INFLUENCE OF SAMPLE SIZE ON THE CURRENT-VOLTAGE CHARACTERISTIC IN MEDIA WITH AN AMBIGUOUS DEPENDENCE OF ELECTRON TEMPERATURE ON FIELD STRENGTH

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The energy balance equation is solved and all possible stationary distributions of the electron temperature over the cross section of a finite-size sample are determined. A classification of the distributions is presented. Only one of them is stable with respect to small perturbations; depending on the sample size and on the electric field strength, this may be either a uniform or monotonic distribution. For sufficiently thick samples there exist field-strength ranges for which not a single solution exists. This leads to hysteresis in the current-voltage characteristic. Current-voltage characteristics for samples with different transverse dimensions are plotted.

NONLINEAR effects connected with heating of the electron gas becomes significant in semiconductors even in relatively weak electric fields. One of the most interesting nonlinear effects, interest in which has been increasing of late, is the occurrence of a decreasing section on the current-voltage characteristic. This property is possessed by the so-called S- and N-shaped current-voltage characteristics. There are many reasons for the appearance of a decreasing section on these characteristics[1]. In this communication we consider an S-shaped current-voltage characteristic due to the presence of superheat mechanisms in semiconductors.

Our purpose is to investigate the true form of the current-voltage characteristic in bounded samples as functions of the dimensions of the latter. The limiting case of large transverse dimensions (relative to the applied voltage) of the sample was investigated qualitatively in[1-3]. We note, however, that to obtain the true form of the current-voltage characteristic it is necessary to take a correct account of the boundary conditions, something not done consistently in the cited papers.

1. FORMULATION OF PROBLEM AND HOMOGENEOUS SOLUTIONS

We consider a semiconductor in which the mean free path connected with the energy transfer is much larger than either the mean free path connected with momentum transfer or the Debye radius. In addition, we assume that the frequency of the interelectron collisions exceed the electron-lattice collision frequency connected with the energy transfer. Under these assumptions, the symmetrical part of the electron distribution function is Maxwellian with an effective temperature Θ.

Let the semiconducting sample have the form of a parallelepiped, in which a constant electric field E is applied along the x direction, and the contacts in the directions y and z are open ($j_y = j_z = 0$, j—current density), and on the boundary planes (y = 0, b and z = 0, a)

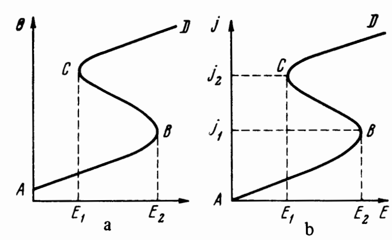

FIG. 1

it is assumed that there are no specific energy-dissipation mechanisms. Then the equation for the temperature and the boundary conditions are written in the form:

$$\frac{3}{2}N\frac{\partial\Theta}{\partial t}=\frac{\partial}{\partial y}\varkappa(\Theta)\frac{\partial\Theta}{\partial y}+\frac{\partial}{\partial z}\varkappa(\Theta)\frac{\partial\Theta}{\partial z}+\sigma(\Theta)E^2-A(\Theta), \quad (1)$$

$$\left.\frac{\partial\Theta}{\partial y}\right|_{y=b;\,0}=\left.\frac{\partial\Theta}{\partial z}\right|_{z=a;\,0}=0, \quad (2)$$

where N is the carrier concentration, κ is the thermal conductivity, σ is the specific electric conductivity, $A \sim N\nu(\Theta)(\Theta - T)$ is a term describing the transfer of heat to the lattice, and $\nu(\Theta)$ is the frequency of the collisions causing this transfer. Equation (1) for the temperature, written in this form, presupposes that the problem is homogeneous with respect to x.

We note that Eq. (1) admits of a stationary homogeneous solution that satisfies automatically the boundary conditions (2). In this case Eq. (1) takes the form

$$\sigma(\Theta)E^2-A(\Theta)=0. \quad (3)$$

If Eq. (3) has three roots with respect to Θ, then an S-shaped dependence of the temperature on the field is obtained (Fig. 1a), and the corresponding current-voltage characteristic (Fig. 1b) is connected with the temperature by the formula

$$j=\sigma(\Theta)E. \quad (4)$$

The decreasing section on the curve of Fig. 1a (and consequently also on Fig. 1b) is unstable against small

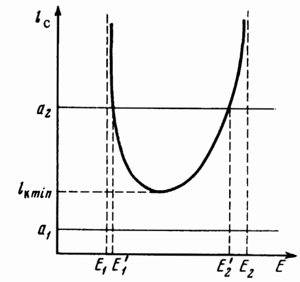

FIG. 2

perturbations[4,5]. Thus, if we confine ourselves to perturbations of the form $\delta\Theta(y, z)e^{-i\omega t}$, which leave the carrier density constant ($\omega \ll 4\pi\sigma/\epsilon$, where ϵ is the dielectric constant of the lattice), then we obtain for ω the equation

$$\omega = i\cdot\frac{2}{3}\frac{\varkappa}{N}\pi^2\left[l_\kappa^{-2}(E)-\left(\frac{n^2}{a^2}+\frac{m^2}{b^2}\right)\right],$$
$$l_\kappa^{-2}(E)=\frac{1}{\pi^2\varkappa}\left(\frac{d\sigma}{d\Theta}E^2-\frac{dA}{d\Theta}\right)\equiv\frac{2}{\pi^2}\frac{\sigma E}{\varkappa}\left|\frac{d\Theta}{dE}\right|^{-1}. \tag{5}$$

A homogeneous fluctuation (m = n = 0) corresponds to a frequency $\omega_0 = i\cdot(2/3)(\kappa/N)(\pi/l_c)^2$. Therefore such a homogeneous fluctuation always increases in time, as a result of which there is realized one of two possible stable branches[1] (AB or CD). In other words, there exists a range of currents $j_1 < j < j_2$, which are never reached in any regime (field regime), and the decreasing branch drops out of consideration in general in this case.

If we connect the sample in a circuit with a serious ballast resistance R of sufficient magnitude (the current regime), then the homogeneous will attenuate with time (Im ω shifts towards negative values). The ballast resistor, which stabilizes the homogeneous perturbation, should satisfy the inequality[4]

$$R > \frac{L}{S}\left|\frac{dj}{dE}\right|^{-1}, \tag{6}$$

where L is the length of the sample and S is the transverse cross section area.

If follows from (5) that if $l_c(E) > a, b$, then inhomogeneous perturbations (m, n $\neq$ 0) in the given field E attenuate with time. In the case of the inverse inequality we have Im $\omega > 0$, and since the inhomogeneous perturbations are not connected with the external circuit, the homogeneous distribution of the temperature is unstable in all regimes. It is clear therefore that at fixed dimensions of the sample, the sign of the imaginary part of ω, corresponding to the inhomogeneous perturbation, is determined by the function $l_c(E)$ when the field is varied. The character of this dependence follows from the definition (5) of l_c and is shown in Fig. 2. As seen from this figure, in fields E_1 and E_2 the quantity $d\Theta/dE$, and with it also l_c, vanishes, and in some intermediate field assumes a minimum value $l_{c\,min}$.

Thus, we can conclude that if a, b $< l_{c\,min}$ (a_1 in Fig. 2), and if $R > (L/S)\{|dj/dE|^{-1}\}_{max}$, then the homogeneous distribution of the temperature is stable in the entire interval of fields $E_1 < E < E_2$, and the form of the current-voltage characteristic is given by Fig. 1b. If the foregoing condition on the transverse dimension is violated (a_2 in Fig. 2), then there are two regions adjacent to E_1 and E_2, whereas before the homogeneous distribution of the temperature is stable. The limits of these regions E_1' and E_2' are determined from the equation

$$l_c(E) = \max(a, b). \tag{7}$$

The region of the fields E, in which the homogeneous distribution is unstable ($E_1' < E < E_2'$) is the larger the thicker the sample. This region of fields calls for an additional investigation, which will be carried out below.

In concluding this section we note that the problem was regarded as homogeneous in x from the very outset. This is connected with the fact that the fluctuations that depend on x decrease the growth increment of the perturbation (the fluctuations that depend only on x generally do not lead to instability of the homogeneous temperature distribution[1]).

2. INHOMOGENEOUS STATIONARY SOLUTIONS AND THEIR STABILITY

Let us consider the intermediate interval of fields, where the condition $a < l_c(E)$ is not satisfied, and let us construct inhomogeneous stationary solutions. With respect to the dimension in the direction of the y axis, we shall assume that the condition $b < l_{c\,min}$ is satisfied. Since in this case the solution that is homogeneous with respect to y is stable, the problem can be regarded as one-dimensional, and the stationary equation (1) for the temperature takes the form

$$\frac{d}{dz}\varkappa(\Theta)\frac{d\Theta}{dz} = -\sigma(\Theta)E^2 + A(\Theta). \tag{8}$$

We shall henceforth consider only the one-dimensional problem, for in this case we can carry out a complete investigation of the solutions of Eq. (8) (unlike the two-dimensional problem, where the equation cannot be integrated exactly).

By making the change of variable $w = \int^{\Theta}\kappa(\Theta)d\Theta$ Eq. (8) and the boundary condition (2) are reduced to the form

$$\frac{d^2w}{dz^2}+\frac{dU(w)}{dw}=0,\quad \left.\frac{dw}{dz}\right|_{z=a,0}=0,$$
$$U(w)=\int^{w}\{\sigma(w)E^2 - A(w)\}\,dw. \tag{9}$$

Equation (9) coincides in outward appearance with the equation of motion of the particle in a potential field, the function U(w) having the meaning of the potential energy of the "particle," and the roles of the time and of the coordinate are played by z and w, respectively.

The form of the potential energy in a fixed field E is shown in Fig. 3. When the field changes from E_1 to E_2, the values of the two maxima change in such a way that the function U(w) has only one maximum in the field $E = E_1$ (or E_2), and the other two extrema coalesce at the inflection point located on the right of the single extremum (or on the left in the field E_2). The boundary conditions imposed on the temperature correspond to finite motion of the "particle" in the potential well and can be satisfied only when the sample subtends an integer number of half-oscillations.

The minimum period of motion of the "particle" corresponds to motion near the minimum of the potential energy, where the particle behaves like a linear oscillator with a period $2l_c(E)$. We note that $l_c^{-2} = \pi^{-2}(d^2U(w)/dw^2)$. It is obvious that when $l_c(E) > a$ there exist no inhomogeneous solutions in the given field[1], and the criterion for the absence of inhomogeneous solutions coincides with the criterion for the stability of the homogeneous solution. Thus, if the homogeneous solution is stable, then it is unique. Conversely, when there exists at least one inhomogeneous solution ($a > l_c(E)$), the homogeneous solution is unstable (see Sec. 1). Let us examine the character of the behavior of the solutions with increasing thickness of the plate in a given field E.

When $2l_c > a > l_c$, there exists only one inhomogeneous solution corresponding to one half-oscillation. When $3l_c > a > 2l_c$ there occurs one more solution, corresponding to two half-oscillations, etc. The total number of inhomogeneous solutions at given a and E is thus equal to $[a/l_c(E)]$ (square brackets denote the integer part of the argument).

Let us renumber the solutions of (9), using the explicit form of the solution, which is written as follows:

$$\sqrt{2}z = \int_{w_1'}^{w} \frac{dw}{\sqrt{U(w_1') - U(w)}}, \tag{10}$$

where w_1' is the value of w on the boundary z = 0, and $U(w_1')$ has the meaning of the total energy of the particle. The following formula is obvious:

$$\sqrt{2}a = p\int_{w_1'}^{w_3'} \frac{dw}{\sqrt{U(w_1') - U(w)}}, \tag{11}$$

where w_3' is the value of w(z) closest to (but not coinciding with) the extremum at the point z = 0, and p = 1, 2, 3, ... is the number of half-oscillations of the p-th solution.

Let us consider now the character of the behavior of the solutions with increasing field for a given sample thickness. In analogy with the solution of the problem of particle motion in a potential well, each solution corresponds to an energy level $U^p(w_1')$. If at a fixed dimension we vary the field in the interval $E_1' < E < E_1^{(1)}$, where $E_1^{(1)}$ is determined from the relation $2l_c(E_1^{(1)}) = a$, then in each well from this region there is one energy level corresponding to a monotonic solution. In the interval $E_1^{(1)} < E < E_1^{(2)}$, where $E_1^{(2)}$ is determined from the condition $2l_c(E_1^{(2)}) = a$, there appears a second level, etc. In some interval of fields from the region $E_1' < E < E_2'$, including the field corresponding to $l_{c\,min}$, the number of levels will be maximal and equal to $[a/l_{c\,min}]$. With further increase of the field, the number of levels begins to decrease, so that in the field range $E_2^{(1)} < E < E_2'$, where $E_2^{(1)}$ is determined from the formula $2l_c(E_2^{(1)}) = a$, there again takes place only one level corresponding to a monotonic solution. We note that a monotonic inhomogeneous solution exists in the entire interval of fields $E_1' < E < E_2'$, and corresponds to the "uppermost" energy level.

Let us investigate the stability of the inhomogeneous solutions against perturbations of the form δw

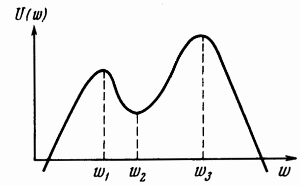

FIG. 3

$= \delta w(z)e^{-i\omega t}$. Let us linearize Eq. (1). Then we obtain from (1) and from the boundary conditions (2)

$$H^p\delta w + {}^3/_2 iN\omega\delta w/\varkappa = -2\sigma(w)E\delta E, \tag{12}$$

$$\frac{d}{dz}\delta w\bigg|_{z=0;\,a} = 0, \qquad H^p = \frac{d^2}{dz^2} + \frac{d\sigma(w^p(z))}{dw}E^2 - \frac{dA(w^p(z))}{dw}. \tag{13}$$

Here $w^p(z)$ is one of the possible inhomogeneous solutions of the equation with a number of half-oscillations equal to p (this circumstance is denoted by the superior index).

Equation (12) and boundary conditions (13) must be supplemented by Kirchhoff's law (the law of conservation of the total current), which takes the form

$$EL + bRE\int_0^a \sigma(w^p)\,dz = \mathscr{E},$$

where $\mathscr{E}$ is the generator emf and is constant. Variation of this relation yields

$$\left[L + bR\int_0^a \sigma(w^p)\,dz\right]\delta E = -bRE\int_0^a \frac{d\sigma(w^p)}{dw}\delta w\,dz. \tag{14}$$

The solution of (12) is sought in the form of an expansion in the eigenfunctions of the same equation without the right hand side. Eq. (12) without the right hand side corresponds to the condition $\delta E = 0$ or, as is seen from (14), to the absence of ballast (R = 0, given-field regime).

As already mentioned above, in this case the homogeneous solution on the decreasing section of the current-voltage characteristic is unstable, i.e., hysteresis occurs. A similar type of instability is expected also for the inhomogeneous solutions. This would correspond to the appearance of at least one negative eigenvalue (i.e., Im $\omega > 0$) in the spectrum of the eigenvalues of the operator H^p.

The method of determining the number of negative eigenvalues of the operator H^p is based on the idea of[6]. Differentiation of Eq. (9) with respect to z gives the following equation for dw^p/dz:

$$H^p\frac{dw^p}{dz} = 0. \tag{15}$$

Thus, the function dw^p/dz, corresponding to the zero eigenvalue, but satisfying boundary conditions differing from the boundary conditions (15), is an eigenfunction of the operator H^p. (The function itself, rather than the derivative, vanishes on the boundary.)

As is well known, from any solution of a second-order differential equation it is possible to construct a second solution that is linearly independent of the first. Thus, knowing dw^p/dz, we can find the solution of Eq. (15) with a derivative that vanishes at the point z = 0. An investigation shows that at the point z = a the deriva-

[1]We note that the existence of only a homogeneous solution if the foregoing conditions satisfied follows from the character of the behavior of the function U(w) and is connected with the assumption that U(w) is a single-valued function of w.

tive of this solution vanishes automatically. Such a solution is an eigenfunction of the operator H^p corresponding to a zero eigenvalue and satisfying the boundary conditions (13). According to the Sturm theorem, the zeroes of two linearly independent solutions mutually alternate with one another. Therefore in the open interval (0, a) the function corresponding to the zeroth eigenvalue of H^p and satisfying the boundary conditions (13) has one more zero than the function dw^p/dz (since two zeroes of the latter are located at the end points of the interval). It is clear that, for example, the function corresponding to the zeroth eigenvalue of the operator H^p with p = 1 and satisfying the condition (13) has one zero, since dw^p/dz with p = 1 has no zeroes in the open interval (0, a). It is easy to verify that the index p determines the number of zeroes possessed by the eigenfunction of the operator H^p corresponding to the zero eigenvalue and satisfying the condition (13).

We apply the oscillation theorem, from which it follows that the ground-state function has no zeroes and that the number of zeroes of each function determines the number of the state, and the corresponding eigenvalues are arranged in an increasing sequence. It is clear from the foregoing that the operator H^p has exactly p negative eigenvalues, and in accordance with the advanced hypothesis, none of the stationary solutions is stable in the given-field regime[2)].

In the current regime ($R \neq 0$), Eq. (12) has a solution only if the right hand side of Eq. (12) is orthogonal to the eigenfunction of the operator H^p corresponding to the zeroth eigenvalue. It is easy to see that this requirement reduces to the existence of a solution of an equation for dw/dE, since the equation for this function is obtained from Eq. (12) by formally letting the frequency ω go to zero. The corresponding solution is constructed by differentiating formula (10) with respect to E. On the other hand, in order for this solution to exist it is necessary to have the same orthogonality relation. Thus, the sought orthogonality does take place.

Taking this circumstance into account, the solution of Eq. (12) takes the form

$$\delta w^p = 2Ei \sum_{n=0}^{\infty} \frac{\varphi_n{}^p \int_0^a \sigma(w^p)\varphi_n{}^p\, dz}{\omega + i\lambda_n{}^p}\,\delta E. \tag{16}$$

Here λ_n^p are the eigenvalues of the operator H^p, $\varphi_n^p(z)$ are the corresponding eigenfunctions, which are assumed to be normalized by the condition

$$\left({}^3/_2 N \int_0^a \frac{1}{\varkappa} \varphi_n{}^p \varphi_m{}^p\, dz = \delta_{nm} \right).$$

We substitute (16) in (14) and write the result as follows:

$$F^p(\omega) \equiv R^{-1} + \frac{b}{L}\left[\int_0^a \sigma(w^p)\, dz + 2E^2 \sum_{n=0}^{\infty} \frac{iT_n}{\omega + i\lambda_n{}^p}\right], \tag{17}$$

where

$$T_n = \left(\int_0^a \sigma(w^p)\varphi_n{}^p\, dz\right)\left(\int_0^a \frac{d\sigma(w^p)}{dw}\varphi_n{}^p\, dz\right).$$

This is the dispersion equation for ω.

The inhomogeneous solution is stable if the function $F^p(\omega)$ of the complex variable ω does not vanish in the upper half-plane. In order to establish the number of zeroes possessed by the function $F^p(\omega)$ in the upper half-plane, we note that the number of poles in this function in the upper half-plane is equal to the number of negative eigenvalues, i.e., it is equal to p. Then, in accordance with the principle of the argument, we have

$$n = p + \frac{1}{2\pi}\Delta_c \arg F^p(\omega). \tag{18}$$

Here n is the number of zeroes in the upper half-plane, and the second term in the right side is the increment of the argument of the function $F^p(\omega)$, after following the contour C and closing the upper half plane of ω, divided by 2π. We choose these contours in the form of a semicircle of infinite radius. We consider the mapping of the chosen contour in the complex $F^p(\omega)$ plane. The entire infinite semi-circle ($\omega \to \infty$) is mapped into a point on the real axis. This point lies to the right of the origin $\left(R^{-1} + (b/L)\int_0^a \sigma(\omega P)dz > 0\right)$. The real positive (negative) axis is mapped into a curve lying above (below) the real axis of the $F^p(\omega)$ plane. The image of the origin of the ω plane lies on the real axis. We can have two cases.

1. The origin of the ω plane is mapped into a point lying on the real axis to the right of the origin, if the inequality $R^{-1} > -d\bar{j}/dE$ is satisfied, where

$$\bar{j} = \frac{1}{a}\int_0^a j(z)\, dz.$$

In this case the image of the contour C does not enclose the origin, i.e., the increment of the argument of $F^p(\omega)$ is equal to zero. From (18) we get n = p. This means that none of the solutions are stable.

This result shows that if sections with positive differential conductivity appear on the current-voltage characteristic, then these sections are unstable. Sections with negative differential conductivity are also unstable if the ballast is chosen to be insufficiently large.

2. The origin of the ω plane is mapped into a point lying on the real axis to the left of the origin, if the following inequality is satisfied.

$$R^{-1} < -\frac{b}{L}\frac{d\bar{j}}{dE}. \tag{19}$$

Since in this case the image of the counter C encloses the origin, which is then circled in the opposite direction, then the argument of $F^p(\omega)$ acquires an increment equal to -2π. Expression (18) yields

$$n = p - 1. \tag{20}$$

This result must be taken to mean that if the ballast resistance is chosen sufficiently large (see (19)), then the only stable inhomogeneous solution on the decreasing section of the current-voltage characteristic is the solution with p = 1, i.e., a monotonic solution. We note that the criterion (19) for an inhomogeneous solution coincides with the criterion (6) for a homogeneous solution[3)].

2) This contradicts the result of [2], where it was erroneously deduced that the spectrum of the operator H^p with p = 1 has no negative eigenvalue.

3) For the limiting case of a cylindrical sample of infinitely large radius, a similar inequality was obtained in [3].

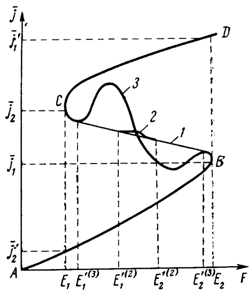

FIG. 4

3. CURRENT-VOLTAGE CHARACTERISTIC

From the results of[4,5] and the preceding section it follows that if there is no balanced resistance (the field regime), then at arbitrary transverse dimensions of the specimen only the branches AB and CD on Fig. 4 are stable. This means that when the field is increased from zero to $E > E_2$, at the point $E = E_2$ the current changes jumpwise from j_1 to j_1', and when the field is decreased from $E > E_2$ to zero the jump of the current from a value j_2 to j_2' occurs at the point $E = E_1$ (hysteresis). Thus, the decreasing section of the current-voltage characteristic can be investigated only if a ballast resistor is used.

If the transverse dimensions a and b of the sample are $< l_{c\,min}$ and the ballast resistor satisfy the inequality

$$R > \frac{L}{S}\left\{\left|\frac{dj}{dE}\right|^{-1}\right\}_{max}$$

(see (16)), then Eq. (3) for the temperature has a homogeneous solution, that is stable against small perturbations. The form of the current-voltage characteristic for this case is shown by Fig. 4, curve 1 (see also Fig. 1b).

On the other hand, if $a > l_{c\,min}$ and $b < l_{c\,min}$ then, as indicated in Sec. 1, there are regions $E_1 < E < E_1'$ and $E_2' < E < E_2$ where the homogeneous distribution of the temperature is stable and is unique as before (the fields E_1' and E_2' are determined by the condition $l_c(E_{1,2}') = a$). In these regions, the current-voltage characteristic coincides with curve of Fig. 4, for which, naturally, it is necessary to have

$$R > \frac{L}{S}\max\left\{\left|\frac{dj(E)}{dE}\right|^{-1}\right\}_{E=E_1,\,E_2}.$$

Let us consider now the variation of the current-voltage characteristic compared with curve 1 of Fig. 4 in the region $E_1' < E < E_2'$, where only a monotonic inhomogeneous distribution of the temperature can be stable. To this end, it is convenient to investigate the function

$$\overline{\Theta} = \frac{1}{a}\int_0^a \Theta(z)\,dz$$

representing the average temperature, since the character of variation of this function duplicates qualitatively the form of the current-voltage characteristic. We can assume approximately that

$$\bar{j} = \frac{E}{a}\int_0^a \sigma\,dz \approx \sigma(\overline{\Theta})E.$$

The average temperature is a function of the field E and of the transverse dimension of the sample a. It therefore depends on the form of the potential energy U(w), which varies with variation of the field E, and on the position assumed by the energy level relative to the bottom of the well. The position of this level in a fixed field changes with changing dimension of the sample. Thus, in regions of fields directly adjacent to E_1' or E_2', the energy level is located near the bottom of the well and the average temperature is close to the branch BC of the homogeneous distribution shown in Fig. 1. It may turn out that in a certain field integral, when the maxima of the potential energy still greatly differ in magnitude, the energy level will be located near the smaller maximum and then the average temperature will be close to the temperature determining this maximum. Therefore the course of the curve of the average temperature in such an integral field will be close to the branch EB or CD of Fig. 1. Such a case takes place in a sufficiently thick sample with dimension $a \gg l_{c\,min}$ (the limiting case is that of an infinite sample, for which the foregoing considerations are obvious).

We note also the following circumstance. Regardless of the dimension of the sample, the $\overline{\Theta}$ curve should cross the branch BC of the homogeneous distribution. This follows from the fact that in measuring the field the value of the smaller maximum increases, and that of the larger one decreases, and ultimately the smaller maximum becomes large. Therefore, by virtue of the continuity, there is found a field $E_0(a)$ from the interval $E_1' < E < E_2'$, in which the average temperature coincides exactly with the temperature of the homogeneous distribution in this field.

The foregoing considerations make it possible to describe certain limiting cases of the qualitative course of the current-voltage characteristic in the field integral $E_1' < E < E_2'$.

1. If the sample is thin ($a \gtrsim l_{c\,min}$), then the field region in which the homogeneous notion is unstable is small. We can therefore expect in this field integral the energy level to be always located at the bottom of the corresponding potential well. Curve 2 in Fig. 4 shows the approximate form of the current-voltage characteristic in the section $E_1'^{(2)} < E < E_2'^{(2)}$.

2. If $a \gg l_{c\,min}$, then the integral (E_1', E_2') is large, of the order of (E_1, E_2). The energy level can approach the maximum in such fields, when both maxima still differ significantly in magnitude. Therefore, when the field increases from E_1' to E_2', the average temperature will first decrease, in accordance with curve BC of Fig. 1, and then increase, tending to the curve CD. Subsequently it again begins to decrease, and intersects at a certain field $E_0(a)$ the curve BC. The function $\overline{\Theta}(E)$ behaves similarly when the field is decreased from E_2'. Thus, the function $\overline{\Theta}(E)$ has in the interval (E_1', E_2') four extrema. An approximate form of the current-voltage characteristic in the section $E_1'^{(3)} < E < E_2'^{(3)}$ is shown in Fig. 4 (curve 3). The variation of curve 3 of Fig. 4 with decreasing dimension, can be easily understood. The two clearly pronounced extrema on curve 3 of Fig. 4 were located over BC, start to come together with decreasing dimension, until at a certain thickness a_1 they coalesce into a single inflection point. The two other extrema located under BC behave in similar fashion,

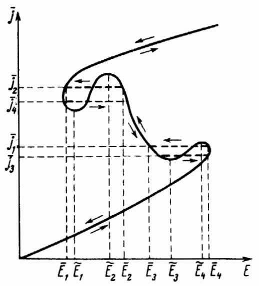

FIG. 5

and the coalescence into a single inflection point takes place, generally speaking, at a different thickness $a_2 \neq a_1$. With further decrease of the thickness, the current-voltage characteristic assumes the form of the curve 2 of Fig. 4, and finally when $a < l_{c\,min}$ it takes the form of curve 1 in Fig. 4.

Let us stop to discuss in greater detail the current-voltage characteristic shown by curve 3 (Fig. 4). The sections of the current-voltage characteristic in the field intervals ($\tilde{E}_1$, $\tilde{E}_2$) and ($\tilde{E}_3$, $\tilde{E}_4$) (see Fig. 5) are unstable, since the condition (19) is not satisfied in these sections. Nor is it satisfied near the points $\tilde{E}_1$, $\tilde{E}_2$, $\tilde{E}_3$, and $\tilde{E}_4$, since at these points $d\bar{j}/dE = 0$, and in order to satisfy the condition (19) the ballast R must be infinite. Therefore when the current increases, the field experiences a discontinuity at the point $\bar{j} = \bar{j}_1$, changing jumpwise from the value $\bar{E}_4$ to $\bar{E}_3$, and at the point $\bar{j} = \bar{j}_2$ it changes jumpwise from $\bar{E}_2$ to $\bar{E}_1$. When the current is decreased, similar jumps occur at the point $\bar{j} = \bar{j}_3$ and $\bar{j} = \bar{j}_4$ (in Fig. 5 the jumps are shown by thick dashed lines). Thus, the current-voltage characteristic of the thick sample has two hysteresis sections. In a sufficiently thin sample, the current-voltage characteristic has no hysteresis sections.

In conclusion we note that we assumed above that there is no constant magnetic field. It is not difficult to take into account the presence of a magnetic field, if the latter is directed along the current. In a weak magnetic field, the results remain the same as before. Since the magnetic field is sufficiently strong (but not quantizing), then this leads to a change in the value of l_c, which decreases in proportion to the reciprocal of the magnetic field. In this case the homogeneous distribution becomes unstable in much thinner samples (compared with the case when there is no magnetic field).

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