DEPENDENCE OF MULTIPLE-MODE EXCITATION IN SEMICONDUCTOR LASERS ON THE ABSORPTION NONLINEARITY

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The effect of two-photon absorption on the excitation of axial modes in semiconductor lasers (SL) is considered. It is shown that the absorption nonlinearity associated with two-photon transitions can lead to simultaneous excitation of several modes near the generation threshold even in the case of a spectrally homogeneous line and a spatially homogeneous medium. The effect of two-photon absorption on mode excitation in SL made of various semiconducting materials, and also the effect of nonlinearity of the absorption coefficient on the watt-ampere characteristic of the SL, are discussed.

FAST collisions between electrons lead to the simultaneous line broadening of SL radiation in a significant range of energies;[1,2] therefore, the energy inhomogeneity cannot serve as the reason for the excitation of several modes in SL near the generation threshold. Spatial inhomogeneity of the active medium and incomplete overlap of the fields of the different modes, which was considered in [3,4], in the account of diffusion of the free carriers, allow us to explain the excitation of several modes for pumping power that is larger by a factor of 1.5–2 in comparison with the generation threshold.[3] Usually the excitation of two or three axial modes in a single spatial generation channel takes place upon increase in the pumping by 2–10% from the excitation threshold of the first mode.[5] Thus, the spatial inhomogeneity of the active medium also fails to explain the complete picture of multiple-mode generation in SL near the autoexcitation threshold. In the present work we consider the effect of nonlinear absorption on the excitation of the various axial modes in the stationary generation regime.

The effective mechanism of nonlinear absorption in gallium arsenide, which is very widely used for the preparation of SL, is two-photon absorption. The expression for the probability of the two-photon transition in gallium arsenide was obtained in [6]: $w = bn_\omega^2$. Here n_ω is the density of photons, b the coefficient which depends on the frequency of the field ω. In the case of interaction of a semiconductor with the set of fields of different modes, by following [6,7] we can assume that the transition probability is proportional to the total photon density: $w = b\left(\sum_i n_{\omega_i}\right)^2$. Correspondingly, the coefficient of two-photon absorption is proportional to the total photon density:

$$\alpha_2 = a\sum_i n_{\omega_i}. \qquad (1)$$

The dependence of the coefficient a on the frequency can be neglected in the limits of the width of the generation spectrum.

The condition for the excitation of each mode requires the equality of amplification and losses in each mode. As the radiation density increases in the first excited mode, the nonlinear losses increase in the frequencies of all the modes. However, because of the different spatial distribution of the fields of the various modes, the losses of the excited mode increase more strongly than the losses of the other modes. Part of the pumping goes to the increase in the amplification coefficient, which is necessary for compensation of the nonlinear increase in the losses of the excited mode. Since an increase in the amplification coefficient takes place here at all frequencies near the maximum, excitation of the axial modes of nearby frequency is possible. The radiation density in each excited mode is regulated by the nonlinear losses in such a way that the losses of the mode are equal to the amplification at its frequency.

Let us consider a generator in which the concentration of holes p is much greater than the concentration of electrons n, which is usually the case in injection SL.

In contrast with the excitation mechanism of several modes associated with the spatial inhomogeneity of the active medium, for which the essential role is played by diffusion of free carriers,[3] the mechanism considered here is sufficiently effective even in a spatially homogeneous medium (that is, for very strong diffusion). Furthermore, on the basis of the results of [3], one can show that in strongly doped gallium arsenide of p-type diffusion of the electrons within their lifetime τ manages to equalize the density of electrons at the nodes and loops of the standing waves. Correspondingly, the rate equations of the stationary generation[3] have the following form:

$$J = \frac{n}{\tau} + \frac{c}{L}\sum_{i=0}^{M}\left[g(\omega_i, n)\int_0^L n_{\omega_i}(z)\,dz\right], \qquad (2)$$

$$[g(\omega_k, n) - \alpha_1]\int_0^L n_{\omega_k}(z)\,dz = a\int_0^L\left[\sum_i n_{\omega_i}(z)\right]n_{\omega_k}(z)\,dz. \qquad (3)$$

Here J is the pumping density, c the light velocity in the semiconductor, $g(\omega_i, n)$ the amplification coefficient at the frequency ω_i, M the number of excited modes, α_1 the loss coefficient, which does not depend on the radiation density. The first equation describes the balance of pumping and interband (or impurity–band) transitions, the remaining M equations, the balance of amplification and losses. Equations (2) and (3) are valid in the weak field approximation, when one can neglect the components proportional to n_ω^2.

The solution of the wave equation for the one-dimensional model of SL with a homogeneous amplifying medium between the mirrors ($0 \le z \le L$) gives the following distribution of photon density along the resonator axis:[8]

$$n_\omega(z) = N_\omega \ln R^{-1}\,\mathrm{sh}^{-1}\left(\frac{\ln R^{-1}}{2}\right)\left[\cos^2 q'\left(z-\frac{L}{2}\right)+\mathrm{sh}^2 q''\left(z-\frac{L}{2}\right)\right],$$

$$q' = \frac{2\pi m}{L}+\frac{L(q'')^2}{2\pi m}, \quad q'' = \frac{\ln R^{-1}}{2L}. \tag{4}$$

Expression (4) is normalized to the average photon density N_ω, R is the coefficient of reflection from the resonator boundary. The solution of the equation was joined with the plane wave on the boundary of the semiconductor—the exterior medium. At a typical value of the reflection coefficient R = 0.3, the value of the first component of the right side of (4), averaged over z, is much greater than the mean value of the second component. Therefore one can use the expression

$$n_\omega(z) = 2N_\omega \cos^2 qz, \quad q = \pi m / L. \tag{5}$$

in what follows for the photon density. Substituting Eq. (5) in Eqs. (2) and (3), we get the equation for the mean values of N_ω:

$$J = \frac{n}{\tau} + c\sum_{i=0}^{M} g(\omega_i, n) N_{\omega_i}, \tag{6}$$

$$g(\omega_k, n) = \alpha_1 + a\sum_{i=0}^{M} N_{\omega_i} + \frac{a}{2} N_{\omega_k}. \tag{7}$$

In this case, a graphic solution of Eqs. (6) and (7) can be obtained if the amplification coefficient near the frequency maximum has symmetric shape, not depending on the electron density;

$$g(\omega, n) \approx g(\omega_0, n_0) + \frac{dg}{dn}(\omega_0, n_0)\Delta n + \frac{1}{2}\frac{d^2 g}{d\omega^2}(\omega_0, n_0)(\Delta\omega)^2. \tag{8}$$

Here $g(\omega_0, n_0)$ is the amplification coefficient at the threshold of excitation of the first mode; ω_n is the frequency at which the amplification coefficient is a maximum (i.e., it is assumed that the frequency of the first excited mode coincides with ω_0); $n_0 = J_0\tau$ is the threshold electron density, J_0 the threshold pumping density; $\Delta n = n - n_0$, $\Delta\omega = \omega - \omega_0$.

Substituting Eq. (8) in Eqs. (6) and (7), we find the solution of these equations:

$$\frac{\Delta J_m}{J_0} = -\frac{m(\Delta\omega_1)^2}{2n_0}\frac{d^2 g}{d\omega^2}\left(\frac{dg}{dn}\right)^{-1}\left[m + \frac{2}{3}(4m^2-1)\left(1+\frac{c\tau\alpha_1}{a}\frac{dg}{dn}\right)\right], \tag{9}$$

$$N_{\omega_k} = -\frac{d^2 g}{d\omega^2}(\Delta\omega_1)^2\frac{m^2-k^2}{a}. \tag{10}$$

In Eqs. (9) and (10), $J_0 + \Delta J_m$ is the threshold of excitation of the m-th mode, $\Delta\omega_1$ the distance between neighboring eigenfrequencies of the resonator, m the number of excited, equidistant modes from the longwave or the shortwave side of the first excited mode. The total number of excited axial modes is equal to $M = 2m + 1$. Measurements of the coefficient of two-photon absorption in gallium arsenide at long wavelength $\lambda = 1.06\mu$ show that in a wide range of intensities of the light flux ($100\ \mathrm{kW/cm^2} \le I \le 15\ \mathrm{MW/cm^2}$), the coefficient of two-photon absorption is proportional to the intensity of the flux $\alpha_2 = b'I$, where $b' = 2\ \mathrm{cm/MW}$[9] (the coefficient $a \approx 5\times10^{-5}\ \mathrm{cm^2}$).

To obtain numerical estimates by Eqs. (9) and (10), we can use the expression for the amplification coefficient,[10] which corresponds to the band model of a strongly doped semiconductor with an exponential "tail" of density of electron states $\rho_e = \rho$, $\exp(E_e/\epsilon_0)$ (E_e is the energy of the electron) and with a δ-shaped acceptor level.[11] Such a band model is ordinarily used for injection of SL. One can show that in this case the following relations are valid for the amplification coefficient:

$$\frac{dg}{dn}(\omega_0, n_0) = \frac{g(\omega_0, n_0)}{n_0},$$
$$\frac{d^2 g}{d\omega^2}(\omega_0, n_0) = -\sqrt{1+4\xi^2}\, g(\omega_0, n_0)\frac{\hbar^2}{\varepsilon_0^2}, \tag{11}$$

where $\xi = \epsilon_0/2kT$. Substituting (11) in (9) and (10), we get

$$\frac{\Delta J_m}{J_0} = \frac{\sqrt{1+4\xi^2}}{2}\left(\frac{\hbar\Delta\omega_1}{\varepsilon_0}\right)^2 m\left[m+\frac{2}{3}(4m^2-1)\left(\frac{c\tau\alpha_1^2}{an_0}+1\right)\right], \tag{12}$$

$$N_{\omega_k} = \sqrt{1+4\xi^2}\frac{\alpha_1}{a}\left(\frac{\hbar\Delta\omega_1}{\varepsilon_0}\right)^2(m^2-k^2) \tag{13}$$

The threshold density of electrons depends on the value of the losses. The connection between them in injection SL, for example, can be found through the relation $g(\omega_0, n) = \beta j$, where $j = eJl$ is the current density, β the amplification factor, l the width of the region of inverted population. At the generation threshold, $g = \alpha_1$ and $n_0 = J_0\tau$. Thus, Eq. (12) for injection SL has the following form:

$$\frac{\Delta J_m}{J_0} = \frac{\sqrt{1+4\xi^2}}{2}\left(\frac{\hbar\Delta\omega_1}{\varepsilon_0}\right)^2 m\left[m+\frac{2}{3}(4m^2-1)\left(1+\frac{ec\alpha_1\beta l}{a}\right)\right] \tag{14}$$

Using typical numerical values of the parameters $\alpha_1 = 50\ \mathrm{cm^{-1}}$, $\beta = 10^{-2}\ \mathrm{cm\text{-}A^{-1}}$, $l = 1\ \mu$,[12] $\hbar\omega_1/\epsilon_0 = 4\times10^{-2}$, ($\Delta\lambda_1 = 3$ Å, $\epsilon_0 \approx 13$ meV), $\xi = 1$ ($T \approx 80^\circ$K), we find $\Delta J_1/J_0 \approx 6\times10^{-2}$ and $\Delta J_m/J_0 \approx 8\,m^3\times10^{-2}$ for $m \ge 2$. Thus, the resonance nonlinear absorption gives an excellent explanation of the picture of multimode generation near the threshold of SL self-excitation. The experimental dependence of the number of excited axial types of oscillations can be represented in the form $m \sim [(j-j_0)/j_0]^{1/2}$,[5] which is also in good agreement with Eq. (12).

The non-axial modes can also be included in the obtained sequence of excitation of modes. For this purpose, one must know the values of the amplification coefficients and the losses of each nonaxial mode in the specific cases, since these quantities can change as a function of the spatial distribution of the field of the mode and of the spatial inhomogeneity of the pumping.

Estimates of the value of the coefficient of two-photon absorption show that in SL prepared on the basis of indium arsenide ($\hbar\omega_0 \approx 0.4$ eV), the coefficient a is approximately seven times that in gallium arsenide. Therefore in SL prepared on a base of indium arsenide, the number of excited modes for otherwise equal conditions ought to be larger than in SL prepared on a base of gallium arsenide. For example, it follows from Figs. 5 and 6 of [13] that in injection SL on indium arsenide for $\Delta\lambda = 34\,\text{Å}$ and $\Delta J/J_0 = 100\%$, the total number of excited modes $M = 9$, which agrees with Eq. (14) if we assume that $a = 3\times10^{-14}\ \text{cm}^2$ (approximately the same numerical values were used for the other parameters as for SL on gallium arsenide). For cadmium selenide b′ (equal to 0.43–0.9 cm/MW[14]) is much less than for gallium arsenide. Therefore, the mechanism of excitation of some modes in cadmium selenide, considered here, ought to be less effective.

We estimate the value of the power P_1 of single-mode SL radiation at the excitation threshold of the second mode:

$$P_1 = cN_{\omega_0}\hbar\omega_0 s \ln R^{-1}. \tag{15}$$

Here s is the area of the transverse cross section of the spatial generation channel, R is the reflection coefficient from the boundary of the resonator. Such an expression for the power P_1 is obtained from the second component in the right side of Eq. (6) after multiplication by $\hbar\omega$, by the function of the external output $(L\alpha_1)^{-1} \times \ln R^{-1}$ and by the volume of the spatial generation channel $v = sL$ (L is the distance between the boundaries of the resonator). Substituting Eq. (13) in (15), we get the value of the power $P_1 \approx 20$ MW at the excitation threshold of the second mode ($m = 1$, $k = 0$) for $R = 0.3$, $\hbar\omega = 1.5$ eV, $s = 2\times10^{-7}\ \text{cm}^2$ [12] (the numerical values of the other parameters needed for the calculation of P_1 are given above).

The power of single-mode radiation can be greater than the presented value when the dimensions of the coherently generating region of the semiconductor significantly exceed the typical dimensions of the spatial generation channel. Increase in the frequency separation between modes, use of a composite resonator,[15] and also an increase in the loss coefficient, especially in the illumination of the reflecting boundaries of the resonator, all lead to an increase in the power of single-mode generation. The dependence of the amount of excited modes on the coefficient of linear losses and on the amplification factor (14) can be used for the experimental test of the effect of nonlinear absorption on the excitation of modes in SL.

We proceed to the derivation of the expression for the watt–ampere characteristics of SL. The amplification coefficient maximum, as a function of frequency, increases linearly with increase in pumping. Therefore, the quantity $\Delta n = n - J_0\tau$ in Eq. (6) can be eliminated by means of the relation

$$\frac{dg}{dn}\Delta n \approx a\sum_i N_{\omega_i},$$

in which the derivative dg/dn does not depend on the change in concentration of the electrons. The amplification coefficient $g(\omega_i)$ in Eq. (6) must be replaced by the total loss coefficient $\alpha_1 + a\sum_i N_{\omega_i}$ and we must add the term

$$\frac{1}{2}ca\Big(\sum_i N_{\omega_i}\Big)^2,$$

on the left side of (6), corresponding to an increase in the electron concentration in the conduction band due to two-photon absorption. The value of the radiation power (P) which comes outside is proportional to the next term in the right-hand side of Eq. (6). This component must be multiplied by a function of the external output $(\alpha_1 L + aL\sum_i N_{\omega_i})\ln R^{-1}$ and by the total volume of the spatial generation channels $V = LS$ and the quantity $\sum_i N_{\omega_i}$ can be expressed, by means of (6), in terms of the increase in the pumping density $\Delta J = J - J_0$. We finally obtain

$$P = \hbar\omega S \ln R^{-1}\frac{c\alpha_1}{a}\left(\sqrt{1+2\Delta Ja/c\alpha_1^2}-1\right). \tag{16}$$

In the numerical example given above, $c\alpha_1^2/aJ_0 = 16$, which allows us to expand the root in Eq. (16) in powers of $\Delta J\ J_0$:

$$P = \hbar\omega S\ln R^{-1}\left[1-\frac{1}{4}\left(\frac{aJ_0}{c\alpha_1^2}\right)\frac{\Delta J}{J_0}\right]\frac{\Delta J}{\alpha_1}.$$

The decrease in the power because of the nonlinearity of absorption in the example considered amounts to 10% for exceeding the threshold by a factor of seven. Such a weak nonlinearity of the watt–ampere characteristics will evidently be masked by the increase in the temperature, which also produces a departure from the linear dependence of the radiation power on the pumping.

In the case of spatially inhomogeneous medium, the quantity $\Delta J_m/J_0$ can be found in the same way as was done in [3], by adding to the equation of the present paper terms associated with nonlinear absorption. As a result, we get an expression analogous to (9) but one which takes into account the effect of spatial "burning out" of the inversion population

$$\frac{\Delta J_m}{J_0} = -\frac{(\Delta\omega_1)^2\, d^2g/d\omega^2}{2n_0\, dg/dn}m\left[m+\frac{2}{3}(4m^2-1)\right.$$
$$\left.\times\left(a+c\tau\alpha_1\frac{dg}{dn}\right)\Big/\left(a+\frac{c\tau\alpha_1}{1+4D\tau q^2}\frac{dg}{dn}\right)\right], \tag{17}$$

where D is the coefficient of ambipolar diffusion, q the wave vector in the semiconductor. Equation (17) transforms into (9) as $D\to\infty$, when the active medium is spatially homogeneous. In the case of very weak nonlinearity of absorption ($a\to 0$), Eq. (17) is identical with the similar expression of [3].

Thus the nonlinearity of the absorption coefficient leads to the excitation of several modes even when the active medium of the SL is energetically and spatially homogeneous.

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