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EFFECTIVE RESISTANCE OF AN IMPERFECT TYPE II SUPERCONDUCTOR IN AN OSCILLATING MAGNETIC FIELD

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The effect of an external oscillating magnetic field on direct current flowing in an imperfect type II superconductor is investigated. It is found that when the alternating component of the external field amplitude exceeds a certain threshold value, an effective dc resistance arises. The dependence of this quantity on the oscillating field frequency, on the amplitude of the alternating component, on the magnitude of the constant component, and on the value of the direct current flowing in the superconductor is determined. The dependence of the threshold amplitude of the external field on the current and field strength is investigated. A theory is proposed which explains the experimental results.

1. INTRODUCTION

It is known that hysteresis losses arise in a sample of an imperfect type II superconductor placed in an alternating magnetic field. In the absence of an outside current flowing through the sample the source of the energy dissipated in it is the device creating the alternating magnetic field. If a current is passed through the sample from any external source, at a value less than critical, then the values of the hysteresis losses should be changed somewhat, inasmuch as the distribution of the induction in the material of the sample is changed in this case. The problem as to whether some effective resistance for the flowing direct current exists in this case is important, i.e., whether the source of the current performs any total work per cycle of change of field, and whether the transport current will be damped in a closed superconducting circuit upon change of the magnetic induction in the material of which the circuit is composed.

Earlier,[1] we discovered in our experiments the existence of such a resistance. Measurements were made of the damping of the direct current in a closed superconducting circuit, part of which, representing a bifilar winding of a coil of imperfect type II superconductor, was placed in magnetic field whose value alternated cyclically about a zero mean value. It was found that the effective resistance under these conditions appears only when the amplitude of the external field exceeds some threshold value. This value is significantly above the value of the first critical field for the material used. The hysteretic nature of the effective resistance was also established, i.e., it was found that its value did not depend on the waveform of the pulses of the field and was directly proportional to the frequency of the pulses.

At the present time, this same phenomenon has been investigated in an external magnetic field which is changing cyclically around some non-zero mean value.

The effect of the value of the current flowing through the superconducting sample on the effective resistance of the value of the variable and constant components of the field strength of the external magnetic field has been investigated. Measurements were also made of the frequency dependence of the effective resistance of the sample and the measurements of its critical current as a function of the value of the external field.

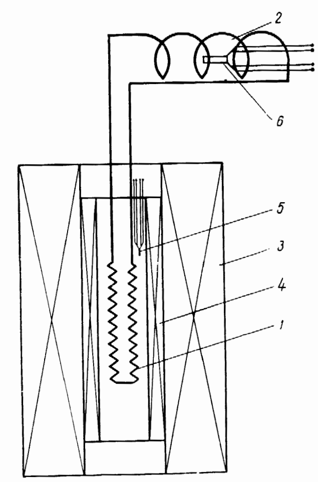

FIG. 1. Schematic diagram of apparatus (explanation in text).

2. EXPERIMENT

1. The method of measurement used is similar to that described in[1]. The circuit arrangement is shown in Fig. 1.

The studied sample 1 is a sufficiently long, double-layer coil, wound on a bakelite form in such a way that the direction of the current is opposite in the layers. Such a system of winding was chosen for convenience in the subsequent theoretical consideration. We used a non-copper-plated wire of the triple alloy 65BT (Nb-Ti-Zr), of diameter 0.25 mm. Its total length was 8.85 m. The measured dependence of the critical current on the external field for this sample is shown in Fig. 2.

The sample was connected in a closed superconducting circuit, the second element of which was a superconducting solenoid 2 with a known inductance, equal to L = 0.97 mH. The solenoid 2 was wound from wire of the same alloy 65BT with increased thickness of copper

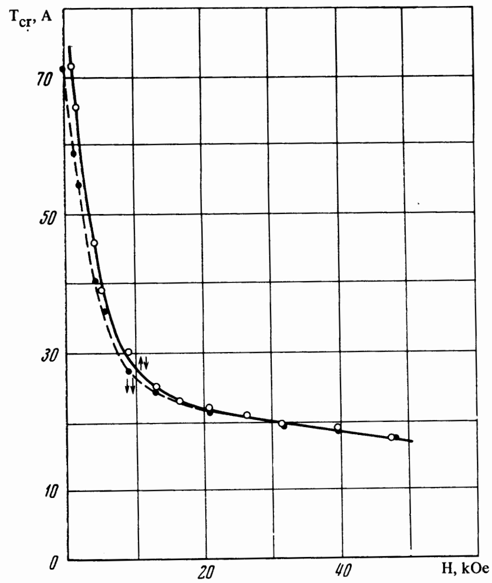

FIG. 2. Experimental dependence of the critical current of the specimen on the field for field of the current parallel (↓↓) and antiparallel (↓↑) to the external field.

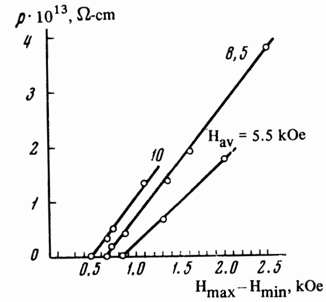

FIG. 3

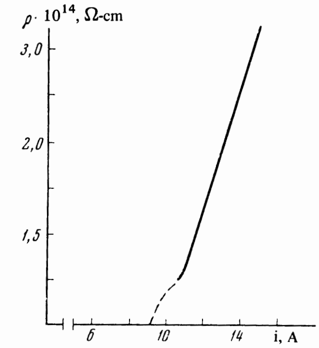

FIG. 4

FIG. 3. Dependence of the effective resistance on the value of the variable field (f = 0.05 Hz) in the case in which the transport current is equal to 15 A.

FIG. 4. Dependence of the effective resistance on the current for H_{av} = 10 kOe, H_{max}-H_{min} = 600 Oe, f = 0.05 Hz.

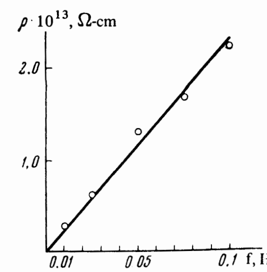

FIG. 5. Frequency dependence of the effective resistance for i = 19.5 A, H = 5.5 kOe, H_{max} - H_{min} = 1500 Oe.

plating (diameter of the superconductor 0.27 mm, diameter over the plating, 0.35 mm). In winding the measurement solenoid, we took into account the fact that the value of the inductance L used in the calculation of the effective resistance of the sample generally depends on the character of the process of current change in it and on its magnetic pre-history.[2] In this connection, its geometry was chosen so that the value of its so-called external inductance was significantly greater than its internal, determined by the magnetic current flowing through the strictly superconducting material. The current in the circuit under consideration was excited in the usual way using a thermal switch.

The constant external magnetic field (up to 50 kOe in a gap of 30 mm) was created by the superconducting solenoid 3. The solenoid 4 served as the source of the alternating magnetic field. This solenoid was supplied from a special low-frequency source of sinusoidal current. The instantaneous values of the external field were recorded by means of a bismuth resistance detector 5, previously calibrated in static fields.

The formation of the effective resistance in the sample studied led to damping of the current in the superconducting circuit, which was recorded by use of the bismuth pickup 6 placed in the solenoid 2. The resistance of the sample was calculated from the rate of fall of the current in the circuit. In order that heating of the sample have no effect on the results of the experiment, changes of the magnetic field were carried out with a frequency of 0.01–0.1 Hz.

By means of the apparatus thus described, one could record sufficiently small values of the resistance. The sensitivity with respect to the specific resistance amounted to 10^{15} ohm-cm in the mean for the duration of the experiment used. The threshold value of the variable field was determined with an accuracy to within 20–30 Oe.

2. Figure 3 shows some results of measurement of the dependence of the effective resistance on the value of the variable field for different values of the constant component; the current in the sample in all cases was the same and amounted to 15 A. The frequency of the variable current was 0.05 Hz. Such measurements show that the effective resistance to the constant current in the sample is observed in the case when the difference of the maximum and minimum values of the external magnetic field exceeds some threshold value. The effective resistance increases linearly with increase in the difference shown.

3. The effective resistance depends also on the current in the circuit. The character of the dependence is seen in Fig. 4. The curve thus introduced was obtained in a single experiment, i.e., the change of the effective resistance was computed from the change in the rate of the continuous decrease in the current in the circuit, from an initial value of i = 15 A to a value for which decrease ceased (i = 9 A).

It is evident that for this value of the current (i = 9 A) the amplitude of the alternating magnetic field used is threshold for the given mean external field. With increase in the current in the circuit, the value of the effective resistance falls off linearly. Close to threshold (the dashed part of the curve) the error in the determination of the resistance is comparable with the value of the resistance itself; in this connection, it has not been possible to establish the exact form of the curve for the value of the current close to threshold.

4. Figure 5 shows the dependence of the value of the effective resistance on the frequency of the variable field. The difference in the maximum and minimum values of the field was maintained equal to 1500 Oe, the

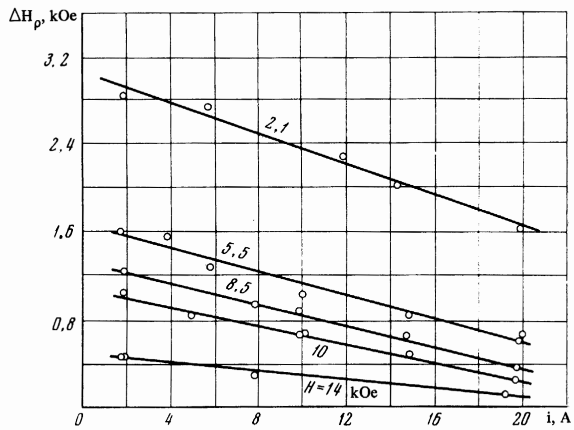

FIG. 6. Threshold values of the difference in the maximum and minimum values of the variable field as a function of the current.

mean constant field was 5.5 kOe. Measurements were carried out for a current of 19.5 A in the circuit.

In the range of frequencies studied, this dependence has a linear character. This circumstance confirms the hysteretic nature of the effective resistance and also indicates that the heat release in the specimen during the time of the experiment does not affect the results of the experiment.

5. The dependence of the threshold values of the difference in the maximum and minimum values of the alternating field ΔH_ρ on the current in the circuit is shown in Fig. 6. Upon increase in the current in the circuit, ΔH_ρ decreases linearly.

3. THEORY

In this experiment, we are essentially dealing with a multiply connected type II superconductor, placed in a variable magnetic field. The problem is to explain under what conditions and for what law the time average of the total current, originally generated in this superconductor, will be damped. The solution of this problem is given below for a simple model. We shall then assume that the distribution of the induction inside the specimen is the same as in a plane layer with effective thickness 2a.

We shall assume that the entire system is completed so that the fluxes of the magnetic field $\Phi_0 = c^{-1}Li$ in the measurement solenoid and Φ_1 through the surface, which is extended on some contour Γ_1 chosen inside the conductor on which the specimen is wound, can be clearly distinguished. From the induction law,

$$-\frac{1}{c}\frac{\partial\Phi_0}{\partial t}-\frac{1}{c}\frac{\partial\Phi_1}{\partial t}=\int\limits_{\Gamma_0}\mathbf{E}\,d\mathbf{l}+\int\limits_{\Gamma_1}\mathbf{E}\,d\mathbf{l}, \tag{1}$$

where Γ_0 and Γ_1 form a closed circuit. We can assume that the electric field intensity along the line Γ_0 is equal to zero, since the total current changes only slightly under the conditions given, and therefore the distribution of the induction in the wire, which is wound on the measuring solenoid, will change only near its surface. Conversely, $E \neq 0$ along the line Γ_1 in general, since the specimen is in a variable external field. Thus

$$-\frac{1}{c}\frac{\partial\Phi_0}{\partial t}=V_S\equiv\frac{1}{c}\frac{\partial\Phi_1}{\partial t}+\int\limits_{\Gamma_1}\mathbf{E}\,d\mathbf{l}. \tag{2}$$

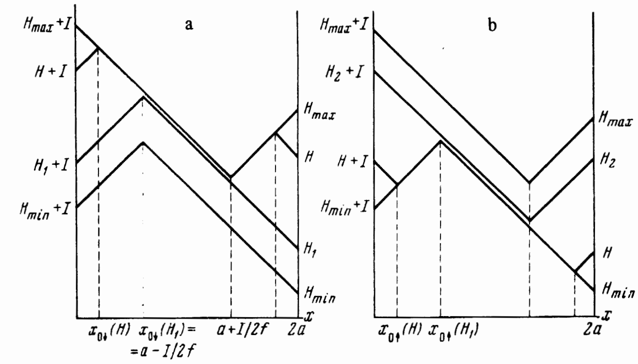

FIG. 7. Distribution of the magnetic induction in a layer for decrease (a) and for increase (b) of the external field in the case in which the external field and the field of the current are parallel.

The quantity V_S is evidently the voltage at the ends of the sample, while the total current is less than critical, the distribution of the induction inside the specimen is such that there is at least one line along which B has a maximum or a minimum, or B = 0. Along such a line, which we shall denote by Γ', we have $\mathbf{E} = c^{-1}\mathbf{B}\times\mathbf{v} = 0$ either because the velocity of motion of the vortex filaments is v = 0, or because B = 0. Therefore, the integration curve Γ_1 in (2) can be closed along Γ' and we obtain

$$V_S=\frac{1}{c}\int\limits_{S'}\frac{\partial\mathbf{B}}{\partial t}\,dS, \tag{3}$$

where S′ is the surface subtending the closed contour, which is identical with Γ' inside the specimen. In general, it is not possible to take the derivative outside the integral sign, since the line Γ' may shift upon change of the external field and current. However, if there is a whole region inside the superconductor in which the distribution of the induction does not change upon periodic variation of the external field, then the line Γ' can be chosen inside this region and then the mean value of V_S over the period T will be:

$$\langle V_S\rangle=\frac{1}{T}\int\limits_0^T V_S\,dT=\frac{1}{cT}\int\limits_{S'}[\mathbf{B}(T)-\mathbf{B}(0)]\,dS=0$$

and, consequently, the total current will not be damped.

The distribution of the induction inside the layer for cases in which the external field increases from H_{min} to H_{max} or decreases from H_{max} to H_{min}, is shown schematically in Fig. 7. Here

$$I=4\pi i/cda \tag{4}$$

is the magnetic field (in the region between the layers) of the current i flowing in the specimen, d the diameter of the wire and the coefficient $\alpha \approx 1$ takes into account the effect of the geometric factors and the magnetization of the specimen. Figure 7 shows the case in which the external field H and the field of the current are parallel.

The fact that the specimen has almost no inductance allows us to assume that the external magnetic field has the same value in the space between the layers as outside.

If, as is usual,[3] the law for the penetration of the field in a hard superconductor is written in the form

$$\partial H(B)/\partial x=\pm f(B), \tag{5}$$

then the field of the critical current $I_{c1}(H)$ for an external field H will be determined by the equation

$$\int\limits_{H}^{H+I_{c1}(H)} \frac{dH}{f(B)} = 2a. \qquad (6)$$

On the other hand, the value of the external field H_1 at which the region vanishes with constant distribution of the induction, can be found in the case of a decrease of the external field from H_{max}, from the equation

$$\int\limits_{H_1}^{H_{max}+I} \frac{dH}{f(B)} = 2a. \qquad (7)$$

Comparing this expression with (6), we find

$$H_{max} + I = H_1 + I_{c1}(H_1).$$

In order that the sample have a resistance that differs somewhat from zero, we must satisfy the condition $H_{min} \leq H_1$, i.e.,

$$H_{max} - H_{min} + I \geqslant I_{c1}(H_{min}), \qquad (8)$$

if the external field and the field of the current are parallel ($\uparrow\uparrow$). If the external field and the current field are antiparallel ($\downarrow\uparrow$), then the field of the critical current will have a different value, I_{c2}, and we have in place of (8)

$$H_{max} - H_{min} + I \geqslant I_{c2}(H_{max}). \qquad (8a)$$

For the value of the effective resistance, if we can assume $f(B) \approx$ const and $H \gg H_{c1}$, we get from (5) for the case of a decrease of the external field from H_{max} (see Fig. 7a)

$$B(x) = H + I + fx, \quad 0 < x < x_{0\downarrow},$$

where $x = x_{0\downarrow}(H)$ is the place on which the induction distribution has a maximum.

For $H_{max} > H > H_1 = H_{max} + I - 2fa$

$$x_{0\downarrow}(H) = (H_{max} - H) / 2f,$$

and for $H_1 > H > H_{min}$

$$x_{0\downarrow}(H) = a - I / 2f.$$

In the case of an increase of the field from H_{min} (see Fig. 7b)

$$B(x) = H + I - fx, \quad 0 < x < x_{0\uparrow},$$

where $x = x_{0\uparrow}$ is the plane on which B(x) has a relative minimum.

For $H_{min} < H < H_2 = H_{min} + 2fa - I$,

$$x_{0\uparrow} = (H - H_{min}) / 2f,$$

and for $H_2 < H < H_{max}$

$$x_{0\uparrow} = a + I / 2f.$$

Using these results from Eq. (3), we get

$$\langle V_S \rangle = \frac{l}{cT} \int\limits_{H_{min}}^{H_{max}} [x_{0\uparrow}(H) - x_{0\downarrow}(H)]\, dH = \frac{lI}{cTf}(H_{max} - H_{min} + I - I_c) = R_{eff}\, i, \qquad (9)$$

where l is the length of the wire on which the sample is wound, $I_c = I_{c1} = I_{c2} = 2fa$ is the field of the critical current (when f = const, we have $H_{c1} = I_{c2}$). We have

$$R_{eff} = \frac{4\pi l}{c^2 dTfa}(H_{max} - H_{min} + I - I_c). \qquad (10)$$

Substituting (9) in (3) and taking the time average, we get the equation determining the current damping:

$$\frac{L}{c^2}\frac{\partial i}{\partial t} = \frac{4\pi i l}{c^2 dTfa}\left[H_{max} - H_{min} + \frac{4\pi}{cda}(i - i_c)\right]. \qquad (11)$$

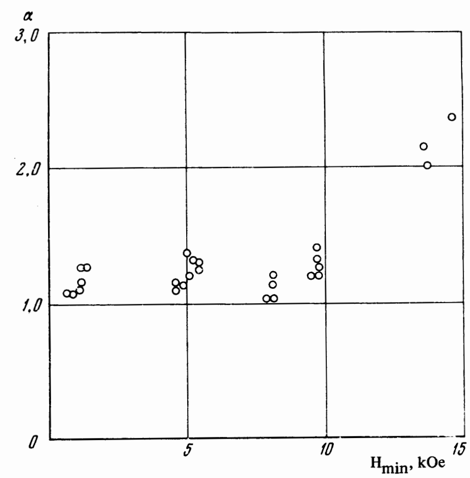

FIG. 8. Dependence of the dimensionless parameter α on the minimum value of the field for different i, H_{av} and $H_{max} - H_{min}$.

4. DISCUSSION OF RESULTS

The proposed theory qualitatively explains the experimental results:

a) the existence of a threshold value $H_{max} - H_{min} = \Delta H_\rho$;

b) the linear dependence of ΔH_ρ on the current in the sample (see Fig. 6);

c) the linear dependence of the effective resistance on the frequency, on the current and on $H_{max} - H_{min}$ (see Figs. 3–5).

For a quantitative comparison of theory with experiment, it is necessary to take it into account that there are two parameters in the theory (α and a), the values of which are known only approximately. The parameter α (see Eq. (4)) should be close to unity. It enters implicitly in Eqs. (8) and (8a), which determine the threshold values of ΔH_ρ.

The quantities $H_{max} - H_{min}$ and $i - i_c$ are known from experiment, so that

$$\alpha = \frac{4\pi(i_c - i)}{cd(H_{max} - H_{min})}.$$

Figure 8 shows the results of calculation of the parameter α from these formulas for different values of the external field and the current in the sample. For H ≤ 12 kOe and for the investigated values of the current, the parameter α has been found to be close to 1.2, which is in complete agreement with the theoretical prediction. For H > 12 kOe, a significant increase is observed in the parameter α with increase in the field, which indicates the inapplicability of Eqs. (8), (8a). The threshold values of $H_{max} - H_{min}$ are shown to be much less than follows from these formulas. For an explanation of this effect, we note that generally the penetration of the vortex filaments in the hard superconductor has a statistical character.[4]

In this connection, Eq. (5) describes only the mean induction distribution. At the same time, for the effects considered here, the fluctuations (microscopic current jumps), which accompany magnetic reversal of the specimen, should have a significant value, since the effective resistance in the final analysis arises from the fact that the induction distribution is different for the growth and for the decay of the external field. The role of the fluctuations increases when the induction is distributed more or less uniformly, i.e., when the external field is sufficiently large in comparison with the field of the critical current.

So far as the value of the effective resistance is concerned, it is necessary to keep in mind that Eq. (10) was introduced under the assumption that the function f(B) = const. In the region $H < 12$ kOe, this condition is not satisfied. Therefore, Eq. (10) should give the absolute values for R_{eff} that are valid only in order of magnitude.

The deviation of the effective resistance, obtained experimentally from the rate of damping of current in the circuit, from the effective resistance calculated from (10) for different values of the transport current, amounts in the mean to $\pm 30\%$.

It follows from what has been set forth above that the given theoretical considerations describe correctly the experimental results over a width range of values of the external field.

In conclusion, the authors consider it their pleasant duty to express their thanks to M. G. Kremlev for useful discussions, and to V. V. Titov for help in preparing the experimental apparatus and operation of the experiment.

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