ELECTROMAGNETIC WAVE ECHO IN A PLASMA

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An effect of the spatial-plasma-echo type for transverse electromagnetic waves in the absence of an external magnetic field is considered in the third order of perturbation theory. The wave frequency is assumed lower than the Langmuir plasma frequency and the effect leads to nonlinear penetration of the waves into the plasma. The amplitude increases with growth of the distance between the grids producing the field.

MANY recent papers have been devoted to theoretical and experimental investigations of the so-called plasma echo. Such an echo was investigated for the case of ion-acoustic waves subject to Landau damping[1], and for the case of analogous waves in a magnetized plasma[2,3] (see also the review[4]).

We shall show in this paper that a phenomenon of the plasma-echo type should be observed also for ordinary transverse electromagnetic waves in a plasma under the condition that their frequency is much smaller than the plasma frequency ω_0. The experimental setup consists of the following: a grid 1 is placed in a plasma in the plane x = 0, and a second grid is placed in the plane x = d. Grid 1 produces an alternating electric field directed along the y axis

$$E_1 e^{-i\omega_1 t}.$$

If $\omega_1 < \omega_0 = 4\pi Ne^2/m$ (N is the number of electrons per unit volume), then the field will attenuate exponentially in both sides of the grid, with a penetration depth

$$\delta_1 = \frac{1}{\varkappa_1} = \frac{c\sqrt{\omega_0^2/\omega_1^2 - 1}}{\omega_1}. \qquad (1)$$

But if fields $E_2 \exp(-i\omega_2 t)$ and $E_3 \exp(-i\omega_3 t)$ are applied to the second grid, which is located at a distance d from the first, then a field burst

$$E_e e^{-i\omega_e t}, \qquad (2)$$

where $\omega_e = \omega_1 + \omega_2 + \omega_3$, is produced in the plasma at a distance $x_e \approx d(\omega_3 + \omega_2)/\omega_e$. The amplitude of the echo is $E_e \sim E_1E_2E_3$. Thus, in the case of transverse waves the echo is produced only in third order in the amplitude. This is clear beforehand from symmetry considerations—it is impossible to make up a third transverse vector E_e out of the two transverse vectors E_1 and E_2.[1)]

Proceeding to the calculation of the field E_e, we make an additional simplifying assumption. Namely, we assume that the normal skin effect situation exists for each of the frequencies ω_1, ω_2, ω_3, and ω_e, i.e.,

$$r_1 = \sqrt{\frac{2T}{m}}\frac{1}{\omega_1} \ll \delta_1, \qquad (3)$$

where T is the electron temperature (this condition was actually used already in formula (1) for the depth of penetration). We start from the kinetic equation for the distribution function in a collisionless plasma

$$\frac{\partial f}{\partial t} + v_x\frac{\partial f}{\partial x} - \frac{e}{m}\left(\mathbf{E} + \frac{1}{c}[\mathbf{vH}]\right)\frac{\partial f}{\partial \mathbf{v}} = 0. \qquad (4)^*$$

We shall show below that the influence of the magnetic field of the wave is appreciable. The effect of interest to us is connected with the free flight of the particles along the x axis. It will therefore suffice to consider henceforth only electrons with $v_x > 0$. In first order, the equation can be rewritten in the form

$$-i\omega_1 f_1 + v_x\frac{\partial f_1}{\partial x} - \frac{e}{m}E_{1y}\frac{\partial f_0}{\partial v_y} = 0. \qquad (5)$$

The field E_1, with allowance for inequality (3), is given by

$$E = E_{01}e^{-i\omega_1 t - |x|/\delta_1}$$

(and analogously for the fields E_2 and E_3, with the substitutions $\omega_1 \to \omega_2$, ω_3 and $x \to x - d$). Solving the equation, we get

$$f_1 = \left(\frac{i\omega_1}{v_x} + \frac{1}{\delta_1}\right)^{-1}\frac{eE_{01}}{mv_x}e^{-i\omega_1 t}\frac{\partial f_0}{\partial v_y}\left(e^{ix\omega_1/v_x} - e^{-x/\delta_1}\right). \qquad (6)$$

In the next approximation, the equation becomes

$$-i(\omega_1 + \omega_2)f_2 + v_x\frac{\partial f_2}{\partial x} - \frac{e}{mc}\left(v_yH_2\frac{\partial f_1}{\partial v_x} - v_xH_2\frac{\partial f_1}{\partial v_y}\right) = 0. \qquad (7)$$

We have omitted from (7) terms with E_2, since they make a small contribution to the final effect. We note also that when f_1 is differentiated with respect to v it is necessary to differentiate only the rapidly varying exponential $\exp[ix\omega_1/v_x]$.

As a result

$$f_2 = \frac{e^2}{m^2}\frac{E_{01}H_{02}}{c}\frac{2d}{\delta_2\omega_2^2}\frac{v_y}{v_x}\frac{\partial f_0}{\partial v_y} \times \exp\left\{i\frac{\omega_2}{v_x}x - i\frac{\omega_2 + \omega_1}{v_x}d\right\}\exp\{-i(\omega_1 + \omega_2)t\}. \qquad (8)$$

analogously, for the third-order correction of interest to us we have

$$f_3 = \frac{e^3}{m^3}\frac{E_{01}H_{02}H_{03}}{c^2\delta_2\delta_3}\frac{4i\omega_1 d^2}{\omega_3^2\omega_2^2}\frac{v_y^2}{v_x^2}\frac{\partial f_0}{\partial v_y} \times \exp\left\{i\frac{\omega_e}{v_x}x - i\frac{\omega_3 + \omega_2}{v_x}d\right\}\exp\{-i\omega_e t\}. \qquad (9)$$

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1)For simplicity we consider only one of the possible formulations of the problem. We could, for example, apply each of the fields E_1, E_2, and E_3 to separate grids, or else the fields E_1 and E_2 to the first grid and E_3 to the second grid, etc.

*$[\mathbf{vH}] \equiv \mathbf{v} \times \mathbf{H}$.

(Of course, there is also a term that differs from (9) by the substitution $\omega_2 \to \omega_3$. It is obtained if the correction with E_3 is calculated first, and then with E_2.)

Integrating f_3 with respect to d^3v, we find the corresponding correction to the current. As usual, in the theory of plasma echo the current differs significantly from zero only near the plane

$$x_e = \frac{\omega_3 + \omega_2}{\omega_e} d,$$

in which the cancellation of the exponentials that oscillate rapidly with velocity takes place. Integrating with respect to dv_y and dv_z, we obtain

$$j = \frac{e^3}{m^3} \frac{E_{01} H_{02} H_{03}}{c^2 \delta_2 \delta_3} d^2 \frac{6 i \omega_1}{\omega_2^3 \omega_3^3} \left(\frac{2T}{\pi m} \right)^{1/2}$$
$$\times e^{-i\omega_e t} \int_0^\infty \frac{dv_x}{v_x^2} \exp\left\{ -\frac{m v_x^2}{2T} + i \frac{\omega_e}{v_x} x - i \frac{\omega_3 + \omega_2}{v_x} d \right\}. \quad (10)$$

The electric field produced by this current satisfies the equation

$$\frac{d^2 E_e}{dx^2} + \varkappa_e^2 E_e = -\frac{4\pi i \omega_e}{c^2} j, \quad \varkappa_e^2 = \frac{\omega_e^2}{c^2} \left(\frac{\omega_0^2}{\omega_e^2} - 1 \right). \quad (11)$$

A solution of this equation can be written out by using the corresponding Green's function

$$E_e = -\frac{i\omega_e}{c^2} \int_{-\infty}^{\infty} 2\pi i \frac{\exp\{-\varkappa_e |x - x'|\}}{\varkappa_e} j(x') dx'. \quad (12)$$

By reversing the order of integration with respect to dv_x and dx', integrating with respect to dx', and taking into account the inequality

$$\varkappa_e r_3 \ll 1, \quad r_3 = \sqrt{\frac{2T}{m}} \frac{1}{\omega_e},$$

we obtain ultimately

$$E(\xi) = \frac{e^4}{m^4} \frac{E_{01} E_{02} E_{03}}{i c^2} \frac{48 \sqrt{\pi}\, \omega_1 d^2 e^{-i\omega_e t}\, TN}{\delta_2^2 \delta_3^2 \omega_e \omega_2^3 \omega_3^3} \Phi\left(\frac{\xi}{r_3} \right), \quad (13)$$
$$\xi = x - x_e,$$

where

$$\Phi(\eta) = \int_0^\infty dy \exp\left\{ -y^2 + i \frac{\eta}{y} \right\}, \quad \eta = \frac{\xi}{r_3}. \quad (14)$$

Let us write out the asymptotic expression for $\Phi(\eta)$ at large values of η. By the saddle-point method we easily get

$$\Phi(\eta) \approx \sqrt{\frac{\pi}{3}} \exp\left\{ -\frac{e^{i\pi/3}}{2^{2/3}} |\eta|^{2/3} \right\}, \quad |\eta| \gg 1, \quad (15)$$
$$\Phi(0) = \sqrt{\pi}/2.$$

It is interesting to note that the amplitude of the echo increases with increasing distance between grids d. Of course, actually the amplitude is limited to collisions. In the case of Coulomb collisions, their number increases as a result of the oscillating character of the distribution function. Thus, the condition for applicability of the obtained formulas turns out to be quite stringent.

It can be shown that the formulas obtained above for a collisionless plasma are valid under the condition

$$d \ll l (d / r_3)^2,$$

where l is the mean free path of the electrons.

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