than or smaller than (3/2)kT). If the quantity given by formula (53) is larger than that given by (57), then

$$\bar{v} \approx \frac{e\tau(\varepsilon_0)}{2m}\left(e^{\eta_0/6} - 1\right)^{-1}. \qquad (59)$$

The result for the opposite case is given in[10]. Thus, "saturation" of $\bar{\epsilon}$ signifies the reaching of the second "ohmic" section which in impurity scattering is characterized by low mobility.

In conclusion we note that effects analogous to those considered here should take place also for other mechanisms of essentially inelastic scattering, namely intervalley scattering with emission of a phonon, excitation or ionization of an impurity center, excitation of a quasilocal or local phonon, etc.

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