ENERGY DISTRIBUTION OF PHOTOELECTRONS IN A QUANTIZED MAGNETIC FIELD AND GUREVICH-FIRSOV TYPE PHOTOMAGNETIC OSCILLATIONS

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We consider the relaxation of non-equilibrium electrons produced by light in a semiconductor when there is a quantized magnetic field present. The kinetic equation for the energy distribution function of the photoelectrons that interact with the optical phonons and the equilibrium electrons is reduced to a set of algebraic equations with shifted arguments. The quantity η, which is the ratio of the Fermi energy (or temperature for non-degenerate electrons) of the equilibrium electrons to the Larmor frequency $\hbar\Omega$ (ultra-quantal case), is then the small parameter. We find an exact solution of the algebraic set of equations for the distribution function with small quantum numbers. We calculate the characteristic relaxation time in a magnetic field when there are electron-electron interactions. We show that electrons with energies less than $h\Omega$ do not at all suffer Coulomb relaxation.

We use the solutions obtained to analyze Gurevich-Firsov type photomagnetic oscillations which were experimentally observed in a paper by Shalyt and coworkers[7] in the ultra-quantal case. We show that the oscillations are caused by the interaction between the photoelectrons and the optical phonons and, at the same time, the equilibrium electrons. (The oscillations do not occur in a pure semiconductor.) The depth of the oscillations may reach a magnitude of order unity. We show the condition imposed upon the concentration of the equilibrium electrons in a magnetic field for which the depth of the oscillations reaches a maximum.

FOR a number of problems it is necessary to know the distribution function of the non-equilibrium electrons originating under the influence of light in the conduction band of a semiconductor. Apparently the first solution of the kinetic equation for photoelectrons interacting with acoustic phonons was given by Landau and Lifshitz[1] (see also[2]). After that the distribution functions of photoelectrons interacting with optical phonons[3,4], equilibrium electrons[5], and with acoustic phonons in a quantized magnetic field[6] have been found.

In the present paper we obtain a solution of the kinetic equation for the photoelectrons in a quantized magnetic field, which are interacting simultaneously with the equilibrium electrons and with optical phonons. In the limiting case of large magnetic fields, when the distance $\hbar\Omega$ between Landau levels exceeds the Fermi energy $\epsilon_F = \hbar\Omega/2$ and the temperature T of the equilibrium electrons, it is possible to reduce the kinetic integral equation to a set of algebraic equations with shifted arguments and to find a solution for small quantum numbers.

The solution obtained is of particular interest in connection with the well-known experiments by Shalyt and coworkers[7] on Gurevich-Firsov (G-F) type photomagnetic oscillations. In those experiments G-F oscillations of the photomagnetic effect were observed at helium temperatures in impurity semiconductors InSb and InAs while the normal G-F resistivity oscillations occur only when $T \sim 100°K$ and in rather pure semiconductors.

Qualitative interpretations were considered in[8,9]. The effect observed is connected with the periodic changes in the probability for the emission of optical phonons by the photoelectrons when the magnetic field increases. However, it was shown in[8] that the photoelectron distribution function vanishes every time when the probability for a resonance emission of optical phonons (and hence the departure of the photoelectrons) is a maximum (i.e., when $\omega_0 = N\Omega$) and exactly compensates the oscillating probability. It was therefore assumed that the interaction with the equilibrium electrons smoothes out the photoelectron distribution function. To verify this idea it is necessary to know the distribution function of the photoelectrons which interact simultaneously with the optical phonons and the electrons. The solution obtained in the present paper enabled us to show that the interaction with the equilibrium electrons leads to G-F oscillations and to evaluate explicitly the oscillating term.

In the following we assume a quadratic dispersion law for the electrons with an effective mass m; we neglect the electron spin. The energy is measured in units $\hbar\Omega$, the momentum in units $\hbar/\Lambda$, length in units $\Lambda = [\hbar c/eH]^{1/2}$.

1. COULOMB RELAXATION FOR NON-EQUILIBRIUM ELECTRONS IN A SEMICONDUCTOR IN A QUANTIZED MAGNETIC FIELD

1. We shall assume that the change in the electron concentration Δn_e in the conduction band caused by the action of a light source is small compared with the concentration n_0 of the equilibrium electrons. We write the electron distribution function in the form

$$\rho_n(p) = f_n^0(p) + f_n(p), \tag{1}$$

where $f_n^0(p)$ is the equilibrium distribution function, $f_n(p)$ the non-equilibrium correction due to the action of the source $J_n(p)$, n the magnetic quantum number, and p the electron momentum component in the direction of the magnetic field.

The function $f_n(p)$ satisfies the equation

$$\left(\frac{\partial f_n(p)}{\partial t}\right)_{ee}+\left(\frac{\partial f_n(p)}{\partial t}\right)_{\text{ph}}-\frac{f_n(p)}{\tau^R{}_n(p)}=-J_n(p). \quad (2)$$

The first and second terms are the collision integrals of the non-equilibrium electrons with, respectively, equilibrium electrons and phonons. The third term takes recombination into account.

In the present section we consider the relaxation of the photoelectrons due only to the Coulomb interaction, dropping the term $(\partial f_n(p)/\partial t)_{\text{ph}}$.

The electron-electron collision integral for the function $f_n(p)$ has in a magnetic field[10] the form

$$\left(\frac{\partial f_n(p)}{\partial t}\right)_{ee}=\sum_{n'n_1n_1'}\int dp_1dk_zW^{nn'}_{n_1n_1'}(k_z,p)\,\delta(\Delta^{nn'}_{n_1n_1'}+k_z(p-p_1)-k_z^2)$$
$$\times\{f_{n'}(p-k_z)f^0_{n_1'}(p_1+k_z)+f_{n_1'}(p_1+k_z)f_{n'}{}^0(p-k_z)-$$
$$-f_n(p)f_{n_1}{}^0(p_1)-f_{n_1}(p_1)f_n{}^0(p)\},$$
$$W^{nn'}_{n_1n_1'}(k_z,p)=\frac{e^4}{\pi}\int_0^\infty\frac{dk_\perp{}^2F_{nn'}(k_\perp{}^2)\,F_{n_1n_1'}(k_\perp{}^2)}{(k_\perp{}^2+k_z{}^2)^2\,|\varepsilon(n'-n-k_zp+{}^1/_2\,k_z{}^2,\ \mathbf{k})|^2},$$
$$F_{nn'}(x)=\frac{(\bar n!)^2}{n'!n!}e^{-x/2}\left|L_{\bar n}^{|n-n'|}\left(\frac{x}{2}\right)\right|^2\left(\frac{x}{2}\right)^{|n-n'|},$$
$$\bar n=\min(n,n'),\quad L_n{}^k(x)\ \text{— Laguerre polynomial}$$
$$\varepsilon(\omega,\mathbf{k})=$$
$$=\varkappa\left[1-\frac{e^2}{\varkappa k^2\pi}\sum_{nn'}F_{nn'}(k_\perp{}^2)\int dp_1\frac{f_{n'}{}^0(p_1+k_z)-f_n{}^0(p_1)}{n'-n+k_z^2/2+k_zp_1-\omega-i\Delta}\right] \quad (3)$$

is the dielectric constant, $i\Delta\to 0$,

$$\Delta^{nn'}_{n_1n_1'}=n+n_1-n'-n_1'.$$

In the collision integral (3) we dropped factors such as $[1-f^0_n(p)]$ as one can show that when $\epsilon_F-{}^1\!/_2<0$ they are unimportant. The usual method for solving the equation consists in reducing it to a Fokker-Planck type differential equation. In the quantal case when small n are important, one can not use such an approximation for the collision integral (3) as scattering occurs with a large momentum transfer $k_z\sim 1$, and energy transfer $\Delta\epsilon\sim 1$. All the same, one can appreciably simplify Eq. (2) in the case of a very strong magnetic field and reduce it to a set of algebraic equations with shifted arguments. This turns out to be possible because of the conditions imposed by the conservation laws in a magnetic field and the conditions T, $(\epsilon_F-{}^1\!/_2)<1$ which mean that the equilibrium electrons are concentrated near p = 0. We note that the condition $\epsilon_F-{}^1\!/_2<1$ is not stringent. For example, in degenerate InSb with $n_0 = 10^{15}\ \text{cm}^{-3}$, $H>10^4$ Oe

$$\xi=\varepsilon_F-{}^1/_2=2\pi^4n_0{}^2,\quad \xi<0.06.$$

2. We first of all turn our attention to the absence of relaxation due to Coulomb interaction if the electrons are in the lowest Landau level with n = 0 and energy $\epsilon<1$. We can check this by putting in the collision integral $\rho_n(p)=\delta_{n0}\rho(p)$, where

$$\delta_{nn'}=\begin{cases}1, & n=n'\\ 0, & n\neq n'\end{cases}.$$

The collision integral then vanishes identically, independent of the form of $\rho(p)$ since as a result of a "one-dimensional collision" the electrons are scattered over a zero angle or exchange momenta according to the conservation law

$$p'=\begin{cases}p\\ p_1\end{cases},\quad p_1'=\begin{cases}p_1\\ p\end{cases},$$

where p, p_1; p', p_1' are the initial and final electron momentum components. In this connection we must note that the sometimes used approximation of an effective electron temperature[9,11] which is based upon the smallness of the electron-electron relaxation time may turn out to be invalid in the case of a strong magnetic field.

3. We transform Eq. (2) for small ξ and T. We then have for the distribution function of the equilibrium electrons

$$f_n{}^0(p)=\delta_{n0}f^0(p),\quad f^0(p)=\left[\exp\frac{p^2/2-\xi}{T}+1\right]^{-1}, \quad (4)$$

where ξ is determined by the normalization condition

$$F_{-1/2}\left(\frac{\xi}{T}\right)=\frac{4\pi^2n_0}{\sqrt{2T}},\quad F_n(z)=\int_0^\infty\frac{dx\,x^n}{e^{x-z}+1}. \quad (5)$$

It is convenient for what follows to denote a characteristic energy of the equilibrium electrons by η where $\eta\approx\xi$ for degenerate electrons and $\eta\approx T$ for non-degenerate electrons.

We use the δ-function to integrate in (3) over k_z. We can split all terms occurring in (3) into two groups. In the terms corresponding to transitions with large k_z, which occur when $\Delta^{nn'}_{n_1n_1}\neq 0$, we can set the initial momentum of the electron equal to zero. We get then instead of integrals over $f_n(p)$ functions $f_n(p')$ with shifted values of the argument, $p' = p + \Delta/p$. In the second group for which $\Delta^{nn'}_{n_1n_1'}=0$, k_z can be small and the integral terms remain.

Equation (2) becomes

$$f_n(p)\left[\frac{1}{\bar\tau_n{}^{ee}(p)}+\frac{1}{\tau_n{}^R(p)}\right]=J_n(p)+\sum_{n'n_1}{}'\frac{4\pi^2n_0}{|p|}\omega^{nn'}_{n_10}(p,p)f_{n'}\left(p+\frac{\Delta^{nn'}_{n_10}}{p}\right)$$
$$+\sum_{n'>n}f_{n'}(p)\int dp_1\frac{f^0(p_1)}{|p-p_1|}\omega^{nn'}_{n'-n0}(p-p_1,p)+\sum_{n'\geqslant n}f^0(p)\int dp_1\frac{f_{n'}(p_1)}{|p-p_1|}$$
$$\times\omega^{n0}_{n'-nn'}(p-p_1,p)-\delta_{n0}f^0(p)\sum_{n'n_1n_1'}\int dp_1dk_zf_{n_1}(p_1)\,W^{0n'}_{n_1n_1'}(k_z,p)$$
$$\times\delta(\Delta^{0n'}_{n_1n_1'}+k_z(p-p_1)-k_z{}^2), \quad (6)$$
$$\omega^{nn'}_{n_1n_1'}(p',p)=W^{nn'}_{n_1n_1'}\left(-\frac{\Delta^{nn'}_{n_1n_1'}}{p'},\ p\right)+W^{nn_1'}_{n_1n'}(p',p),$$
$$\frac{1}{\bar\tau_n{}^{ee}(p)}=4\pi^2n_0\sum_{\substack{n'n_1\\ +,-}}{}'\frac{W^{nn'}_{0n_1}(p/2\pm\sqrt{p^2/4+\Delta^{nn'}_{0n_1}},\ p)}{2\sqrt{p^2/4+\Delta^{nn'}_{0n_1}}}$$
$$+\sum_{n>n'\geqslant 1}\int\frac{dp_1}{|p-p_1|}f^0(p_1)\,[W^{nn'}_{0n-n'}(p-p_1,p)+W^{nn'}_{0n-n'}(0,p)]$$
$$+(1-\delta_{n0})\int\frac{dp_1f^0(p_1)}{|p-p_1|}\omega^{n0}_{0n}(p-p_1,p). \quad (7)$$

The prime on the summation sign indicates that the summation must be taken over all indices for which $\Delta^{nn'}_{n_1n_1'}\neq 0$.

4. It is clear from Eq. (6) that in front of the integral terms we have functions $f^0(p)$ which vanish except near p = 0.

This is connected with the fact that for transitions with $\Delta^{nn'}_{n_1n_1'}=0$ the final values of the photoelectron momentum p' are equal to the initial value of the equilibrium electron momentum. Such transitions therefore produce a function near p = 0 almost repeating $f^0(p)$. We can thus look for a solution of the set (6) in the form

$$f_n(p) = \bar{f}_n(p) + f^0(p) A_n(p), \tag{8}$$

where $A_n(p)$ is a function which is smooth near p = 0. The functions $\bar{f}_n(p)$ and $A_n(p)$ satisfy the set of equations

$$\bar{f}_n(p)\left[\frac{1}{\bar{\tau}_n{}^{ee}(p)}+\frac{1}{\tau^R{}_n(p)}\right]=\sum_{n'n_1}{}' \frac{4\pi^2 n_0}{|p|}\,\omega_{n_10}^{nn'}(p,p) f_{n'}\left(p+\frac{\Delta_{n_10}^{nn'}}{p}\right)$$
$$+J_n(p)+\sum_{n'>n}\bar{f}_{n'}(p)\int\frac{dp_1}{|p-p_1|}f^0(p_1)\,\omega_{n'-n0}^{nn'}(p-p_1,p), \tag{9}$$

$$A_n(p)\left[\frac{1}{\tau_n{}^{ee}(p)}+\frac{1}{\tau_n{}^R(p)}\right]=\sum_{n'>n}A_{n'}(p)\int\frac{dp_1}{|p-p_1|}f^0(p_1)$$
$$\cdot\left[\omega_{n'-n0}^{nn'}(p-p_1,p)+\omega_{n'-nn'}^{n0}(p-p_1,p)\right]$$
$$+\sum_{n'\geqslant n}\int\frac{dp_1}{|p-p_1|}\bar{f}_{n'}(p_1)\,\omega_{n'-nn'}^{n0}(p-p_1,p),\quad n\neq 0,$$

$$A_0(p)=-\frac{\tau_0{}^R(p)}{4\pi^2 n_0}\left[\sum_n\int dp_1\frac{\bar{f}_n(p_1)}{\bar{\tau}_n{}^{ee}(p_1)}+\sum_{n\geqslant 1}\int dp_1\frac{A_n(p_1)f^0(p_1)}{\tau_n{}^{ee}(p_1)}\right], \tag{10}$$

where

$$\frac{1}{\tau_n{}^{ee}(p)}=\frac{1}{\bar{\tau}_n{}^{ee}(p)}-\int\frac{dp_1}{|p-p_1|}f^0(p_1)\,\omega_{0n}{}^{n0}(p-p_1,p). \tag{11}$$

The integral equation (6) is thus reduced to the set of algebraic equations (9), (10) with shifted arguments.

5. Of most interest is the solution for small quantum numbers as for $n \gg 1$ the quasi-classical approach is valid. In what follows we restrict ourselves to considering the case when the photoelectrons are produced with energies $\epsilon_n(p) < 7/2$, i.e., occupy three Landau levels. In that case

$$f_n(p)=0 \quad\text{for}\quad \varepsilon_n(p)=n+\frac{1}{2}+\frac{p^2}{2}>\varepsilon_0,$$

where $\epsilon_0 < 7/2$ is the maximum energy of the photoelectrons produced. The indices n and n′ in Eqs. (9), (10) take on the values from zero to two. The arguments of the functions $f_{n'}(p')$ on the right-hand side of (9) for $\bar{f}_2(p)$ take such values that $\epsilon_{n'}(p') > \epsilon_0$. All sums in (9) therefore vanish as

$$\bar{f}_2(p) = \bar{\tau}_2{}^{ee}(p) J_2(p). \tag{12}$$

Similarly we get from (10)

$$A_2(p)=\tau_2{}^{ee}(p)\int\frac{dp_1}{|p-p_1|}\omega_{02}{}^{20}(p-p_1,p)\,\bar{\tau}_2{}^{ee}(p_1)J_2(p_1). \tag{13}$$

(We neglect here recombinations.) Apart from known terms $\bar{f}_0(p + 1/p)$ occurs in the equation for $\bar{f}_1(p)$. As the minimum of the expression $|p + 1/p|$ equals 2, to find $\bar{f}_1(p)$ it is necessary to know $\bar{f}_0(p)$ for $|p| > 2$. For such p we find

$$\bar{f}_0(p)=\bar{\tau}_0{}^{ee}(p)\left[J_0(p)+\frac{4\pi^2n_0}{|p|}\omega_{00}{}^{01}(p,p)\bar{f}_1\left(p-\frac{1}{p}\right)\right.$$
$$\left.+\frac{4\pi^2n_0}{|p|}\omega_{00}{}^{02}(p,p)\bar{f}_2\left(p-\frac{2}{p}\right)\right]. \tag{14}$$

Substituting (14) into (9) we get for $\bar{f}_1(p)$ an equation in which the function

$$\bar{f}_1\left(\frac{p^2+1}{p}-\frac{p}{p^2+1}\right).$$

occurs.

The solution of this equation can be constructed by splitting the range of |p| values from zero to $\sqrt{[2(\epsilon_0 - 3/2)]}$ into regions in each of which the term with the shifted argument is determined in terms of the value of the function in the preceding region, which lies at a higher energy. If we restrict ourselves to $\epsilon_0 < 5/2 + 1/8$ we do not need this division as the two last terms in (14) occur with values $\epsilon_1(p - 1/p) > \epsilon_0$ and $\epsilon_2(p - 2/p) > \epsilon_0$ and thus vanish. We find thus from (14)

$$\bar{f}_0\left(p+\frac{1}{p}\right)=\bar{\tau}_0{}^{ee}\left(p+\frac{1}{p}\right)J_0\left(p+\frac{1}{p}\right).$$

Finally we get

$$\bar{f}_1(p)=\bar{\tau}_1{}^{ee}(p)\left[J_1(p)+\frac{4\pi^2n_0}{|p|}\omega_{00}{}^{10}(p,p)J_0\left(p+\frac{1}{p}\right)\bar{\tau}_0{}^{ee}\left(p+\frac{1}{p}\right)\right.$$
$$\left.+\frac{4\pi^2n_0}{|p|}\omega_{00}{}^{12}(p,p)f_2\left(p-\frac{1}{p}\right)+\bar{f}_2(p)\int\frac{dp_1}{|p-p_1|}f^0(p_1)\,\omega_{10}{}^{12}(p-p_1,p)\right],$$

$$A_1(p)=\tau_1{}^{ee}(p)\left[2A_2(p)\int\frac{dp_1}{|p-p_1|}f^0(p_1)\,\omega_{10}{}^{12}(p-p_1,p)\right. \tag{15}$$
$$\left.+\sum_{n'=1}^{2}\int\frac{dp_1}{|p-p_1|}\bar{f}_{n'}(p_1)\,\omega_{n'-1n'}^{10}(p-p_1,p)\right],$$

$$\bar{f}_0(p)=\frac{\bar{\tau}_0{}^{ee}(p)\tau_0{}^R(p)}{\bar{\tau}_0{}^{ee}(p)+\tau_0{}^R(p)}\left[J_0(p)+\frac{4\pi^2n_0}{|p|}\omega_{1\,0}^{\,00}(p,p)\,\bar{\tau}_0{}^{ee}\left(p+\frac{1}{p}\right)\right.$$
$$\times J_0\left(p+\frac{1}{p}\right)+\sum_{n_1}{}'\sum_{n'=1}^{2}\frac{4\pi^2n_0}{|p|}\omega_{n_10}^{0n'}(p,p)f_{n'}\left(p+\frac{n_1-n'}{p}\right)$$
$$\left.+\sum_{n'=1}^{2}\bar{f}_{n'}(p)\int\frac{dp_1}{|p-p_1|}f_0(p_1)\,\omega_{n'0}^{0n'}(p-p_1,p)\right].$$

By substituting (12) to (15) into (10) we find the function $A_0(p)$.

6. The times $\bar{\tau}_n^{ee}(p)$ and $\tau_n^{ee}(p)$ determine the relaxation of photoelectrons with quantum number n and momenta, respectively, $p \gtrsim \sqrt{(2\eta)}$, $p \lesssim \sqrt{(2\eta)}$. We get the expression for $1/\tau_n^{ee}(p)$ by subtracting from $1/\bar{\tau}_n^{ee}(p)$ the probabilities for photoelectron scattering without changing the quantum number n and also subtracting the "arrival" of an equilibrium electron with level n = 0 without change of momentum which compensates the corresponding "departure":

$$\frac{1}{\tau_n{}^{ee}(p)}=4\pi^2n_0\sum_{\substack{n'n_1\\+,-}}\frac{W_{0n_1}^{nn'}\left(p/2\pm\sqrt{p^2/4+\Delta_{0n_1}^{nn'}}\,p\right)}{2\sqrt{p^2/4+\Delta_{0n_1}^{nn'}}}$$
$$+\sum_{1\leqslant n'<n}\int\frac{dp_1f^0(p_1)}{|p-p_1|}\left[W_{0n-n'}^{\;nn'}(p-p_1,p)+W_{0n-n'}^{\;nn'}(0,p)\right]. \tag{16}$$

The relaxation times $\bar{\tau}^{ee}$ and τ^{ee} can be evaluated if we assume that the Debye screening radius is larger than Λ. In that case we can neglect the dispersion of the dielectric constant in (7) and (16) for transitions with $k_z \sim 1$, and put $\epsilon(\omega, \mathbf{k}) = \kappa$. The matrix elements $W_{n_1n_1'}^{nn'}$ can then be evaluated by writing the Laguerre polynomials in series and can be expressed in terms of the exponential integral function. We shall not give here the corresponding formulae. The analysis shows that the main transitions are those in which the quantum numbers n either do not change at all or change for only one electron. Physically this is connected with the fact that the matrix element for the Coulomb interaction is small for large changes in momentum. In $\bar{\tau}_n^{ee}(p)$ we must thus leave terms with $\Delta = \pm 1$, $n' = n \pm 1$; $\Delta = 0$, $n' = n$. For small values of the momentum, transitions without changes in n dominate so that

$$\frac{1}{\bar{\tau}_n^{ee}(p)} \approx 4\pi^2 n_0 \frac{W_{00}^{nn}(p)}{|p|}, \quad W_{00}^{nn}(p) \approx \frac{e^4}{\pi\varkappa^2}\frac{1}{|p|^2}, \quad |p| < 1. \quad (17)$$

We find $\tau_n^{ee}(p)$ which we clearly need only for $p \lesssim \sqrt{(2\eta)}$. In that case the first sum in (16) turns out to be smaller by a factor $\sqrt{\eta}$ than the second term which describes transitions with small changes in k_z so that we get instead of (16)

$$\frac{1}{\tau_n^{ee}(p)} \approx \int \frac{dp_1}{|p-p_1|} f^0(p_1) W_{01}^{nn-1}(p-p_1, p), \quad p \leqslant \sqrt{2\eta}. \quad (18)$$

When evaluating the integral in (18) it is necessary to take into account the dispersion of $\epsilon(\omega, \mathbf{k})$ as otherwise we get a logarithmically divergent expression. The divergence is caused by scattering without changes in p, p_1 of two electrons with the same momentum $p = p_1$. Such electrons do not fly away from one another after the collision and interact for an infinitely long time. There occurs therefore a divergence in the Born approximation similar to the case of scattering by a static potential.[12]

Calculating $\epsilon(\omega, \mathbf{k})$ from (3) and substituting it into (18) we find for a degenerate electron gas

$$\frac{1}{\tau_n^{ee}(p)} = \frac{e^4}{2\pi\varkappa^2}\int_0^\infty \frac{du}{u^2} F_{n\,n-1}(u) F_{01}(u) \ln\left\{\left[\frac{(\sqrt{2\xi}-p)^2 64 d^4}{2\xi\pi^2} e^u + 1\right]\right.$$
$$\times\left.\left[\frac{(\sqrt{2\xi}+p)^2 64 d^4}{2\xi\pi^2} e^u + 1\right]\right\}, \quad d^{-2} = \frac{4\pi n_0 e^2}{\varkappa\xi}. \quad (19)$$

By writing the Laguerre polynomials as series we can perform the integration. For instance, for n = 2, restoring dimensions (we neglect unity under the logarithm sign)

$$\frac{1}{\tau_2^{ee}(0)} = \frac{2\pi n_0 e^4}{\varkappa^2 \hbar\Omega\sqrt{2m\xi}} \cdot \frac{5}{16} \ln\left(\frac{64 d^4}{\pi^2\Lambda^4} e^{0,6}\right). \quad (20)$$

When $n \gg 1$ we can estimate the integral over n using the asymptotic formula[10] $F_{nn-1}(u) \approx J_1^2(\sqrt{2nu})$, where J_1 is a Bessel function;

$$\frac{1}{\tau^{ee}(0)} = \frac{2\pi n_0 e^4 L}{\varkappa^2\hbar\Omega\sqrt{2m\xi}} = \frac{e^4 m}{\pi\varkappa^2\hbar^3} L, \quad L = \left(\frac{1}{8}+\frac{1}{\pi}\right)\ln\frac{64 d^4}{\pi^2\Lambda^4}. \quad (21)$$

Comparing (20) and (21) it is clear that $\tau_n^{ee}(p)$ depends little on n. Moreover, it is clear from (21) that the relaxation time τ_n^{ee} depends only logarithmically on the concentration of the equilibrium electrons. This effect is caused by the fact that the probability for a transition with $k_z \approx 0$ is inversely proportional to the momentum of the cold electrons $\sqrt{\eta} \sim n_0$.

For a non-degenerate electron gas we find

$$\frac{1}{\tau_T^{ee}(0)} = \frac{2\sqrt{2\pi} n_0 e^4 L_T}{\varkappa^2\sqrt{mT}\hbar\Omega}, \qquad L_T = \left(\frac{1}{8}+\frac{1}{\pi}\right)\ln\frac{16 d_T^4}{\pi C_1 \Lambda^4},$$
$$d_T^{-2} = \frac{4\pi n_0 e^2}{\varkappa T}, \qquad C_1 = e^{0,577}. \quad (21')$$

7. The expressions obtained for $\bar{\tau}_n^{ee}$ and τ_n^{ee} and also the relations between the $W_{n_1 n_1'}^{nn'}$ enable us to simplify the original set of equations appreciably so that we have instead of (9) and (10), respectively,

$$\bar{f}_n(p)\left[\frac{1}{\bar{\tau}_n^{ee}(p)} + \frac{1}{\tau_n^H(p)}\right] = J_n(p) + W_{10}^{nn}\left(\frac{1}{p}\right)\frac{4\pi^2 n_0}{|p|} f_n\left(p+\frac{1}{p}\right)$$
$$+ \frac{4\pi^2 n_0}{|p|} W_{00}^{nn+1}\left(\frac{1}{p}\right)\left[\bar{f}_{n+1}\left(p-\frac{1}{p}\right) + \delta_{n0} A_1\left(p-\frac{1}{p}\right) f^0\left(p-\frac{1}{p}\right)\right]$$
$$+ \frac{4\pi^2 n_0}{|p|} W_{00}^{n-1n}\left(\frac{1}{p}\right)\bar{f}_{n-1}\left(p+\frac{1}{p}\right) + \delta_{n1}\sum_{n'}\frac{4\pi^2 n_0}{|p|} W_{n'n'}^{10}(p)$$
$$\times \bar{f}_{n'}\left(p+\frac{1}{p}\right) + \delta_{n0}\frac{4\pi^2 n_0}{|p|}\left[\sum_{n'\geqslant 1}\sum_{n_1=n'-1}^{n'+1} W_{n_1 n'}^{00}(p) f_{n'}\left(p+\frac{n_1-n'}{p}\right)\right.$$
$$\left. + W_{10}^{00}(p)\bar{f}_0\left(p+\frac{1}{p}\right)\right], \quad (22)$$

$$A_n(p) = \tau_n^{ee}(p)\left[\sum_{n'\geqslant n}\int\frac{dp_1}{|p-p_1|}\bar{f}_{n'}(p_1) W_{00}^{n'n'}(p-p_1)\right.$$
$$\left. + \delta_{n1}\sum_{n'\geqslant 2}\int\frac{dp_1}{|p-p_1|}\bar{f}_{n'}(p_1) W_{00}^{n'n'}(p-p_1)\right]. \quad (23)$$

The second term in (23) and the last two in (22) take into account transitions of equilibrium electrons to the level n = 0 and n = 1 during collisions with non-equilibrium electrons. We note that (23) is an exact expression for $A_n(p)$ in terms of the functions $\bar{f}_n(p)$. The quantities $W_{n_1 n_1'}^{nn'}(p)$ are defined by Eq. (3) in which we must put $\epsilon(\omega, \mathbf{k}) = \kappa$.

2. KINETIC EQUATION TAKING THE INTERACTION WITH OPTICAL PHONONS INTO ACCOUNT. PHOTOMAGNETIC GUREVICH-FIRSOV OSCILLATIONS

1. We now take into account in Eq. (2) the integral of the collisions of the photoelectrons with optical phonons and we shall then assume that the phonon frequency ω_0 and the matrix element of the electron-phonon interaction M are independent of the momentum. We must then replace $\bar{\tau}_n^{ee}(p)$ in Eqs. (9), (10) by $\bar{\tau}_n(p)$:

$$\bar{\tau}_n(p) = \frac{\bar{\tau}_n^{ee}(p)\tau_n^{ph}(p)}{\bar{\tau}_n^{ee}(p)+\tau_n^{ph}(p)}, \quad \frac{1}{\tau_n^{ph}(p)} = \frac{1}{\tau^{ph}}\sum_{n'}\frac{1}{\sqrt{p^2+2(n-n'-\omega_0)}} \quad (24)$$

and add the term

$$\frac{1}{\tau^{ph}}\sum_{n'}\frac{f_n'(\sqrt{p^2+2(n-n'+\omega_0)})}{\sqrt{p^2+2(n-n'+\omega_0)}}, \quad (25)$$

corresponding to the "arrival" of electrons from a state with energy $\epsilon_n(p) + \omega_0$ into a state with energy $\epsilon_n(p)$. Here $1/\tau^{ph} = |M|^2 V/\pi\Lambda^3\hbar^2\Omega$ (in the form with dimensions). We can simplify the solution of Eqs. (9) and (10) with the changes (24) and (25) for small n by dividing the integral over the energy from zero to ϵ_0 into bands of width ω_0 ($\omega_0 > 1$). The extra term in (25) in each band will be determined by the value of $f_n(p)$ in the preceding band which lies higher in energy. We shall not write down the complete expression for the solution as it is too cumbersome. We only give the solution for the function $A_n(p)$:

$$A_n(p) = \tau_n^{ee}(p)\sum_{n'\geqslant n}\prod_{n''=n}^{n'}\left[1+\frac{\tau_{n''}^{ee}(p)}{\tau_{n''}^{ph}(p)}\right]^{-1}\int\frac{dp_1}{|p-p_1|}\bar{f}_{n'}(p_1) W_{00}^{n'n'}(p-p_1) \quad (26)$$

and the expression for $\bar{f}_n(p)$ which is valid for the upper occupied Landau level:

$$\bar{f}_n(p) = \bar{\tau}_n(p) J_n(p). \quad (27)$$

2. Expressions (26) and (27) which describe the distribution function of the non-equilibrium electrons which interact with electrons and phonons enable us to study the problem of photomagnetic G-F oscillations which

were observed in ref.[7]. Two unusual facts need explanations: the presence of oscillations at helium temperatures and the absence of oscillations in pure samples. We have suggested[8] to connect this effect with the oscillations in the transverse diffusion coefficient $D_\perp$ for photoelectrons which emit optical phonons and interact with equilibrium electrons. In[9], the photomagnetic-effect oscillations were interpreted as the heating up of equilibrium electrons to some effective temperature due to the transfer of energy from the photoelectrons. As the energy transferred by the optical phonons P_{ph} oscillates with the magnetic field, the effective temperature must then also show G-F oscillations.

The expressions for $D_\perp$ and P_{ph} are of the form

$$D_\perp = \frac{|M|^2 V}{(2\pi)^3 \Delta n_e} \sum_{nn'} (n+n'+1) \int dp f_n(p) [p^2 + 2(n-n'-\omega_0)]^{-1/2}, \quad (28)$$

$$P_{ph} = \frac{2\omega_0 |M|^2 V}{(2\pi)^3} \sum_{nn'} \int dp f_n(p) [p^2 + 2(n-n'-\omega_0)]^{-1/2}. \quad (29)$$

The oscillations in $D_\perp$ and P_{ph} are caused by the periodic change in the probability to emit an optical phonon which is proportional to $[p^2 + 2(n - n' - \omega_0)]^{-1/2}$ and which reaches a maximum when $\omega_0 = N$ (N an integer). It is clear from (28) and (29) that the integrands in $D_\perp$ and P_{ph} are the same so that the character of the oscillations must be similar for small n. For the sake of simplicity we consider in what follows only P_{ph}.

3. After substituting the distribution function into (28) we have for the energy emitted by electrons in level n

$$P^n_{ph} = \bar{P}^n + \delta P^n, \qquad \bar{P}^n = \frac{\omega_0}{(2\pi)^2} \int dp \frac{\bar{f}_n(p)}{\tau^{ph}_n(p)}, \quad (30)$$

$$\delta P^n = \frac{\omega_0}{(2\pi)^2} \int dp \frac{f^0(p) A_n(p)}{\tau^{ph}_n(p)}. \quad (31)$$

If we exclude the interaction with the equilibrium electrons we find for P^n_{ph}

$$\bar{P}^n = \frac{\omega_0}{(2\pi)^2} \int dp J_n(p), \qquad \delta P^n = 0, \qquad n_0 \to 0.$$

Hence, the oscillations are absent as $n_0 \to 0$. This result, first found in[8], is connected with the periodic vanishing of the photoelectron distribution function $f_n(p)$ when $p \approx 0$ due to the resonance emission of an optical phonon by the photoelectrons

$$f_n(p) \sim \tau_n^{ph}(p), \qquad n_0 \to 0.$$

The interaction with the electrons leads to a smoothing out of $f_n(p)$. We calculate P^n_{ph} assuming $J_n(p) = J$ to be independent of n and p. The integration over p in (30) is split into two intervals: $|p| < 1$ and $|p| > 1$. The integral over the second interval does not contain a singularity and is a smooth function of H. The integration over the first interval can be reduced by using (17) and (27). When $\alpha \lesssim 1$ we can neglect τ^{ee} in $\bar{f}_n(p)$ as compared to τ^{ph} and we then get, including the dimensions,

$$\int_{|p|<1} dp \frac{\bar{f}_n(p)}{\tau^{ph}_n(p)} \approx J\alpha \sum_{N=0}^{[n-\omega_0/\Omega]} \left\{ \frac{1}{3}[1+2(N+\delta)]^{3/2} + \frac{2}{3}[2(N+\delta)]^{3/2} - 2(N+\delta)[1+2(N+\delta)]^{1/2} \right\}, \quad (32)$$

where

$$\alpha = \frac{\sqrt{m}\varkappa^2(\hbar\Omega)^{3/2}}{2\pi n_0 e^4} \frac{1}{\tau^{\Phi}}, \quad \delta = n - \frac{\omega_0}{\Omega} - \left[n - \frac{\omega_0}{\Omega}\right], \quad 0 \leqslant \delta < 1,$$

[x] is the integer part of the number. The smallness of the oscillations in $\bar{P}^n$ is connected with the fact that the function $\bar{f}_n(p)$ is small in the region $p \sim 0$ ($\bar{f}_n(p) \sim p^3$).

4. The integration in (31) is over the range of small p because of the function $f^0(p)$ so that we put p = 0 in W^{nn}_{00}. Substituting $A_n(p)$ for the upper level (with n' = n) in (31) we get

$$\delta P^n = \frac{\omega_0}{(2\pi)^2} I_n \int dp_1 \frac{4\pi^2 n_0 W^{nn}_{00}(p_1)}{|p_1|} \bar{f}_n(p_1), \quad (33)$$

$$I_n = \frac{1}{4\pi^2 n_0} \int dp f^0(p) \frac{\tau^{ee}_n(p)}{\tau^{ee}_n(p) + \tau^{ph}_n(p)}, \quad (34)$$

and for small p and δ we get from (24)

$$\tau_n^{ph}(p) = \tau^{ph}\sqrt{p^2 + 2\delta}.$$

As in (30), the integral in (33) oscillates little with the magnetic field and when $\alpha \lesssim 1$ is approximately equal to J. On the other hand, I_n is a function which oscillates with a large amplitude and period $\Delta(1/H) = e/mc\omega_0$. One can check this most simply by considering the limiting case $\eta = 0$. Taking as the distribution function

$$f^0(p) = 4\pi^2 n_0 \delta(p),$$

we find that $\tau^{ee}_n(p) = |p|\tau^{ee}_n$, $\tau^{ee}_n = 4\pi^2 n_0 W^{nn-1}_{01}(0)$,

$$I_n = \begin{cases} 0, & \delta \neq 0 \\ \tau^{ee}_n/(\tau^{ph}_n + \tau^{ee}_n), & \delta = 0 \end{cases}. \quad (35)$$

δP^n is thus an oscillating function of H which is nonvanishing only for strict resonance $\omega_0 = N$.

In the case of finite η we put in (34) $\tau^{ee}_n(p) = \tau^{ee}(0)$, as $\tau^{ee}_n(p)$ changes slowly in the interval $p \lesssim \sqrt{(2\eta)}$.

We find I_n for degenerate electrons:

$$I_n = \beta \left[\ln \frac{1+\sqrt{1+\delta_1}}{\sqrt{\delta_1}} + \begin{cases} \mathscr{L}, & \beta^2 > \delta_1 \\ \mathscr{A}, & \beta^2 < \delta_1 \end{cases} \right],$$

$$\mathscr{L} = \frac{1}{\sqrt{1-\delta_1/\beta^2}} \ln \frac{\delta_1 + \beta\sqrt{1+\delta_1} - \sqrt{1-\delta_1/\beta^2)}}{\sqrt{\delta_1}(\sqrt{1+\delta_1}+\beta)},$$

$$\mathscr{A} = \frac{1}{\delta_1/\beta_2 - 1} \left(\arcsin \frac{\delta_1 + \beta\sqrt{1+\delta_1}}{\sqrt{\delta_1}(\beta + \sqrt{1+\delta_1})} - \frac{\pi}{2} \right),$$

$$\delta_1 = \frac{\delta}{\xi}, \qquad \beta = \frac{\tau^{ee}(0)}{\tau^{ph}\sqrt{2\xi}} = \left(\frac{\varkappa^2 |M|^2 V}{2\pi^2 \hbar^2 c^2 e^2 L} \right) \frac{H^2}{n_0}. \quad (36)$$

The quantity I_n reaches its maximum value when δ = 0:

$$I_n = \beta \ln \left(\frac{1+\beta}{\beta} \right), \quad (37)$$

and its minimum value $I_n \approx \beta\sqrt{\eta}$ when $\delta \sim 1$.

We study how δP^n depends on the concentration of the equilibrium electrons when δ = 0. As $n_0 \to 0$, the quantity $\beta \to \infty$ and $I_n \to 1$, and as a whole the function δP^n tends to zero in accordance with the results obtained above. In the opposite limiting case when $n_0 \to \infty$, $\beta \to 0$, the transferred power tends to zero as $\delta P^n \sim I_n \sim \beta \ln(1/\beta) \to 0$ although the logarithm becomes infinite. This means that the departure of the photoelectrons is extremely fast due to the electron-electron interaction and the distribution function vanishes.[8]

The largest value is thus reached for values of β of the order unity:

$$\beta \approx 1, \quad \frac{1}{\tau^{ph}} \approx \frac{1}{\tau^{ee}(0)} \sqrt{\frac{2\xi}{\hbar\Omega}} = \frac{2\pi n_0 e^4 L}{\varkappa^2 \sqrt{m} (\hbar\Omega)^{3/2}}, \quad (38)$$

i.e., when the time for Coulomb relaxation in the magnetic field is equal to the time for emission of an optical phonon. In the case of a non-degenerate electron gas the condition $\beta \approx 1$ takes the form

$$\frac{1}{\tau^{\mathrm{ph}}} \approx \frac{1}{\tau_T^{ee}(0)}\sqrt{\frac{2T}{\hbar\Omega}} = \frac{2\sqrt{2\pi}n_0 e^4 L_T}{\varkappa^2\sqrt{m}(\hbar\Omega)^{3/2}}. \quad (39)$$

We must emphasize that as β is a function of H^2/n_0 the optimum concentration n_0 is a function of the magnetic field.

Comparing Eqs. (32), (33), and (37) we find that $\delta P^n \approx \overline{P}^n$. If $\eta \sim 1$, the depth of the oscillations is small and, as an estimate, does not exceed 10%. When we decrease $\eta \to 0$ the depth of the oscillations may reach a magnitude of the order of unity due to δP^n.

We can thus take it for proven that the interaction of photoelectrons with the equilibrium carriers leads to oscillations in P_{ph} and $D_\perp$ with period $1/\omega_0$.

It is interesting to note that an "oscillation amplification" effect occurs as $\eta \to 0$. The physical reason for this lies in the fact that the interaction with the electrons not only smoothes out the distribution function, but also leads to a localization of the photoelectrons near p = 0 and they also make a considerable contribution to the oscillation term.

5. One can generalize the results of the last section if we bear in mind the way $|M|^2$ depends on the phonon momentum, i.e., if we put

$$|M|^2 = \frac{A}{q^2}, \quad A = \frac{2\pi\hbar\omega_0 e^2}{V}\left(\frac{1}{\varepsilon_\infty} - \frac{1}{\varkappa}\right).$$

In that case we must replace in Eqs. (36) to (39) $|M|^2$ by the quantity $A\hbar/2m\omega_0$. We calculate also δP using the general Eq. (26) for $A_n(p)$:

$$\delta P = \frac{\omega_0}{(2\pi)^2} \sum_{n,n' \geqslant n} I_{nn'} \int \frac{dp_1}{|p - p_1|} 4\pi^2 n_0 W_{00}^{n'n'}(p_1) \bar{f}_{n'}(p_1),$$

$$I_{nn'} = \frac{(-1)^{n'-n}}{4\pi^2 n_0} \frac{\beta^{n'-n}}{(n'-n)!} \frac{\partial^{n'-n}}{\partial\beta^{n'-n}} \int \frac{dp f^0(p)\beta\sqrt{2\eta}}{(\sqrt{p_\parallel^2 + 2\delta} + \beta\sqrt{2\eta})}. \quad (40)$$

CONCLUSION

We have shown in this paper that the transverse diffusion coefficient $D_\perp$ and the power P_{ph} transferred from the photoelectrons to the optical phonons oscillate with a period $e/mc\omega_0$ and that the oscillations are connected not only with a "smoothing out" of the distribution function, as was suggested in[8,9] but also with the localization of the photoelectrons near p = 0 due to the electron transitions inside one Landau zone.

To evaluate the relative contribution caused by $D_\perp$ and P_{ph} it is necessary to evaluate the strength of the photomagnetic effect and the Nernst effect. However, since the effective temperature approximation may turn out to be invalid because of the absence of Coulomb relaxation to the zeroth Landau level the calculation of the kinetic coefficients must be done with exact distribution functions which were found above. However, this problem goes outside the framework of the present paper.

Nonetheless, in view of the fact that the expressions for $D_\perp$ and P_{ph} are the same, the character of the oscillations will be the same in the two cases and, hence, the results obtained are valid and open to experimental verification.

Of most importance is the condition (38) for the oscillations to be maximal; it formally is the same as the one used in[8,9] but with an explicitly defined relaxation time in a magnetic field. It follows from (38) that the optimum concentration is a function of the magnetic field. For a constant concentration the depth of the oscillations will depend on the magnetic field, and if the parameter β passes through unity, the dependence is not monotonic. This fact makes it possible to verify condition (38) experimentally by varying n_0 and H.

We note also that we can directly estimate from (38) the matrix element of the electron-phonon interaction, determining n_0 and H experimentally.

In general we must emphasize that a study of Gurevich-Firsov type photomagnetic oscillations offers great perspectives to determine the parameters of the electron-electron and electron-(optical)-phonon interactions as these interactions compete in this effect, and also of the large experimental depth of the oscillations and of the existence of the exact solution.

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