EFFECT OF FLUCTUATIONS ON THE DEPENDENCE OF THE JOSEPHSON CURRENT ON THE MAGNETIC FIELD

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Submitted August 28, 1969

Zh. Eksp. Teor. Fiz. 58, 1497–1510 (May, 1970)

The anomalous dependence of the critical Josephson current on the magnetic field strength is investigated in extended Sn-Sn junctions. It is shown that the observed peculiarities are due to thermal fluctuations and inhomogeneities of the barrier layer (structural fluctuations). The effect of coupling energy, junction temperature, and of the external circuit resistance on the shape of the $I_c(H)$ characteristics is studied. Both thermal and structural fluctuations can lead to a decrease of the oscillation amplitude of $I_c(H)$ which is more rapid than $1/H$. However, whereas thermal fluctuations destroy the superconductivity of the junction, structural fluctuations will lead to the appearance of a "background" superconducting current which depends only weakly on the magnetic field.

1. INTRODUCTION

THE effect of fluctuations on the properties of superconducting systems, which has recently been at the focus of attention of the physics of superconductivity,[1] is particularly great when one or two of the linear dimensions of the sample become small compared with the coherence length ξ_0.[2] Consequently, fluctuations should have an appreciable effect on the properties of plane Josephson junctions which are in essence a two-dimensional superconducting system.[3] The transition energy E_0 of the junction into the superconducting state is much less than the condensation energy for a bulk superconductor. In the presence of weak magnetic fields (~ 1 Oe) parallel to the plane of the junction, or when a current flows through the junction, the "coupling" energy of two superconductors through a thin barrier layer decreases even further. In the event when the coupling energy averaged over the junction becomes comparable with the energy of the thermal fluctuations, the stationary superconducting state is destroyed and a finite voltage appears at the barrier.

In the first publications[4] on the observation of anomalous characteristics of the Josephson current their origin was not as yet related to the effect of fluctuations. However, Vant-Hull and Mercereau[5] already showed that the tunnel current for $V \neq 0$ includes a constant component that oscillates in a magnetic field, although the critical Josephson current in zero field vanished for such junctions. Subsequent theoretical[6,7] and experimental[8,9] papers considered, basically, the effect of thermal fluctuations on the form of the current-voltage characteristics of Josephson junctions. A theory[6,7] based on ideas due to Anderson[10] explained qualitatively the decrease of the critical Josephson current under the influence of fluctuations and the presence of a constant component of the Josephson current for $V \neq 0$ in zero magnetic field. Quantitative discrepancies between the theory of the current-voltage characteristics[6,7] and the experimental curves[9] were explained by the fact that the theory considered the properties of point junctions for which one could neglect the nonuniform distribution of fluctuations over the area of the junction. For real film Josephson tunnel junctions it is difficult to attain dimensions so small that one could consider them to be point junctions.

The purpose of this work is an experimental investigation of the effect of fluctuations upon the dependence of the critical Josephson current on the magnetic field in extended junctions. One must differentiate between thermal or thermodynamic fluctuations which decrease with decreasing temperature, and structural fluctuations which constitute random inhomogeneities of the properties of the barrier layer and which are always to some extent present in real junctions. In this work, after a presentation of the results of an experimental investigation of the $I_c(H)$ dependence for junctions under the influence of fluctuations (Sec. 2), we present a comparison of the obtained characteristics with the calculated curves contained in[11,12] (Sec. 3). A comparative analysis of the effect of thermal and structural fluctuations on the $I_c(H)$ dependence, as well as some additional results are included in Sec. 4.

2. EXPERIMENTAL RESULTS

As the object of our investigation we used Sn-Sn Josephson tunnel junctions of crosslike geometry[13] whose resistivity was between several hundredths of an ohm up to several ohms per square millimeter of junction area. The magnitude of the area of the junctions was varied within the limits 0.1–1.0 mm². As is well known,[14] the resistance R of such junctions decreases monotonically with time when the samples are stored at room temperature. During the first stage this decrease occurs uniformly over the entire area of the junction and is accompanied by an increase of the Josephson current in zero magnetic field $I_c(H=0) \equiv I_0$:

$$I_0 = \frac{\pi}{2}\frac{\Delta}{e}\frac{1}{R} \qquad (T=0,\ \Delta_1=\Delta_2=\Delta). \tag{1}$$

The coupling energy of the junction increases in proportion to the critical current $E_0 = \hbar I_0/2e$. At later stages of annealing there is an appreciable increase in the inhomogeneities of the properties of the barrier layer. This is accompanied by an increase in the probability of the appearance of a short circuit whose presence is readily observed from the change in the shape of the current-voltage characteristics and of the $I_c(H)$ dependence.[14] The properties of shorted tunnel junctions are not considered in this paper.

In order to investigate the changes in the $I_c(H)$ dependences with increasing E_0, we used both different tunnel junctions prepared under various regimes and having different resistances R, as well as the effect of the uniform decrease in the resistance of the junction observed at the initial stages of sample annealing at room temperature. The latter method is particularly suitable since it makes it possible to exclude the effect of certain not fully reproducible factors in the condensation and oxidation of thin films. The qualitative results coincided in both instances.

The automatic recording of the $I_c(H)$ dependences was carried out with the aid of the system described in[13]. If the voltage on the junction jumped from $V_1 < V_0$ to $V_2 \gg V_0$ on increasing the current through the junction[1)], then the indicated system yielded the $I(H)|_{V=V_1}$ dependence. In particular, if $V_1 = 0$, the $I_c(H)$ curve was recorded. If, on the other hand, $V_1 > V_0$ or the transition from the superconducting tunneling to the normal characteristic proceeded smoothly, then the change of the constant component of the tunnel current for $V = V_0$ was recorded. Both cases differed clearly on the experimental curves; however, since this difference is immaterial for our further discussion it is not noted on the graphs presented below.

Figures 1b–1e show the I_c/I_0 dependences for a Sn-Sn tunnel junction obtained as a result of prolonged (about a month-long) anneal of a sample at room temperature. In order to carry out measurements, the annealing was periodically interrupted and the sample was cooled down to $T = 1.5°K$. The dimensions of the junction were 0.39×1.1 mm^2. The resistivity before the annealing was $\rho = 0.45$ ohm-mm^2. The Josephson penetration depth λ_j calculated from formula[3]

$$\lambda_j^2 = \hbar c^2 \rho \,/\, 8\pi^2 \Delta \lambda_L, \tag{2}$$

for the given junction was 1.09 mm. The period of $I_c(H)$ oscillations was determined by the narrow film; consequently, the Josephson penetration depth exceeded the characteristic dimension of the junction by a factor of 2.8. The experimental curves shown in Figs. 1b–1e correspond to resistances of the junction in the normal state of 1.05, 0.5, 0.4 and 0.32 ohm; the values of the critical current in zero magnetic field at $T = 1.5°K$ are 0.45, 1.0, 1.2, and 1.9 mA, respectively. The experimentally observed critical current in zero external field was between 50 and 70 percent of its theoretically calculated value which was obviously due to "frozen-in" small magnetic fluxes in the sample which appear when it is cooled[2)]. These "frozen-in" fluxes do not exert any appreciable influence on the oscillations of $I_c(H)$, since the external magnetic flux which links the sample exceeds their magnitude even near the first minimum. Consequently, the effect of parasitic "frozen-in" fields is concentrated in a narrow region near $H = 0$; the satisfactory symmetry of the characteristics of $I_c(H)$ relative to a change in the sign of the magnetic field serves to confirm this.

Since the Josephson penetration depth exceeds the width of the junction, the field dependence of the critical current should follow formula[3]

$$I_c(H) = I_0 \frac{|\sin(\pi H/H_0)|}{\pi H/H_0}, \tag{3}$$

where H_0 is the value of the magnetic field corresponding to a flux quantum penetrating into the junction. This dependence is illustrated in Fig. 1a; it is confirmed by the experimental data for the junctions of lower resistance (see, for instance Fig. 2c). For the high-resistance junction whose characteristics are shown in Figs. 1b–1e, a feature of the $I_c(H)$ dependence is the initial absence of the side maxima in the diffraction pattern, followed by their gradual appearance with increasing coupling energy E_0. The period of the oscillations corresponds, as usual, to a flux quantum penetrating into the region between the films. The decrease in the amplitude of the oscillations and along with it of the quantity I_c to zero on increasing the magnetic field indicates the absence of local short circuits or thinning in the oxide film[3)].

The $I_c(H)$ characteristics shown in Fig. 1 were obtained at constant temperature and consequently at a constant mean energy of the thermal fluctuations $\sim kT$. A decrease of the ratio of the energy fluctuations to the coupling energy of the junction E_0 is attained by a gradual increase of the latter. An analogous change of the $I_c(H)$ characteristics should obviously result from a decrease of the temperature T of the junction, since this is accompanied by a simultaneous decrease in the energy of fluctuations and an increase of the coupling

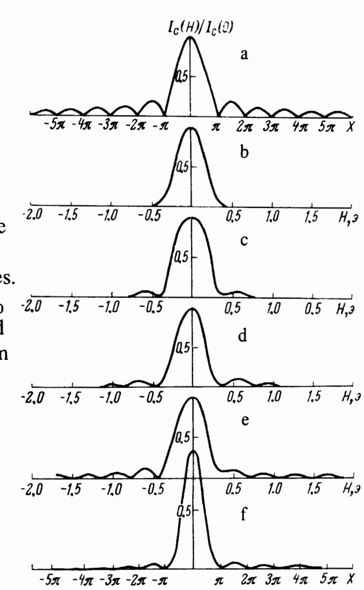

FIG. 1. Dependence of the critical Josephson current on the magnetic field: a, f—theoretical curves; b–e—experimental curves. $T = 1.5°K$; $X = \pi H/H_0$ where H_0 is the value of the magnetic field corresponding to a flux quantum penetrating into the junction.

[1)] The voltage V_0 amounted to several microvolts and corresponded to the sensitivity threshold of the follow-up system.

[2)] Such parasitic magnetic fluxes were produced in the sample, in spite of the use of annealed ferromagnetic screens that attenuated the external fields by a factor of several times ten.

[3)] Figure 1e does not show the magnetic fields at which $I_c^{max} \to 0$.

energy of the junction proportional to $I_0(T)$:

$$\frac{I_0(T)}{I_0(0)} = \frac{\Delta(T)}{\Delta(0)} \operatorname{th} \frac{\Delta(T)}{2kT}. \quad (4)$$

In Fig. 2 we present the characteristics of a relatively low-resistance ($\rho = 1.2 \times 10^{-2}$ ohm-mm^2) junction for which

$$S = 0.172\ \text{mm}^2 \quad R = 0.069\ \text{ohm} \quad \lambda_j = 0.2\ \text{mm}$$

$l = 0.4$ mm and $T_c = 3.8°$K. For H = 0 the magnitude of the obsei ved critical current amounts to 85 percent of its theoretical value. At low temperatures (Fig. 2c) the experimental $I_c(H)$ dependence (curve 1) is in good agreement with the theoretical curve 2 plotted in accordance with formula (3). The value of I_0 for the theoretical curve was obtained from relations (4) and (1). The small asymmetry of the curve relative to the origin near H = 0 is due to the inherent magnetic fields of the Josephson current in a junction of cross-like geometry. With increasing temperature (Figs. 2c, b, a) the side maxima of the diffraction pattern disappear gradually, and only a single, central maximum for H = 0 remains. The period of the oscillations decreases somewhat with increasing temperature in accordance with the growth of the field penetration into the superconductor.[15] In Fig. 2 we present only the constant component due to the superconducting current obtained by subtracting from the observed characteristic the quasiparticle tunnel current. It is seen that the pattern of change of the characteristics $I_c(H)/I_0$ (Fig. 2) taking place when the ratio of the energy of thermal fluctuations and of the coupling energy changes is analogous to that shown in Fig. 1. However, unlike the latter, in this case there is no change in the structure of the barrier layer (all curves were obtained in the same experiment) and the described changes are due solely to the thermal fluctuations. The effect of the latter is only important near T_c in a low-resistance sample when E_0 decreases appreciably.

In addition to fluctuations appearing in the junction itself, fluctuations introduced into the junction from the external circuit which is as a rule at room temperature play an important role in a number of cases. The magnitude of these fluctuations depends on the ratio of the resistance R_1 of the external circuit to the resistance R of the junction. In order to allow for these, one

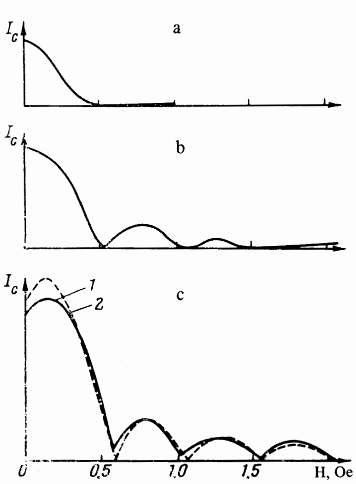

FIG. 2. Dependence of the critical current on the field at various temperatures: a—T = 3.7°K, b—T = 3.4°K, c—T = 2.44°K; curve 1—experiment, 2—theoretical curve plotted in accordance with formula (3).

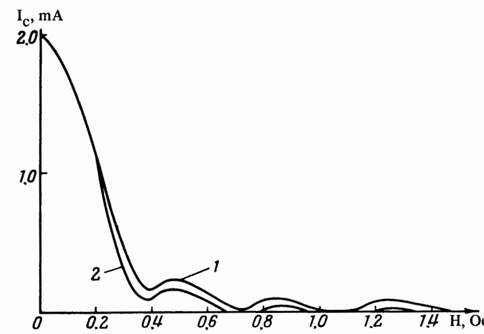

FIG. 3. The effect of the resistance of the external circuit R_1 on the $I_c(H)$ dependence: curve 1—$R_1 > 200$ ohm, 2—$R_1 = 40$ ohm.

can introduce the effective noise temperature T^* which depends both on the junction temperature T and on the temperature of the external circuit T_1:

$$T^* = \frac{T_1 R + T R_1}{R + R_1} \approx T + \frac{R}{R_1} T_1, \qquad R_1 \gg R. \quad (5)$$

In Fig. 3 we show the $I_c(H)$ dependences obtained for a junction with a resistance of 0.3 ohm and a temperature of 1.5°K. Curve 1 corresponds to a resistance of the external circuit of more than 200 ohm and its form exhibiting features[4)] due to fluctuations in the junction itself practically does not depend on the specific value of R_1. Curve 2 corresponds to a resistance of the external circuit $R_1 = 40$ ohm and exhibits a considerable decrease in the magnitude of the Josephson current under the influence of fluctuations introduced from the external circuit. Whereas for curve 1 the addition to the effective sample temperature due to the external circuit was less than 0.5°K, for the lower curve it amounted to 2.25°K, i.e., it exceeded the sample temperature. Thus, the external circuit exerts an overwhelming influence on the Josephson current under the condition that its resistance R_1 satisfies the inequality $R_1 \lesssim RT_1/T$ (where T_1 is room temperature) [see formula (5)]. These numerical estimates are only tentative since the normal resistance of the junction for $V \approx 0$ is appreciably nonlinear. However, they show that it is precisely the thermal fluctuations appearing in the junction and introduced from the external circuit, and not various extraneous causes such as for example the leads[5)], which give rise to the described features.

It is also of interest to trace the changes of the $I_c(H)$ curves due to structural fluctuations, i.e., the inhomogeneities in the oxide layer which cause changes in the density of the Josephson current from point to point inside the junction. The influence of structural fluctuations can be investigated in samples that have been subjected to sufficiently long annealing which is accompanied by an appreciable increase of inhomogeneities of the barrier layer. A typical characteristic

[4)]Foremost among such features is the decrease of the envelope of the oscillation maxima to zero at a rate faster than I(H) in formula (3) (see the discussion in Sec. 3).

[5)]Generally speaking, the external signals picked up by the internal measuring circuit increase the effective noise temperature T* of the sample, causing T_1 to become much higher than room temperature. The estimates presented above show that the influence of the pickup is negligible in the present case.

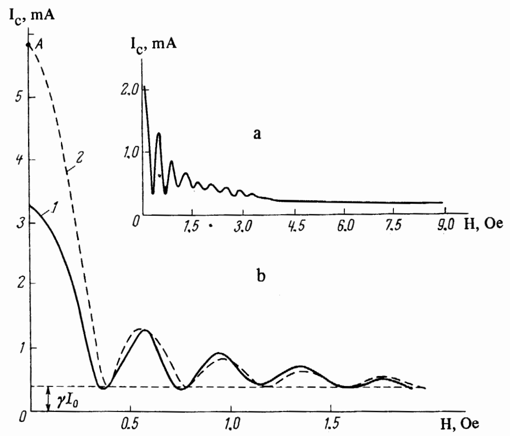

FIG. 4. Field dependence of the critical current for a junction with a nonuniform layer of dielectric. a—experiment, T = 1.5°K; b—1—experimental curve with expanded scale, 2—theoretical.

$I_c(H)$ of such a sample is shown in Fig. 4a. The junction whose characteristics are shown in Fig. 4 has the following parameters: $\rho = 0.061$ ohm-mm^2, $S = 0.37 \times 1.06$ mm^2, $R = 0.16$ ohm, $\lambda_j = 0.41$ mm. Curve 1 of Fig. 4b reproduces the initial section of the characteristic of 4a on an enlarged scale.

The basic difference between these curves and the characteristics shown in Figs. 1—3 consists in the presence of a certain "background" superconducting current towards which the envelope of the oscillation maxima tends asymptotically. The critical current at the oscillation minima is also approximately equal to the same quantity. It is interesting to note that the ratio of the periods of oscillation of $I_0(H)$ for two mutually perpendicular orientations of the magnetic field directed along the superconducting films was inversely proportional to the widths of these films. This indicates a uniform distribution of irregularities of the barrier layer over the area of the junction. In other words, the characteristic distance between inhomogeneities was much smaller than the dimension l of the junction. A detailed discussion of the effect of structural fluctuations on the $I_c(H)$ characteristics is given in the following Section.

3. DISCUSSION OF RESULTS AND COMPARISON WITH THEORY

a) The effect of thermal fluctuations. As has been shown by Anderson and Goldman,[9] regardless of the fact that the tunnel junctions employed by them had a very small area ($S = 1.93 \times 10^{-4}$ cm^2), they did not satisfy the condition for being "point-like":

$$r = RC(2eI_0/\hbar C)^{1/2} \ll 1, \qquad (6)$$

where C is the capacitance of the junction. One cannot all the more consider "point-like" the junctions whose properties have been described above, since for these $r \gg 1$. A theoretical investigation of the effect of thermal fluctuations on the properties of extended junctions was carried out by Ivanchenko and Zil'berman.[11] The dimensions of the junctions were assumed to be not too large so that the distribution of the magnetic field in the junction could be assumed to be almost uniform ($l \lesssim 2\lambda_j$).

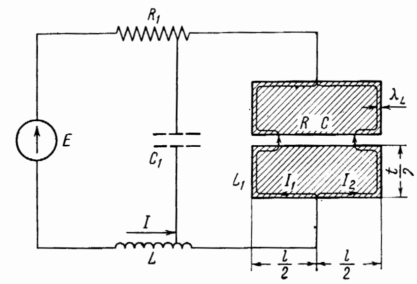

FIG. 5. Equivalent circuit of a Josephson tunnel junction connected to the external circuit. E—external emf, C_1—distributed shunting capacitance of the circuit.

The theoretical analysis of the effect of internal and external thermal fluctuations in[11] was carried out within the framework of the equivalent circuit shown in Fig. 5 and representing a certain idealization of the real experimental situation. In Fig. 5 a source of emf E, a resistance R_1, and an inductance L are connected in series to an external circuit at a temperature T_1. It is assumed that a junction whose normal resistance is R and capacitance C is connected to the circuit symmetrically and the superconducting current flows to the region of the junction over a skin layer of thickness λ_L whose equivalent inductance is L_1. In the experiment a distributed capacitance C_1 (dashed in Fig. 5) turns out to be connected between the current-carrying leads even in the immediate vicinity of the junction; this capacitance decreases considerably the values of R_1 and L for the high-frequency components of the current. In other words, the effective values of R_1 and L which should be substituted in the formulas of[11] can differ considerably from the total resistance and inductance of the external circuit.

Only the case of strong fluctuations is analyzed in[11]. The condition for strong fluctuations is, according to[11], equivalent to the fulfilment of the following inequalities:

$$D \gg \gamma, \gamma_1, \gamma_2, \qquad (7)$$

where the parameter D is

$$D = \frac{I_0^2}{E_0^2} R^* \theta^* = \left(\frac{2e}{\hbar}\right)^2 R^* \theta^* \quad (\theta^* = kT^*), \qquad (8)$$

and the parameters γ, γ_1, and γ_2 are

$$\gamma = \frac{1}{RC}, \quad \gamma_1 = \frac{1}{R_1 C}, \quad \gamma_2 = \frac{L + L_1}{R_1}. \qquad (9)$$

Let us clarify whether the condition of strong fluctuations is fulfilled for the junction whose experimental characteristics are shown in Figs. 1b—ie. Taking $R \sim 1.0$ ohm (before annealing) and $C \sim 5 \times 10^3$ pF, we find that $\gamma \approx 2 \times 10^8$ sec^{-1}. Since usually $R_1 \sim 100$ ohm, $R^* \approx R$ and $T^* \sim 4.5$°K. Substitution of the calculated parameters in (8) yields $D \sim 6 \times 10^8$ sec^{-1}, i.e, the inequality $D \gg \gamma$ is fulfilled without adequate excess. Since $R_1 \ll R_1$, the inequality $D \ll \gamma_1$, is satisfactorily fulfilled. As regards the last inequality ($D \gg \gamma_2$), by virtue of the impossibility of allowing for the effect of the parasitic capacitance C_1 its estimate represents a well-known difficulty. If it is assumed that the capacitance C_1 effectively short circuits the measuring circuit for the high-frequency components of the current, the last inequality will also be fulfilled even in the immediate vicinity (at a distance of several centi-

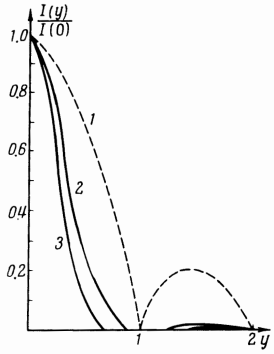

FIG. 6. Theoretical $I_c(H)$ dependence with account of the fluctuations. [11] Curve 1 – in the absence of fluctuations; 2–β = 1.25; 3–β = 2.5; $\pi y = X = \pi H/H_0$.

meters) of the junction. We are consequently entitled to expect that at least the qualitative predictions of the theory should correspond to the observed characteristics (Fig. 1).

The theoretical curves of the dependence of the critical current on the magnetic field for various levels of fluctuations are shown in Fig. 6. The parameter β of the theory which characterizes the level of fluctuations when the inequalities

$$L_1 \lesssim L, \quad \gamma \gg \gamma_1, \gamma_2, \quad R_1 \gg R \tag{10}$$

are fulfilled is approximately

$$\beta \approx DRC(L_1/L)^2. \tag{10a}$$

Making use of the previously obtained estimates for D and RC and assuming that if C_1 effectively short circuits the measuring circuit in the immediate proximity of the junction, then $L_1 \sim L$; we shall then find for the experimental value of β the estimate: $\beta \sim 3$. This estimate coincides approximately with the value of the β parameter used to plot the theoretical curves 2 and 3 in[11] (Fig. 6). A comparison of curves 2 and 3 of Fig. 6 and curves b, c, and d of Fig. 1 shows good qualitative agreement of the observed characteristics with the theoretical ones. The following features of the $I_c(H)$ dependence which appear in the experiment under the influence of thermal fluctuations are qualitatively explained by the theory.

1. At higher relative levels of fluctuations and values of the parameter $\beta \sim 1-10$ only a central maximum remains on the $I_c(H)$ curve; the lateral maxima of the diffraction pattern are suppressed by the fluctuations (Fig. 1b).

2. For small β (large E_0) but when the condition of strong fluctuations (7) is fulfilled, the $I_c(H)$ dependence is according to the theory[11] proportional to $(\sin X/X)^2$ and not to $(\sin X/X)$, as in (3). In Fig. 1f we show the dependence $I_c = I_0(\sin X/X)^2 (X = \pi H/H_0)$ plotted on the same scale as curve 1e. The value of I_0 for the curve 1f was obtained from the value R of the junction in accordance with (1) and (4). One can conclude that the qualitative picture of the change of the form of the dependence $I_c(H)$ is the following: as the coupling energy E_0 increases the envelope of the oscillation maxima of the experimental curves 1c, 1d, and 1e takes on intermediate forms between the case of strong oscillations (curve 1f) and the case in which the oscillations are absent (curve 1a). A more detailed comparison between theory and experiment is prevented, first, by the inaccuracy of the equivalent circuit (Fig. 5) that describes the real measuring circuit and, secondly, by the insufficiently strong fulfilment of inequalities (7) for the investigated junctions.

b) The effect of structural fluctuations. Let us go over to a discussion of the effect of structural fluctuations on the investigated characteristics. The calculations carried out in[12] are based on a simple model in which the dependence of the amplitude of the Josephson current density on the x coordinate which lies in the plane of the junction is of the form

$$I_c(x) = I_0 + I_1(x) = I_0 + \sum_{n=1}^{\infty} I_n \cos\frac{\pi n x}{l}, \tag{11}$$

where I_0 is a "constant component" of the current density amplitude, and the variable addition $I_1(x)$ due to the inhomogeneities of the barrier layer, is of a random nature. We shall assume that the autocorrelation function for $I_1(x)$ is of the form

$$\overline{I_1(x_1)I_1(x_2)} = \overline{I_1^2}e^{-\alpha|x_1-x_2|}, \tag{12}$$

where the bar denotes averaging over the junction and $r = 1/\alpha$ is the correlation radius determined by the characteristic dimensions of the inhomogeneities. The random fluctuations of the amplitude of the Josephson current are thus described with the aid of two parameters: the mean-square of the fluctuations $\overline{I_1^2}$ and the correlation radius r. The dependence of the critical current on the magnetic field with allowance for the inhomogeneities of the barrier layer is described by the formula[12]:

$$I_c^2(H) = \left(\frac{\sin X}{X}\right)^2 \overline{I^2_{\text{even}}} + \left(\frac{\cos X}{X}\right)^2 \overline{I^2_{\text{odd}}}, \tag{13}$$

where $X = \pi y = \pi H/H_0$, and

$$\overline{I^2_{\text{even}}} = I_0^2 + \frac{2\overline{I_1^2}}{\pi N}\sum_{n=1}^{\infty}\left(\frac{X^2}{X^2-\pi^2 n^2}\right)^2 \frac{1}{(1+n^2/N^2)} - \frac{2\overline{I_1^2}}{\pi^2 N^2}\left(\sum_{n=1}\frac{X^2}{(X^2-\pi^2 n^2)}\frac{1}{(1+n^2/N^2)}\right)^2 (1-e^{-2\pi N}), \tag{14}$$

$$\overline{I^2_{\text{odd}}} = \frac{2\overline{I_1^2}}{\pi N}\sum_{n=1}^{\infty}\left(\frac{X^2}{X^2-\pi^2(n-1/2)^2}\right)^2 \frac{1}{(1+(n-1/2)^2/N^2)} - \frac{2\overline{I_1^2}}{\pi^2 N^2}\left(\sum_{n=1}^{\infty}\frac{X^2}{X^2-\pi^2(n-1/2)^2}\frac{1}{(1+(n-1/2)^2/N^2)}\right)^2 (1+e^{-2\pi N}). \tag{15}$$

Here we have introduced the notation $2\pi N = l/r$. Equations (13)–(15) are valid for arbitrary values of N and X, i.e., for arbitrary values of the magnetic field H and dimension of the inhomogeneities r. For large N ($N \gg 1$) corresponding to a small dimension of the inhomogeneities compared with the width of the junction (small-scale fluctuations) the terms $\propto 1/N^2$ in (14) and (15) can be neglected. Formula (13) takes on a simple form if one assumes that the magnetic field is not too large

$$I_c^2(H) = \left(I_0^2 - \frac{\overline{I_1^2}}{\pi N}\right)\left(\frac{\sin X}{X}\right)^2 + \frac{\overline{I_1^2}}{\pi N}, \quad N \gg 1, \quad N \gg \pi X. \tag{16}$$

Figure 7 shows the $I_c(X)/I_0$ dependences plotted in accordance with (16) for various values of the parameter $\gamma^2 = (\overline{I_1^2}/I_0^2)(1/\pi N)$. Curve 1 corresponds to a uniform barrier layer and is plotted in accordance with

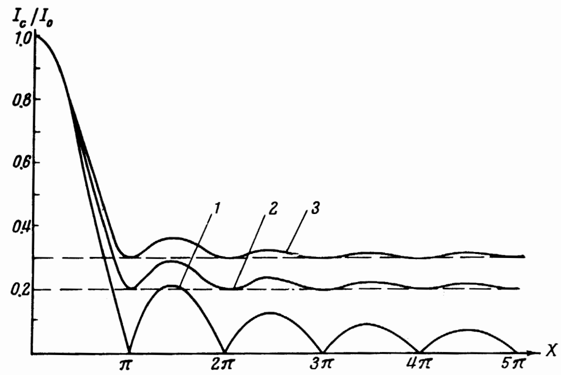

FIG. 7. Theoretical $I_c(H)$ dependence with account of inhomogeneities of the barrier layer. Curve 1—homogeneous barrier; 2—$\gamma = 0.2$; 3—$\gamma = 0.3$; $X = \pi H/H_0$.

formula (3), whereas curves 2 and 3 correspond to a γ parameter of 0.2 and 0.3.

It is seen from formula (16) and Fig. 7 that the effect of inhomogeneities leads to the following peculiarities of the $I_c(H)/I_0$ dependence.

1. For zero magnetic field ($X = 0$) $I_c = I_0$ independently of the value of the parameter γ. The values of I_c at the minima differ from zero and are γI_0. On increasing the magnetic field the critical current approaches this same value asymptotically.

2. Near the minima the form of the curve becomes smoother. There are no jumps of the derivative and the derivative $\partial I_c/\partial H$ itself vanishes.

3. The amplitude of the critical current oscillations noticeable on the "background" of the structural fluctuations is rapidly attenuated. If the fluctuations are large ($\gamma \lesssim 1$) then, as is readily seen, the envelope of the maxima is proportional to $1/X^2$ and not to $1/X$ as for a uniform barrier.

The enumerated features are clearly noticeable on the experimental curve 1 given in Fig. 4. The theoretical curve 2 is also plotted there for comparison. The scale for the theoretical curve was chosen starting from the following considerations. The period of the oscillations of the theoretical curve is equal to the average period of the oscillations of the experimental curve. The ordinate scale of curve 1 was chosen in such a way that it coincide on the average with the experimental curve at the minima, and the value I_c (the point A) was determined from (1) ($\Delta = 0.615$ mV) with account of the temperature dependence $I_0(T)$ [formula (4)]. If one does not consider the region of fields close to $H = 0$, then the agreement between the theoretical and experimental curves is satisfactory. The reasons for the discrepancy near $H = 0$ have been indicated above.

One can thus on the basis of the proposed model[12] determine the parameter γ from the experimental curves, and consequently also the product $\overline{I_1^2}(r/l)$ (in Fig. 4 $\gamma = 0.066$). It is, however, of interest to estimate separately the mean square of the current density fluctuations $\overline{I_1^2}$ and the characteristic dimension of the inhomogeneities r. In principle such an estimate can be made if one investigates the field dependence of the "background" level in sufficiently strong magnetic fields. It follows from (13)—(15) that under the conditions

$$N \gg 1, \quad X \gg \pi, \quad X \ll \pi N \tag{17}$$

the "background" critical current decreases in accordance with the law

$$I_c^2 \approx \frac{\overline{I_1^2}}{\pi N}\left(1 - \frac{X^2}{\pi^2 N^2}\right). \tag{18}$$

If, on the other hand, one assumes, instead of the last inequality that $X \gg \pi N$, then it can be shown that $I_c^2 \sim I/X^2$. We note that conditions (17) mean that the wavelength of the Josephson current density $\mathscr{X} = \pi l/X$ remains much larger than the dimension r of the inhomogeneities. A quantitative estimate of $\overline{I_1^2}$ and r based on experimentally obtained data is rendered difficult by the insufficient accuracy of the experimental curves for large values of the magnetic field. Qualitatively, however, the "background" critical current in the experiment (Fig. 4a) decreases in agreement with the theory.

4. CONCLUSION

Comparing the effect of thermal and structural fluctuations on the dependence of the critical Josephson current on the magnetic field, one can draw the conclusion that both the former and the latter lead to a faster than $1/H$ [formula (3)] decrease of the oscillation maxima. However, whereas the thermal oscillations lead to the destruction of the superconducting state of the junction, the structural fluctuations merely "smear out" the interference pattern. Thus, for a junction whose dielectric layer contains inhomogeneities the envelope of the maxima of the oscillations of $I_c(H)$ does not tend to zero but to some finite value. It is readily seen that the inhomogeneities of the barrier layer should not in themselves lead to a decrease of the critical current in zero magnetic field compared with the quantity given by formula (1). Since the local value of the amplitude of the Josephson current $(1/l)I_c(x)dx$ is proportional to the normal conductivity of the junction at a given point, then in calculating the critical current averaged over the junction formula (1) remains valid if one understands R to be the normal resistance averaged over the junction determined as usual[16] from the inclination of the asymptotic straight line to the current-voltage characteristic for large V ($V \gg 2\Delta/e$). Consequently, inhomogeneities of the tunnel barrier cannot lead to the observed discrepancies of the experimental and theoretical curves near $H = 0$. Except for the most probable reason for such a discrepancy noted above, an analogous effect can be due to the intrinsic magnetic fields of the Josephson current when the inhomogeneities of the barrier layer are taken into account.[17] In this case ($\lambda_j \ll l$) the distribution of magnetic field in the junction is nonuniform and the problem becomes more complicated.

Attention was drawn in[11] to the qualitative difference in the effect of the destruction of the superconducting state in point and extended junctions. Whereas in point junctions fluctuations however small lead to the appearance of the resistive state, in extended junctions the appearance of the resistive state is sudden when the fluctuation energy reaches a certain threshold level. It is known from the experimental[18] and theoretical[19] studies of the effect of microwave radiation on the volt-ampere characteristics of Josephson tunnel junctions and of other weak links[20] that a high-frequency field leads to a decrease of the amplitude of the superconducting current for $V = 0$ and in this sense this action is equivalent to thermal fluctuations (see the discussion of this problem in[21]).

The experimental dependence of the critical Josephson current on the power of the microwave radiation ($\nu = 9600$ Mcs) is shown in Fig. 8. This dependence is of an unusual form and has features that have not been previously observed in tunnel junctions. In Fig. 8 we present the dependence obtained for an extended junction whose characteristics $I_c(H)$ are shown in Fig. 1. It corresponds to measurements at a stage of the annealing intermediate between b and c in Fig. 1. The continuous recording of the $I_c(P)$ curve was carried out automatically with the aid of a special system. The following features of the $I_c(P)$ curve are noteworthy: a) the small region of increase of the critical current with increasing power of the radiation at $P \approx 0$ (−70— −60 dBm); b) the abrupt decrease of the critical current at certain power levels of the radiation up to the total disappearance of the superconductivity of the junction.

Such features were also observed in measurements with other junctions. The first feature is probably connected with an analogous effect observed for weak bridge links[22] and has so far not been explained theoretically. The second feature can be interpreted qualitatively from the point of view of a nonuniform penetration of the high-frequency field into the extended junction. We note that the destruction of the superconductivity of the junction is not accompanied by the destruction of the superconductivity of the films contained in the junction, since the energy gap determined from the one-particle current-voltage characteristic is almost unchanged. Unfortunately, there is no theory of the effect of fluctuations of arbitrary intensity on the characteristics of extended junctions. A quantitative comparison of the experimental data with theory appears therefore, impossible. We note that in the discussion above we have assumed that the overwhelming effect of a high-frequency field on the constant Josephson current for $V = 0$ is qualitatively analogous to the effect of thermal fluctuations.

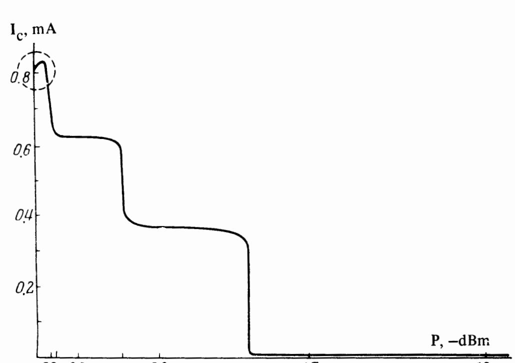

FIG. 8. Experimental dependence of the critical current on the power of the microwave emission. H = O, T = 1.5°K. The power is given in -dBm units, i.e. in negative decibels from the level of 1 milliwatt (for instance, −20 dBm = 10^{-5} watt). The power scale is nonlinear. The dashed circle marks the portion of the characteristic on which the critical current increases with increasing microwave power.

In conclusion we emphasize that although it is in the general case essential to consider the effect of both thermal and structural fluctuations simultaneously, in a specific experimental situation one can observe quite clearly the dominating effect of each type of fluctuation separately.

I express my deep gratitude to I. O. Kulik for considerable help in the theoretical consideration of the problem of structural fluctuations.

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Translated by Z. Barnea
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