SINGLE-WAVE APPROXIMATION FOR PARAMETRICALLY AMPLIFIED WAVES

G. I. FREĬDMAN

Radiophysics Institute of the Gor'kiĭ State University

Submitted October 13, 1969

Zh. Eksp. Teor. Fiz. 58, 1959–1966 (June, 1970)

It is shown that if the amplitudes of parametrically amplified waves whose group velocities differ and lie in a single plane with the group velocity of the pumping wave have a sufficiently weak transverse coordinate dependence, then the propagation of the waves can approximately be described by an equation of the type of the diffusion equation for the amplitude of one of the waves. The main cases of interaction between the waves and pumping-radiation beams or pulses are considered within the limits of applicability of the equation.

THE problem of investigating the interaction between parametrically amplified waves and wave packets of pump radiation arises, in particular, in the analysis of the parametric superluminescence accompanying passage of picosecond pulses or narrow light beams through crystals with a quadratic nonlinearity. An investigation of such processes is necessary also for the determination of the mode locking conditions of parametric light generators (PLG)[1,2], and also for the determination of the conditions of optimal focusing of the pump radiation[3,4] and for the analysis of the influence of the width of the pump beam on the structure of the angular at frequency spectrum of the PLG radiation[4]. However, even if the pump radiation field is specified, the fields of the amplified waves at the output of the nonlinear crystal can be obtained only in certain particular cases. For example, in the case of small gain, the fields at the exit from the interaction region can be determined by using successive approximations[3]. At an interaction close to degenerate, when the difference between the velocities of the amplified waves is negligibly small, and diffraction effects are not significant, the law of transformation of the amplified waves can be readily obtained in the case of large gains[5]. In the case of nondegenerate interaction, when the difference between the group velocities of the amplified wave is appreciable, their fields at the exit from the crystal can be obtained only at special values of the parameters of the problem[4,6,7]. As a result of this, at large values of the gain parameter and different group velocities of the amplified waves, the problem arises of obtaining an approximate analysis of their interaction with wave packets of pump radiation.

The small parameter of the problem, which makes it possible to simplify the system of equations for the amplitudes of the amplified waves[8], is obviously the ratio of the difference of the shift of the wave packets at a distance of the effective interaction and of the diffraction parameter corresponding to this distance, to the characteristic scales of variation of the amplitudes and phases of the interacting waves. A feature of the initial system of equations is that, unlike the wave equation[9] or Maxwell's equations[8], the zeroth-approximation solutions of interest are not close to quasiperiodic waves or waves traveling along the characteristics, with slowly varying amplitudes, but are exponentially growing or damped waves. Consequently, to simplify the initial system of equations one cannot use the average new method widely employed (see, for example[8,10]) in the derivation of equations for the amplitudes of quasiperiodic waves.

In the derivation of the equations of geometrical optics, Kravtsov[11] used a method which makes it possible to simplify the initial equations for the solutions close to the zeroth-approximation eigenwaves without using averaging. In this case, however, interest attaches to allowance for the effects of the spreading of the wave packets owing to the different directions of the group velocities of the parametrically amplified waves, i.e., the effects of second order of smallness. On the other hand, the derivation and investigation of the eikonal equations and of the equations for the amplitudes becomes quite cumbersome in such an approximation. In this paper, using as an example light beams with coplanar group velocities, we show by modifying the method used in[9,11] that the course of a certain class of parametric-amplification processes is determined by an equation of the type of a diffused equation for the amplitude of one of the waves. This equation, relative to the initial system, is the analog of the equation of quasioptics for Maxwell's equations. However, inasmuch as principal interest attaches in this case to the determination of the field near the maximum of the amplitudes, the single-wave approximation equation, unlike the quasioptics equation, remains applicable for a large class of solutions and for large variations of the parameters of the initial equations.

SINGLE-WAVE APPROXIMATION EQUATION

For concreteness, let us consider the interaction between parametrically amplified waves and a beam of harmonic pump radiation in a uniaxial crystal with quadratic nonlinearity[1)]. The equations for the amplitudes $E_{1,2}$ of the waves whose wave vectors lie in one plane with the optical axis of the crystal and satisfy the synchronism condition are conveniently written by introducing the dimensionless parameters and variables[2)]

[1)]The relations determining the interaction of plane non-monochromatic waves with pump-radiation pulses are obtained from those presented below by a simple change of notation (see, for example, [6]).

[2)]We do not introduce special symbols for the dimensionless variables, and will identify specifically expressions with dimensional variables.

$$z\to\gamma_0 z,\quad y\to\gamma_0|\Delta\beta_1\Delta\beta_2|^{-1/2}y,\quad x\to\gamma_0|\Delta\beta_1\Delta\beta_2|^{-1/2}x;$$
$$\mathscr{E}_1=(\omega_3/\omega_1)^{1/2}E_0^{-1}E_1,\quad \mathscr{E}_2=(\omega_3/\omega_2)^{1/2}E_0^{-1}E_2^*;$$
$$\beta_i\to\beta_i|\Delta\beta_1\Delta\beta_2|^{-1/2},\quad k_{1,2}^{-1}\to\gamma_0 k_{1,2}^{-1}|\Delta\beta_1\Delta\beta_2|^{-1},$$
$$\alpha_i\to\gamma_0^{-1}\alpha_i,\quad g(\mathbf{r})=\gamma_0^{-1}\gamma_0(\mathbf{r}).$$

Here β_i are the angles between the directions of the group velocity of the i-th wave and the z axis, $\Delta\beta_{1,2} = (\beta_{1,2}-\beta_3)$ are the angles between the directions of the group velocities of the amplified waves and the group velocity of the pump radiation, and γ_0 is the maximum value of the parameter of interaction between the amplified waves $\gamma_0(\mathbf{r})$ and is proportional to the amplitude of the pump radiation E_0. The equations for $\mathscr{E}_{1,2}$ then take the form

$$\frac{\partial\mathscr{E}_{1,2}}{\partial z}+\Delta\beta_{1,2}\frac{\partial\mathscr{E}_{1,2}}{\partial x}\mp\frac{i}{2k_{1,2}}\Delta_\perp\mathscr{E}_{1,2}+\alpha_{1,2}\mathscr{E}_{1,2}=i\begin{cases}g(\mathbf{r})\mathscr{E}_2\\-g^*(\mathbf{r})\mathscr{E}_1\end{cases}.\quad(1)$$

The solutions of Eqs. (1) will be sought in the form of a super-position of two solutions close to the zeroth-approximation eigenwaves:

$$\mathscr{E}_1(\mathbf{r})=e^{\Psi}\{A_{(+)}(z;\,\mu\mathbf{r}_\perp)+A_{(-)}(z;\,\mu\mathbf{r}_\perp)+\mu u_1(z;\,\mu\mathbf{r}_\perp)+\mu^2u_2(z;\,\mu\mathbf{r}_\perp)+\ldots\},\quad(2a)$$

$$\mathscr{E}_2(\mathbf{r})=e^{\Psi}\{K_{(+)}A_{(+)}(z;\,\mu\mathbf{r}_\perp)+K_{(-)}A_{(-)}(z;\,\mu\mathbf{r}_\perp)+\mu v_1(z;\,\mu\mathbf{r}_\perp)+\mu^2v_2(z;\,\mu\mathbf{r}_\perp)+\ldots\}.\quad(2b)$$

Here μ is a small parameter characterizing the slowness of the variation of the amplitudes with respect to the transverse coordinates, the eikonal Ψ is assumed to be a fast function of only the coordinate x, and is sought in the form

$$\Psi=\int_0^x\Psi_x(\mu^2z;\,\mu\mathbf{r}_\perp)\,dx,$$

which, as seen from the results below, is valid subject to definite limitations on the rate of change of the amplitude and of the phase of the pump radiation.

The equations for the amplitude $A_{(+)}$ and $A_{(-)}$ should change over, as $\mu\to 0$, into the equations for exponentially growing and attenuating waves, respectively. We shall therefore assume that these equations are of the form

$$\frac{\partial A_{(+)}}{\partial z}+\Delta\beta\frac{\partial A_{(+)}}{\partial x}-\gamma_{(+)}A_{(+)}=\mu f_1+\mu^2f_2,\quad(3a)$$

$$\frac{\partial A_{(-)}}{\partial z}+\Delta\beta\frac{\partial A_{(-)}}{\partial x}+\gamma_{(-)}A_{(-)}=0.\quad(3b)$$

$\gamma_{(\pm)}$, f_1, f_2, $K_{(\pm)}$ and Ψ_x will be determined below from the condition of smallness of u_i and v_i compared with $A_{(+)}$.

Assuming the effect connected with the second derivatives in (1) to be not lower than of second order of smallness, substituting (2) and (3) in (1), and equating to zero the sums of terms of equal order in μ, we obtain the zeroth-approximation equations:

$$\pm\gamma_{(\pm)}=-\alpha_1-\Delta\beta_1\Psi_x+ig(\mathbf{r})K_{(\pm)},$$
$$\pm K_{(\pm)}\gamma_{(\pm)}=K_{(\pm)}(-\alpha_2+\Delta\beta_2\Psi_x)-ig^*(\mathbf{r})\quad(4)$$

and the equations for the functions $u_{1,2}$ and $v_{1,2}$.

In order for $u_{1,2}$ and $v_{1,2}$ not to grow more rapidly than $A_{(+)}$, it is necessary that they correspond to excitation of a damped wave, i.e., that they have the form $U_{1,2}=A_{1,2}$; $V_{1,2}=K_{(-)}A_{1,2}$, where the change of $A_{1,2}$ is determined by an equation that differs from Eq. (3b) for the amplitude of the free damped wave $A_{(-)}$ only in having a nonzero right-hand part. From the conditions of the compatibility of the equations for $u_{1,2}$ and $v_{1,2}$ we then obtain relations that make it possible, in conjunction with relations (4) at a given value of the effective propagation angle $\Delta\beta$, to determine the functions $\gamma_{(\pm)}$, $K_{(\pm)}$, and Ψ_x, and also the dependence of f_1, and f_2 on the amplitudes $A_{(\pm)}$. We confine ourselves to the simplest case, when the rate of change of the pump phase is second order of smallness ($\arg g(\mathbf{r})=\Phi(\mu^2\mathbf{r})$). Then the rate of change of the coefficients $K_{(\pm)}$ will be of the same order of smallness, and from the condition for the compatibility of the equations for $u_{1,2}$ and $v_{1,2}$ follow the relations

$$K_{(-)}(\Delta\beta-\Delta\beta_1)=K_{(+)}(\Delta\beta-\Delta\beta_2),\quad f_1=\frac{\Delta\beta_2-\Delta\beta}{\Delta\beta_1-\Delta\beta_2}\frac{\partial A_{(-)}}{\partial(\mu x)},\quad(5a)$$

$$f_2=\frac{K_{(-)}}{K_{(+)}-K_{(-)}}\Big\{(\Delta\beta_1-\Delta\beta_2)\frac{\partial A_1}{\partial(\mu x)}-A_{(+)}K_{(-)}^{-1}\Big[\frac{\partial}{\partial(\mu^2z)}+\Delta\beta\frac{\partial}{\partial(\mu^2x)}\Big]K_{(+)}-i\frac{\mu^{-2}e^{-\Psi}}{2k_1}\Big[\Delta_\perp(A_{(+)}e^{\Psi})+\frac{k_1}{k_2}\Delta_\perp(K_{(+)}A_{(+)}e^{\Psi})/K_{(-)}\Big]\Big\}.\quad(5b)$$

From (4) and (5a), assuming for concreteness that $\Delta\beta_1>\Delta\beta_2$, we get

$$(\Delta\beta_1-\Delta\beta_2)\Psi_x=-2\Delta\alpha_{12}-\frac{2|g(\mathbf{r})|(\Delta\beta-\overline{\Delta\beta})}{\sqrt{\widetilde{\Delta\beta}^2-(\Delta\beta-\overline{\Delta\beta})^2}},$$
$$K_{(\pm)}=-i\frac{\Delta\beta_{1,2}-\Delta\beta}{\sqrt{\widetilde{\Delta\beta}^2-(\Delta\beta-\overline{\Delta\beta})^2}}\exp(i\arg g(\mathbf{r})),\quad(6)$$
$$\gamma_{(\pm)}=\pm|g(\mathbf{r})|\frac{(\Delta\beta-\overline{\Delta\beta})\Delta\beta_1\widetilde{\Delta\beta}^{-1}+(\Delta\beta_{1,2}-\Delta\beta)}{\sqrt{\widetilde{\Delta\beta}^2-(\Delta\beta-\overline{\Delta\beta})^2}}\pm\bar{\alpha},$$

where

$$\Delta\alpha_{1,2}=\frac{\alpha_1-\alpha_2}{2},\quad\overline{\Delta\beta}=\frac{\Delta\beta_1+\Delta\beta_2}{2},\quad\widetilde{\Delta\beta}=\frac{\Delta\beta_1-\Delta\beta_2}{2},$$
$$\bar{\alpha}=\frac{\Delta\beta_1\alpha_2-\Delta\beta_2\alpha_1}{\Delta\beta_1-\Delta\beta_2}.$$

To determine the dependence of f_2 on $A_{(+)}$ it is necessary to find the first-approximation amplitude A_1, which is determined, as can be readily seen from (1) and (2), by the equation

$$\frac{\partial A_1}{\partial z}+\Delta\beta\frac{\partial A_1}{\partial x}+\gamma_{(-)}A_1=(\Delta\beta-\Delta\beta_1)\frac{\partial A_{(+)}}{\partial(\mu x)}.\quad(7)$$

Taking into account the fact that the main contribution to $A_1(z;\,\mathbf{r}_\perp)$ is determined by the interval $(z-(\gamma_{(+)}+\gamma_{(-)})^{-1};\,z)$, the solution of Eq. (7) can be written, accurate to terms of order μ, in the form

$$A_1=(\Delta\beta-\Delta\beta_1)\Big\{(\gamma_{(+)}+\gamma_{(-)})^{-1}\frac{\partial A_{(+)}}{\partial(\mu x)}+(\gamma_{(+)}+\gamma_{(-)})^{-2}\frac{\partial\gamma_{(+)}}{\partial(\mu x)}A_+\Big\}.\quad(8)$$

Thus, it follows from (3a), (5b), and (8) that the equation for the amplitude of the growing wave, accurate to terms of second order of smallness inclusive, is of the form

$$\frac{\partial A_{(+)}}{\partial z}+(\Delta\beta+\delta\beta)\frac{\partial A_{(+)}}{\partial x}-(\gamma_{(+)}+\delta\gamma)A_{(+)}-D\frac{\partial^2A_{(+)}}{\partial x^2}-i\frac{e^{-\Psi}}{2k_1}\tilde{\Delta}_\perp(A_{(+)}e^{\Psi})=(\Delta\beta_2-\Delta\beta_1)\frac{\partial A_{(-)}}{\partial x};$$
$$\tilde{\Delta}_\perp(f)=\Big[\Delta_\perp(f)-\frac{k_1}{k_2}\frac{\Delta\beta_1-\Delta\beta}{\Delta\beta-\Delta\beta_2}\frac{|g(\mathbf{r})|}{g(\mathbf{r})}\Delta_\perp\Big(\frac{g}{|g|}f\Big)\Big].\quad(9)$$

Here the diffusion coefficient D and the coordinate-dependent corrections $\delta\beta$ and $\delta\gamma$ are determined by the relations

$$D = \frac{(\Delta\beta_1 - \Delta\beta)(\Delta\beta - \Delta\beta_2)}{\gamma_{(+)} + \gamma_{(-)}}, \quad \delta\beta = \frac{D}{(\gamma_{(+)} + \gamma_{(-)})}\frac{\partial}{\partial x}(2\gamma_{(+)} + \gamma_{(-)}),$$

$$\delta\gamma = D(\gamma_{(+)} + \gamma_{(-)})\frac{\partial}{\partial x}\left[(\gamma_{(+)} + \gamma_{(-)})^{-2}\frac{\partial\gamma_{(+)}}{\partial x}\right] - i\frac{\Delta\beta_1 - \Delta\beta}{\Delta\beta_1 - \Delta\beta_2}\left(\frac{\partial}{\partial z} + \Delta\beta_2\frac{\partial}{\partial x}\right)\arg g(\mathbf{r}). \quad (10)$$

In the peripheral regions of the pump radiation beam, where the amplitude of its field becomes small, the conditions for the applicability of the equation of the single-wave approximation (9) are violated. Since, with the exception of special cases (for example $\Delta\beta_2 \to 0$), the changes of the fields of the amplified waves in the peripheral regions is small, these changes can be determined by solving Eqs. (1) by successive approximations.

In the zeroth approximation, to which we confine ourselves[3)], at the limit of applicability of the equation of the single-wave approximation (the limit of the interaction region), the radiation conditions are satisfied. Since we assume the diffraction effects to be small, these conditions can be written in the form

$$\mathscr{E}_{1,2}(C_{+1,2}) = \mathscr{E}_{1,2}^{(h)}, \quad (11a)$$

$$\mathscr{E}_{1,2}(C_{-1,2}) = \mathscr{E}_{1,2}^{(u)}. \quad (11b)$$

Here $\mathscr{E}_{1,2}^{(h)}$ are functions specified on the sections $C_{+1,2}$ of the boundary of the interaction region, on which radiation comes from the outside of this region; $C_{-1,2}$ are sections of the boundary from which radiation of the corresponding wave goes into the interaction region.

MAIN TYPES OF INTERACTION OF PARAMETRICALLY AMPLIFIED WAVES IN THE SINGLE-WAVE APPROXIMATION

The character of the process of amplification of parametrically interacting waves, and also the limits in which the solutions of the single-wave approximation equation can be used for the determination of their fields, depend strongly on the relative direction of the differences between the group velocities of the amplified waves and the group velocity of the pump radiation, i.e., on the sign of the product $(\Delta\beta_1\Delta\beta_2)$.

The main regularities can be clarified by using the very simple example of an interaction between one-dimensional beams whose fields depend on the coordinate y as a parameter, and the diffraction spreading or spreading resulting from the finite curvature of the wave front are negligibly small, so that for example the parameter $k_{1,2}^{-1}$ in (9) can be set equal to zero. In addition, the phase front of the pump radiation will be assumed to be plane, and the distribution of its intensity over the section of the beam is assumed independent of the coordinate z. In order not to write out cumbersome equations, we shall also assume that the fields $\mathscr{E}_{1,2}^{(h)}$ of the amplified waves incident on the lateral surface of the interaction region are equal to zero.

At identical directions of the group velocities of the amplified waves relative to the group velocity of the pump radiation $(\Delta\beta_1\Delta\beta_2 > 0)$, the sections of the interaction-region boundary $C_{\pm1}$ and $C_{\pm2}$ coincide ($C_{+1} = C_{+2} = C_{(+)}$, $C_{-1} = C_{-2} = C_{(-1)}$). Consequently, since the conditions (11b) determine the fields of the waves radiated from the interaction region, relations (11a) do not suffice for the determination of the boundary conditions for Eqs. (9) on the boundary of this region. This is connected with the fact that when $\Delta\beta_1\Delta\beta_2 > 0$, in accordance with the initial equations (1), it is necessary to specify the fields of the amplified waves on the boundary $C_{(+)}$ in order to determine the fields in the interaction region. Therefore the solutions of the equation of the single-wave approximation differ little so far from the exact solution of the problem, so long as they depend weakly on the boundary conditions on the section $C_{(-)}$.

To illustrate the character of the solutions and the conditions for their applicability, let us consider the simplest case, when the intensity of the pump radiation is uniformly distributed over the cross section of a pump beam of width 2a ($g(x) = 1$ when $|x| < a$; $g(x) = 0$ when $|x| > a$). If we assume that Eq. (9), with constant coefficients corresponding to $g(x) = 1$, is valid in all of space, then the field in the interaction region $|x| < a$ can be readily determined by taking into account the fact that in this case the Green's function of Eq. (9) takes the form

$$G(\Delta z; \Delta x) = (4\pi D\Delta z)^{-1/2}\exp\left[\gamma_{(+)}\Delta z - \frac{(\Delta\beta\,\Delta z - \Delta x)^2}{4D\Delta z}\right] \quad (12)$$

where $\Delta z = z - z_1$ and $\Delta x = x - x_1$. In the case when the amplitudes $\overline{\mathscr{E}}_1(x_1)$ and $\overline{\mathscr{E}}_2(x_1)$ of the amplified waves have a slow dependence on the transverse coordinates at the input to the crystal, it is necessary to assume $\Psi_x = 0$, i.e., $\Delta\beta = \overline{\Delta\beta}$, $\gamma_{(\pm)} = 1 \mp \alpha$, and $D = \Delta\beta^2/2$. Then comparison of the solutions obtained from relations (2) and (12) with the exact solution of the problem, which for a one-dimensional pump beam can be readily found and is given in[4], shows that the solution of the single-wave approximation equation is close to the exact solution, at least so long as the field at the point (z; x) depends essentially on the values of the fields at the input into the interaction region at the points $(z_1; x_1)$ located not farther than a distance equal to the diffusion radius $\rho = 2\sqrt{D\Delta z}$ from the ray $(x - x_1) = \Delta\beta(z - z_1)$. It is easy to see also that after this the field in the interaction region begins to depend strongly on the boundary conditions on the section $C_{(-)}$ $(x = a)$.

An analogous situation occurs also in the case of inhomogeneous distribution of the intensity and phase over the cross section of the pump beam. We shall not stop to discuss this in detail. We note only that although there is no known solution of the diffusion equation with variable coefficients, the field of the amplified waves, at least until the maximum of the amplitudes goes outside the interaction region, can be found in many cases by using different approximations of the variable coefficients. For example, neglecting the dependence of the diffusion coefficient D on the transverse coordinates[4)]

---

3)Analogous relations can be written also when account is taken of the changes of the fields of the amplified waves in the peripheral regions.

4)The dependence of the diffusion coefficient D on the transverse coordinates can be readily taken into account when the scale of variation in the transverse direction of the variable coefficients of Eq. (9) is much smaller than the diffusion radius ρ. Here, obviously, the Green's function of Eq. (9) satisfies approximately Eq. (12), in which $\gamma_{(+)}\Delta z$ and $D\Delta z$ are replaced by the integrals $\int_{z_1}^{z}\gamma_{(+)}\,dz$, $\int_{z_1}^{z}D\,dz$ along the corresponding ray.

and approximating the functions $(\gamma_{(+)} + \delta\gamma)$ and $(\Delta\beta + \delta\beta)$ by quadratic functions, we can find the solution (9) in analogy with the procedure used in[12] for $(\Delta\beta + \delta\beta) = 0$. It turns out that the equation

$$\frac{\partial\varphi}{\partial\xi} - \beta(\xi)\frac{\partial\varphi}{\partial\eta} - D\frac{\partial^2\varphi}{\partial\eta^2} = 0 \qquad (13)$$

can be reduced, by making the change of variables

$$\eta = q(\xi)x, \quad z = \int_0^z d\xi/q(\xi), \quad \varphi = A_{(+)}(z;x)f(\xi)\exp[a(\xi)\eta + b(\xi)\eta^2]$$

and by an appropriate choice of the functions $q(\xi)$, $f(\xi)$, $a(\xi)$, and $b(\xi)$, to the form (9) with a quadratic dependence of $(\gamma_{(+)} + \delta\gamma)$ and $(\Delta\beta + \delta\beta)$ on the transverse coordinates. In particular, when $\delta\beta = 0$ and $(\gamma_{(+)} + \delta\gamma) = \gamma^0_{(+)}(1 - x^2/a^2)$, the solution (9) is given by

$$A_{(+)}(z;x) = \sqrt{q(\xi)}\exp\left[\gamma^0_{(+)}z - \frac{\Delta\beta^2}{4D}\int_0^\xi \frac{2(1-q) - q(1-q)^2}{q}d\xi - \frac{\Delta\beta}{2D}q(1-q)x - \frac{\gamma^0_{(+)}\xi}{a^2}x^2\right]\int_{-\infty}^{\infty} dx_1 \bar{A}_{(+)}(x_1)$$
$$\times\exp\left\{-(4D\xi)^{-1}\left[\int_0^\xi \beta(\xi)d\xi - (qx - x_1)\right]^2\right\}\Big/\sqrt{4\pi D\xi}. \qquad (14)$$

Here $\bar{A}_{(\pm)}(x)$ is the value of A_+ at the entrance to the crystal;

$$\xi = a\,\mathrm{th}\left(2z\sqrt{D\gamma^0_{(+)}}/a\right)/2\sqrt{D\gamma^0_{(+)}}, \quad q(\xi) = \sqrt{1 - 4D\gamma^0_{(+)}\xi^2/a^2};$$
$$\beta(\xi) = \Delta\beta[1 - q(1-q)]/q.$$

As seen from (14), the inhomogeneity of the distribution of the pump radiation does not change qualitatively the picture of the propagation of the amplified waves. Just as in the homogeneous pump beam, a systematic departure of the waves from the interaction region takes place. Only the distribution of their intensities will change both as a result of the diffusion process and as a result of the dependence of the gain on the transverse coordinates.

We now proceed to the case when the group velocities of the amplified waves are directed in opposition to the group velocity of the pump beam $(\Delta\beta_1\Delta\beta_2 < 0)$. It is clear that until the waves reach the interaction-region boundary, the processes will be qualitatively the same as in the preceding case. However, when $\Delta\beta_1\Delta\beta_2 < 0$ the radiation conditions (11a) are the boundary conditions for Eqs. (9), and the initial equations do not contradict energy transfer to the interior of the interaction region from any point on its boundary. Therefore when $\Delta\beta_1\Delta\beta_2 < 0$ the solutions of the equation of single-wave approximation can be valid also at larger distances from the entrance into the nonlinear crystal.

In the general case, solutions of Eq. (9) satisfying the boundary conditions (11a) and with an arbitrary value of the effective propagation angle $\Delta\beta$ have a rapid dependence on the transverse coordinates, i.e., they do not satisfy the conditions for the applicability of the equation of the single-wave approximation. This indicates that it is necessary to take into consideration solutions of Eq. (9) with different directions of the effective group velocity, which in the general case hinders the use of (9) for the determination of the field after the waves reach the boundary of the interaction region. However, when $\Delta\beta_1\Delta\beta_2 < 0$ there exist a class of fields captured (dragged) by the pump beam[6,7], which in most cases are of principal interest, since their gain is the largest. These fields correspond to small solutions of Eq. (9) on the boundary of the interaction region, for waves with an average group velocity directed along the direction of the pump radiation beam $(\Delta\beta = 0)$. The existence of such solutions can be seen from relation (14), in which we can put $\Delta\beta = 0$ when $\Delta\beta_1\Delta\beta_2 < 0$[5)]. As seen from (14), in this case fields that are small on the boundaries of the interaction region remain small relative to fields in the interaction region during the course of the entire propagation process, and at distances $z > a/2\sqrt{D\gamma^0_+}$ ($z > a(\Delta\beta_1 - \Delta\beta_2)/2\sqrt{2}\,|\Delta\beta_1\Delta\beta_2|$ in dimensional variables) the wave tends to the fundamental mode of the parametric waveguide[13], the amplitude of which increases exponentially

$$A^0_{(+)}(z;x) = \left[\frac{\sqrt{2}}{\sqrt{\pi\rho_0^2}}\int_{-\infty}^{\infty} dx_1\bar{A}_+(x_1)\exp\left(-\frac{x_1^2}{\rho_0^2}\right)\right]\times\exp\left[\gamma^0_{(+)}\left(1 - \frac{1}{a\sqrt{2}}\right)z - \frac{x^2}{\rho_0^2}\right]. \qquad (15)$$

Here $\rho_0^2 = 2a\sqrt{D}/\sqrt{\gamma^0_+}$ (in dimensional variables, $\rho_0^2 = 2a\gamma_0^{-1}\sqrt{|\Delta\beta_1\Delta\beta_2|}$) is the square of the radius of the fundamental mode.

Thus, it follows from the foregoing that the equation of the single-wave approximation (9) at $\Delta\beta_1\Delta\beta_2 > 0$ makes it possible to investigate the process of parametric amplification at least up to the start of the departure of the maximum of the field from the interaction region, and in the case when $\Delta\beta_1\Delta\beta_2 < 0$ it can be used to investigate the amplification of fields close to fields captured by the pump radiation.

---

5)Then, as follows from (5) and (10), $\gamma^0_{(+)} = (\tilde{\Delta}\beta^{-1} - \tilde{\alpha})$, $D = (2\tilde{\Delta}\beta)^{-1}$, $\gamma^0_{(-)} = (\frac{1}{2}(\Delta\beta_1^2 + \Delta\beta_2^2)\tilde{\Delta}\beta^{-1} + \alpha)$.

1S. E. Harris, IEEE, QE-3, 5, 205 (1967).
2V. M. Fortus and G. I. Freidman, Izv. vuzov, Radiofizika, 12, 1200 (1969).
3G. D. Boyd and D. A. Kleinman, J. Appl. Phys. 39, 3597 (1968).
4G. I. Freĭdman, Proc. First Vavilov Conference, Novosibirsk, 1969.
5W. Glann, Appl. Phys. Lett. 11, 333 (1967).
6M. M. Sushchik, V. M. Fortus, and G. I. Freĭdman, Izv. vuzov, Radiofizika 12, 293 (1967).
7M. M. Sushchik, V. M. Fortus, and G. I. Freĭdman, ibid. 13, 251 (1970).
8S. A. Akhmanov and R. V. Khokhlov, Problemy nelineĭnoĭ optiki (Problems of Nonlinear Optics), Nauka, 1964.
9L. A. Ostrovskiĭ, Izv. vuzov Radiofizika 4, 955 (1961).
10M. I. Rabinovich and E. I. Yakubovich, ibid. 9, 987 (1966).
11Yu. A. Kravtsov, Zh. Eksp. Teor. Fiz. 55, 1470 (1968) [Sov. Phys.-JETP 28, 769 (1969)].
12V. I. Talanov, Dissertation, Gor'kiĭ, 1967.
13M. M. Sushchik and G. I. Freĭdman, Izv. vuzov, Radiofizika 13, No. 9 (1970).

Translated by J. G. Adashko
241