THEORY OF SURFACE ELECTROMAGNETIC WAVES IN METALS LOCATED IN A WEAK MAGNETIC FIELD

É. A. KANER and N. M. MAKAROV

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

Submitted November 26, 1969

Zh. Eksp. Teor. Fiz. 58, 1972–1986 (June, 1970)

It is shown that surface electromagnetic waves exist in metals located in a weak magnetic field which is parallel to the interface. The wave spectrum is localized near the electron transition frequencies between various surface levels. The existence of an additional shift and of collisionless broadening of resonances on these levels is established. A quantum theory of surface waves is developed under the condition of specular reflection of electrons from the metal-vacuum boundary. The influence of anisotropy of the Fermi surface is investigated and it is shown that there are no surface waves near the local maxima of the transition frequencies as a function of p_z. These surface oscillations are electron eigenwaves in the Fermi gas of the conduction electrons.

1. INTRODUCTION

NEAR the surface of a metal, the quantum states of the electrons in a magnetic field differ from the known Landau levels. These differences are due to collisions between the electrons and the surface of the sample. If the magnetic field **H** is parallel to the boundary and the scattering of the electrons is specular, then, owing to multiple reflection, the electrons drift along the interface in a direction perpendicular to the field **H**. Their motion along the normal to the boundary is then finite and periodic, and consequently can become quantized. Such quantum states are called magnetic surface levels. A special position among the surface electrons is occupied by the so-called "glancing" electrons, in which the center of the classical orbit is located outside the metal, at a distance approximately equal to the radius of the electron trajectory (Fig. 1). The period of the finite motion of the glancing electrons is much smaller than the cyclotron period, and therefore the frequency of the transitions between the different quantized states lies in the microwave band even in very weak magnetic fields, $H \sim 1-10$ Oe. The special role of the glancing electrons is connected with their significant influence on the high-frequency properties of the metals. The oscillations of the surface impedance of metals in weak fields, discovered by M. Khaĭkin[1] in 1960, is resonant absorption of the external electromagnetic wave in transitions between discrete levels of glancing electrons. Such an explanation of the impedance oscillations was first given by Nee and Prange[2]. The possibility of such a treatment of the oscillations is based on the fact that the attenuation of the surface states is a result of collisions between electrons and the rough boundary of the sample should be small compared with the distance between levels. In other words, the electron reflection should be close to specular. For glancing electrons, the reflection coefficient from a rough surface is close to unity, even if the scattering of all the remaining surface electrons is diffuse. This statement is based on the fact that in the case of glancing incidence the effective "wavelength" of the electron $1/k_x$ is much larger than the average height of the roughness. The attenuation of the surface states was investigated in detail in a paper by Fuks and the authors[3].

Impedance oscillations in weak fields were observed by M. Khaĭkin in tin, indium, and cadmium[1] and also in bismuth[4], and by Fawcett and Walsh[5] and Herrmann[6] in tungsten. Very detailed experimental investigations of these effects were carried out by Koch and his co-workers in tin, indium, and aluminum[7], copper, gallium[8], and bismuth[9]. The nonmonotonic change of the impedance of potassium in weak fields at radio frequencies was described by Gantmakher, Fal'kovskiĭ, and Tsoĭ[10].

The discussed effect is actually cyclotron resonance on the magnetic surface levels. The usual cyclotron resonance[11], due to the transitions between the Landau levels of the volume electrons, occurs in a strong magnetic field at frequencies that are multiples of the cyclotron frequency Ω. Since the quantum levels of the glancing electrons are not equidistant, the oscillations in weak fields have no periodicity in the reciprocal magnetic field and are the result of a superposition of different resonance series. It must be emphasized that this phenomenon, like any other resonance, is a collective effect.

It was established in recent years that collective oscillations, namely weakly damped electromagnetic waves, exist in the vicinity of resonances of various types[12]. By way of an example we can point to cyclotron waves[13,14] near cyclotron resonances, spin waves in alkali metals near paramagnetic resonance[15,16], quantum waves near giant quantum oscillations of Landau damping[17,18], etc. The collective character of the resonances in weak fields gives grounds for assuming that electromagnetic waves should exist near the transition frequencies between the magnetic surface

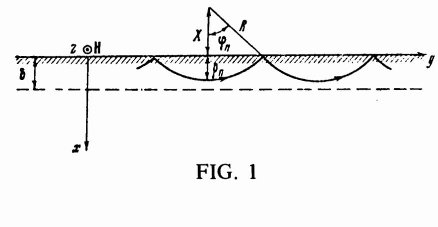

FIG. 1

levels. Since the considered quantum states are localized near the surface of the metal, the corresponding waves should be surface waves.

In this paper we present a theoretical investigation of the possible existence of surface electromagnetic waves in metals. It is shown that such waves indeed exist if the electron mean free path l is sufficiently large. We determine the spectrum and the damping of the surface oscillations, and also investigate the influence of the anisotropy of the electron dispersion law on the properties of the surface waves. A preliminary report of this investigation was published earlier [19].

2. FORMULATION OF PROBLEM

Let a metal be placed in a constant and homogeneous magnetic field **H** parallel to its surface. We direct the x axis along the inward normal to the metal-vacuum interface and the z axis parallel to the vector **H** (Fig. 1). Since the glancing electrons drift along the surface in the direction of the y axis, we seek a surface H-wave in which only the component of the electric field differs from zero. Inside the metal (x > 0) we have

$$E_y(x, z, t) = E(x) \exp[i(qz - \omega t)], \tag{2.1}$$

and in the vacuum (x < 0)

$$E_y(x, z, t) = E(0) \exp[\varkappa x + iqz - i\omega t]. \tag{2.2}$$

From Maxwell's equations in vacuum it follows that

$$\varkappa = (q^2 - \omega^2/c^2)^{1/2}, \tag{2.3}$$

where c is the velocity of light. The nonzero components of the alternating magnetic field are

$$H_x^{(\sim)} = -\frac{cq}{\omega} E_y, \qquad H_z^{(\sim)} = \frac{c}{i\omega}\frac{\partial E_y}{\partial x}. \tag{2.4}$$

The continuity of E_y on the interface x = 0 ensures continuity of $H_x^{(\sim)}$. The condition for the conservation of the z component of the alternating magnetic field on passing through the surface of the metal is

$$E'(0) = (q^2 - \omega^2/c^2)^{1/2} E(0), \tag{2.5}$$

where the prime denotes the derivative with respect to x. The boundary condition (2.5) represents the equality of the surface impedance of the metal

$$Z(\omega, q) = \frac{4\pi i\omega}{c^2}\frac{E(0)}{E'(0)} \tag{2.6}$$

to the impedance of free space $4\pi i\omega/c^2\kappa$.

The surface impedance of the metal (2.6) is determined by solving Maxwell's equations, which we write down for the Fourier component

$$\mathscr{E}(k) = 2\int_0^\infty E(x)\cos(kx)\,dx: \tag{2.7}$$
$$(k^2+q^2)\mathscr{E}(k) + 2E'(0) = 4\pi i\omega c^{-2} j(k, q).$$

Here j(k, q) is the Fourier component of the y-component of the current density of the conduction electrons. Equation (2.7) yields an independent relation between $E'(0)$ and

$$E(0) = \frac{1}{\pi}\int_0^\infty \mathscr{E}(k)\,dk. \tag{2.8}$$

This relation together with (2.5) yields the dispersion equation $\omega = \omega(q)$ of the surface waves.

3. CURRENT DENSITY

To solve Eq. (2.7) it is necessary to know the current density j(k, q). We obtain first an expression for j(k, q) and demonstrate the possibility of the existence of surface waves in an idealized model of the metal, whose Fermi surface is a circular cylinder with axis parallel to the magnetic field **H**. In other words, we assume that the electron dispersion law

$$\varepsilon(\mathbf{p}) = (p_x^2 + p_y^2)/2m \tag{3.1}$$

is independent of p_z. Here ϵ is the energy, **p** the momentum, and m the effective mass of the electron. In this model, the z projection of the electron velocity is $v_z \equiv 0$, and therefore the current density j(k, q) = j(k) does not depend on the wave number q.

It is obvious from physical considerations that independent contributions to the current density are made by the volume (nonresonant) electrons and by the electrons colliding with the surface of the metal. Accordingly, the current j(k) can be represented in the form of a sum of two terms:

$$j(k) = j_{\mathrm{vol}}(k) + j_{\mathrm{sur}}(k). \tag{3.2}$$

Since the impedance oscillations in weak fields and the sought surface waves are the result of quantum transitions between discrete states of the conduction electrons, the current density, generally speaking, must be calculated by quantum theory. However, the quantization of the electron states in weak fields is important only in the calculation of $j_{\mathrm{sur}}(k)$. The current density of the volume electrons must be determined by classical theory, since the quantum effect in $j_{\mathrm{vol}}(k)$ are negligibly small in the weak-field region H ~ 1–10 Oe under consideration. In this region, the following inequalities are satisfied:

$$\delta \ll v/|\nu - i\omega| \ll (\delta R)^{1/2}. \tag{3.3}$$

The physical meaning of the right-hand inequality is that the characteristic path length $(2\delta R)^{1/2}$ of the electron trajectory in the skin layer δ is large compared with the effective mean free path $v/|\nu - i\omega|$. The left-hand inequality is simply the condition for the anomalous skin effect. Here $R = v/\Omega$ is the cyclotron radius, v the Fermi velocity, and ν the frequency of the collisions between the electron and the scatterers.

When the inequalities (3.3) are satisfied, it is possible to disregard completely the dependence on the magnetic field in the expression for $j_{\mathrm{vol}}(k)$. In the limiting case of the anomalous skin effect we can obtain for the current density of the volume electrons the following asymptotic formula:

$$j_{\mathrm{vol}}(k) = \frac{\omega_0^2}{2\pi v}\left\{\frac{\mathscr{E}(k)}{k} - \frac{1}{2\pi}\int_0^\infty \frac{dk'\mathscr{E}(k')}{(kk')^{1/2}}\left[\pi\delta(k'-k) + \frac{1}{k'+k}\right]\right\}, \tag{3.4}$$

where $\omega_0 = (4\pi Ne^2/m)^{1/2}$ is the plasma frequency, e the absolute value of the charge, and N the electron density. The integral term in $j_{\mathrm{vol}}(k)$ is due to the presence of a metal-vacuum interface. As will be shown below, this integral term is offset exactly by the corresponding term in the surface current due to the nonresonant electrons.

We now proceed to calculate the surface current density $j_{\mathrm{sur}}(k)$. We confine ourselves henceforth to the quasiclassical approximation, since the exact quantum mechanical expressions for the wave functions and the matrix elements are quite complicated. Numerical calculations performed in[20] show that the quasiclassical approximation give good results even for small quantum numbers $n \gtrsim 4$. We derive a quasiclassical formula for $j_{sur}(k)$ by quantizing the corresponding classical expression. The latter was found in[21] and in the case of specular reflection of the electrons from the metal-vacuum interface it is given by

$$j_{sur}(k)=\frac{1}{\pi}\int_0^\infty dk' Q_{sur}(k,k')\mathscr{E}(k'). \tag{3.5}$$

For a cylindrical Fermi surface, the kernel of the operator $\hat{Q}_{sur}$ can be represented in the form

$$Q_{sur}(k,k')=i\frac{\omega_0^2 R}{\pi^2}\int_0^\pi d\varphi\frac{\sin\varphi}{\varphi}\sum_{s=-\infty}^{\infty}\left(\omega-\frac{\pi s\Omega}{\varphi}+i\nu\right)^{-1}$$
$$\times\int_0^\varphi d\lambda\cos\lambda\cos(\pi s\lambda/\varphi)\cos[kR(\cos\varphi-\cos\lambda)]$$
$$\times\int_0^\varphi d\mu\cos\mu\cos(\pi s\mu/\varphi)\cos[k'R(\cos\varphi-\cos\mu)]. \tag{3.6}$$

The physical meaning of the variable φ is defined by the formula $\cos\varphi = -X/R$, where X is the projection of the coordinate of the center of rotation of the electron on the x axis. Consequently, φ is the glancing angle of the electron at the instant of collision with the surface of the metal (Fig. 1).

In order to change over from the classical description to the quantum description, it is necessary to quantize the motion of the surface electrons. According to the results of[3], the condition for quasiclassical quantization consists in the fact that the X coordinate of the center of rotation assumes discrete values in accordance with the formula

$$\frac{\varepsilon}{\hbar\Omega}\left\{1+\frac{2}{\pi}\left[\frac{X}{R}\left(1-\frac{X^2}{R^2}\right)^{1/2}+\arcsin\frac{X}{R}\right]\right\}=2\left(n-\frac{1}{4}\right),$$
$$n=1,2,3\dots$$

Thus, the condition for the quantization of the angle φ is

$$2\varphi-\sin(2\varphi)=2\pi\hbar\Omega(n-1/4)/\varepsilon. \tag{3.7}$$

The transition to the quasiclassical expression for the surface density of the current consists in replacing the integral with respect to φ by a sum with respect to n. It is then necessary to replace $d\varphi$ by $\pi\hbar\Omega/\epsilon[1-\cos(2\varphi)]$. As a result we get

$$Q_{qw}(k,k')=i\frac{\hbar}{\pi p}\sum_{n=1}^{M}\frac{1}{\varphi_n\sin\varphi_n}\sum_{s=-\infty}^{\infty}\frac{\omega_0^2}{\omega-\omega_{ns}+i\nu}$$
$$\times\int_0^{\varphi_n}d\lambda\cos\lambda\cos\left(\frac{\pi s\lambda}{\varphi_n}\right)\cos[kR(\cos\varphi_n-\cos\lambda)]\int_0^{\varphi_n}d\mu\cos\mu\cos\left(\frac{\pi s\mu}{\varphi_n}\right)$$
$$\times\cos[k'R(\cos\varphi_n-\cos\mu)], \tag{3.8}$$

where φ_n is the solution of (3.7),

$$\omega_{ns}=(\varepsilon_{n+s}-\varepsilon_n)/\hbar=\pi s\Omega/\varphi_n \tag{3.9}$$

is the frequency of the transition between the levels n + s and n, $2\pi\hbar$ is Planck's constant, and $p = mv$. In the quasiclassical approximation the quantum number n is assumed to be large compared with $|s|$. The summation over n is from unity to M, which is equal to the integer part of the ratio $\epsilon_F/\hbar\Omega$, where ϵ_F is the Fermi energy. The effective collision frequency ν contained in the resonant denominators of formula (3.8) is actually the sum of the damping decrements of the wave functions in the states n + s and n. In the case of strict specular reflection of the electrons from the surface, when the principal role is played by volume scattering, the damping of the surface states is independent of the magnetic field and of the quantum numbers, and ν coincides with the collision frequency in the volume of the metal.

In such a derivation of the quasiclassical expression for the current density of the surface electrons it is tacitly assumed that the total energy of the electrons ϵ can be replaced by the Fermi energy ϵ_F. The possibility of such a substitution in the case of quantum resonances is actually far from obvious. For example, giant quantum oscillations of the absorption of acoustic and electromagnetic waves become strongly smeared out even at low temperatures[22], when the uncertainty of the Fermi energy is quite small. Therefore it is necessary to obtain an exact criterion for the correctness of the substitution of ϵ_F for ϵ. The effective smearing of the Fermi distribution due to the temperature T and the finite quantum energy of the electromagnetic wave $\hbar\omega$ is of the relative order of magnitude $\Delta\epsilon_F/\epsilon_F \sim (T+\hbar\omega)/\epsilon_F \ll 1$. The resonant-frequency uncertainty $\Delta\omega_{ns}$ uncertainty associated with this smearing is of the order of $\omega(T+\hbar\omega)/\epsilon_F$. Consequently, formula (3.8) is valid if the indicated resonant-frequency uncertainty $\Delta\omega_{ns}$ is small compared with the collision frequency ν, i.e.,

$$\omega\frac{T+\hbar\omega}{\varepsilon_F}\ll\nu. \tag{3.10}$$

This inequality, just as expression (3.8), was derived from exact quantum-mechanical calculations. It is well satisfied in the frequency region $\omega \lesssim 10^{11}\ \text{sec}^{-1}$ and at temperatures $T \lesssim 10°K$ even at large mean free paths $l = v/\nu \sim 1$ cm. If the inequality (3.10) is violated, then the expression for j_{sur} becomes much more complicated.

It is seen from (3.8) that in the quantum case the surface current has a resonant character. The resonance occurs at the frequencies ω_{ns}. Near resonance, as $\nu \to 0$, it is possible to separate from the sum (3.8) one resonant term. The remaining sum over n and s is a continuous function of the frequency end of the magnetic field, and therefore the sum over n can be replaced by an integral. Thus, the contribution of the nonresonant electrons to the surface current is described by the classical formula (3.6). The asymptotic behavior of Q_{sur} in the limit of the anomalous skin effect and weak magnetic fields (3.3) can be obtained from (3.6) by the stationary-phase method. It turns out that the contribution to the current from the stationary point $\varphi = \pi$ cancels exactly the integral term in the volume-electron current (3.4). On the other hand, the contribution of the point $\varphi = 0$ is negligibly small compared with the volume current by virtue of the first inequality of (3.3).

Resonance in weak magnetic fields is ensured by glancing electrons with small values of the glancing angle

$$\varphi_n=\left[\frac{3\pi\hbar\Omega}{mv^2}\left(n-\frac{1}{4}\right)\right]^{1/3}, \tag{3.11}$$

for which $n \ll \epsilon_F/\hbar\Omega$. Expression (3.11) follows directly from formula (3.7) at small φ. The resonant term in Q_{qu}, due to the glancing electrons, takes the form

$$Q(k,k') = i\frac{\hbar}{\pi p}\frac{\omega_0^2}{\omega - \omega_{ns} + i\nu}\psi_{ns}(k)\psi_{ns}(k'), \quad (3.12)$$

where

$$\psi_{ns}(k) = \int_0^1 dx \cos(\pi s x)\cos[k\rho_n(1-x^2)] \quad (3.13)$$

is the limiting quasiclassical expression for the matrix element of the plane wave between the wave functions of the surface states with numbers n + s and n. The length

$$\rho_n = (1/2)R\varphi_n^2 \quad (3.14)$$

is the height of the segment made up of the arc of the n-th quantized orbit and the surface of the metal (Fig. 1).

The quanticlassical formulas (3.9), (3.12), and (3.13) are valid when $n \gg |s|$. If they are applied to transitions with small $n \sim |s|$, then a noticeable error will result. To refine these expressions for small quantum numbers it is necessary to use for the transition frequency ω_{ns} and for the functions $\psi_{ns}(k)$ formulas that are not expanded in powers of s/n. In this case

$$\omega_{ns} = (3\pi\Omega/2)(mv^2/3\pi\hbar\Omega)^{1/3}[(n+s-1/4)^{2/3} - (n-1/4)^{2/3}],$$

and $\cos(\pi s x)$ in the integrand of (3.13) is replaced by

$$2\left(1+\frac{\hbar\omega_{ns}}{\varphi_n^2\varepsilon_F}\right)^{-1/4}\left(1+\frac{\hbar\omega_{ns}}{\varphi_n^2\varepsilon_F}x^{-2}\right)^{-1/4}\cos\left[\pi\left(n-\frac{1}{4}\right)x^3-\frac{\pi}{4}\right]$$
$$\cdot\cos\left[\pi\left(n-\frac{1}{4}\right)x^3\left(1+\frac{\hbar\omega_{ns}}{\varphi_n^2\varepsilon_F}x^{-2}\right)^{3/2}-\frac{\pi}{4}\right].$$

Summing the foregoing, the final expression for the Fourier component of the current density j(k) can be represented in the form of two independent terms:

$$j(k) = \frac{\omega_0^2\mathscr{E}(k)}{2\pi k v} + i\frac{\hbar\omega_0^2}{\pi^2 p}\frac{\psi_{ns}(k)}{\omega-\omega_{ns}+i\nu}\int_0^\infty dk'\mathscr{E}(k')\psi_{ns}(k'). \quad (3.15)$$

The first (classical) term in (3.15) is the Fourier component of the current density of an unbounded metal and is due to the volume electrons which form a skin layer near the surface. The second (quantum) component describes the resonant part of the current of the glancing electrons. The relative magnitude of these two terms near resonance is determined by the parameter $\hbar k^2/m\nu$, which is the ratio of the energy of an electron momentum hk to the collision width of the electron levels $\hbar\nu$. If we assume that k is of the order of the reciprocal skin-layer thickness δ^{-1}, then the resonant value of the surface current is much larger than the screening current of the volume electrons under the condition

$$\hbar/m\delta^2\nu \gg 1. \quad (3.16)$$

When this inequality is satisfied, however, it is impossible to neglect the first term in (3.15) because the resonant part of the surface current does not lead to a skin effect, i.e., to a screening of the metal against the external wave. As will be shown below, inequality (3.16) is the necessary condition for the existence of surface waves.

4. SURFACE IMPEDANCE AND DISPERSION EQUATION

To obtain the dispersion equation of the surface waves it is necessary to solve Maxwell's equation (2.7), in which the current density is given by formula (3.15). We introduce the notation

$$\mathscr{E}(k) = F(k)E'(0), \quad B = \frac{2\hbar\omega_0^2}{c^2 p} = \frac{\hbar}{m\omega\delta^3} \quad \delta = \left(\frac{c^2 v}{2\omega_0^2\omega}\right)^{1/3}. \quad (4.1)$$

In this notation, Eq. (2.7) takes the form

$$F(k) = -\frac{2}{k^2+q^2-i(\delta^3 k)^{-1}}\left[1+B\frac{\omega}{\omega-\omega_{ns}+i\nu}\psi_{ns}(k)\Phi_{ns}\right]. (4.2)$$

This relation is an integral equation with respect to F(k) with a degenerate kernel. The unknown constant Φ_{ns} is expressed in terms of F(k) by

$$\Phi_{ns} = \frac{1}{\pi}\int_0^\infty dk' F(k')\psi_{ns}(k'). \quad (4.3)$$

We substitute in (4.3) the value of F(k) from (4.2) and solve the resultant linear equation with respect to Φ_{ns}. As a result we get

$$\Phi_{ns} = -\alpha_{ns}(q)\frac{\omega-\omega_{ns}+i\nu}{\omega[1+B\beta_{ns}(q)]-\omega_{ns}+i\nu}. \quad (4.4)$$

Here

$$\alpha_{ns}(q) = \frac{2}{\pi}\int_0^\infty\frac{dk\psi_{ns}(k)}{k^2+q^2-i(\delta^3k)^{-1}}, \quad \beta_{ns}(q) = \frac{2}{\pi}\int_0^\infty\frac{dk\psi_{ns}^2(k)}{k^2+q^2-i(\delta^3k)^{-1}}. \quad (4.5)$$

Formula (4.4) makes it possible to break down the Fourier component of the electric field in the form

$$\mathscr{E}(k) = -\frac{2E'(0)}{k^2+q^2-i(\delta^3k)^{-1}}\left\{1-\frac{B\omega\alpha_{ns}(q)\psi_{ns}(k)}{\omega[1+B\beta_{ns}(q)]-\omega_{ns}+i\nu}\right\}. \quad (4.6)$$

From this we readily obtain the surface impedance $Z(\omega, q)$ of the metal. We present for it an explicit expression at q = 0:

$$Z(\omega,0) = Z_0 + i\frac{4\pi\omega^2\alpha_{ns}^2(0)c^{-2}B}{\omega[1+B\beta_{ns}(0)]-\omega_{ns}+i\nu}, \quad (4.7)$$

where

$$Z_0 = \frac{16\pi\omega\delta}{3\sqrt{3}c^2}e^{-i\pi/3} \quad (4.8)$$

is the impedance of the metal in the absence of a magnetic field in the case of specular reflection of the electrons from the surface. The surface impedance $Z(\omega, 0)$ determines the absorption and the phase shift (the shift of the frequency of the resonant circuit) at normal incidence of the external electromagnetic wave on the metal.

Formula (4.7) differs from the analogous expression obtained for the impedance by Prange and Nee[23] by a variational method. In their formulas there is no shift and additional damping of the resonance lines due to the complex term $B\beta_{ns}(0)$ in the resonance denominators of (4.7). Indeed, these authors assume that the resonance terms in the current density are small compared with the nonresonant current of the volume electrons. In other words, they consider a limiting case opposite to (3.16). In this case Eq. (4.2) must be solved with the aid of successive approximations, putting in the zeroth approximation $\Phi_{ns} = -\alpha_{ns}(0)$. Formula (4.7) obtained by us in general and is valid for any value of the parameter $B\beta_{ns}(0)$. It is seen from this formula that the resonance in the surface impedance, generally speaking, is shifted towards lower frequencies (stronger fields) relative to the transition frequency ω_{ns}. The relative frequency shift $(\omega_{ns} - \omega_{res})/\omega_{res}$ is equal to $B\,\mathrm{Re}\,\beta_{ns}(0)$. Besides the resonance shift, there is also an additional line broadening, the relative magnitude of which is determined by B Im $\beta_{ns}(0)$. The change in the position and the additional damping of the resonance are due to the change of the amplitude and phase of the electromagnetic field on the trajectories of the glancing electrons.

We shall not analyze here in detail the form of the resonance curve, and present only asymptotic expressions for $\alpha_{ns}(0)$ and $\beta_{ns}(0)$ as functions of the relation between δ and ρ_n. If the resonant trajectories lie entirely inside the skin layer, i.e.,

$$\rho_n \ll \delta, \tag{4.9}$$

then we have

$$\alpha_{ns}(0) = (-1)^s \frac{2\rho_n}{(\pi s)^2}, \quad \beta_{ns}(0) = \frac{\rho_n}{2(\pi s)^2} + i \frac{8[1-6/(\pi s)^2]^2 \rho_n^4}{\pi(\pi s)^4 \delta^3} \ln\left(\frac{\delta}{\rho_n}\right). \tag{4.10}$$

In the opposite limiting case, when only the ends of the resonant orbits lie inside the skin layer,

$$\rho_n \gg \delta, \tag{4.11}$$

we get

$$\alpha_{ns}(0) = i(-1)^s \frac{\pi|s|}{4} \frac{\delta^3}{\rho_n^2}, \quad \beta_{ns}(0) = \frac{\pi}{6\sqrt{3}} \frac{\delta^2}{\rho_n} e^{i\pi/3}. \tag{4.12}$$

It follows from these asymptotic formulas that the minimal values of $|\alpha_{ns}|$ and $|\beta_{ns}|$ are reached at $\delta \sim 2\rho_n/\pi|s|$ and coincide in order of magnitude with the thickness of the skin layer δ. The presence of a maximum of $|\alpha_{ns}(0)|$ as a function of the ratio $2\rho_n/\pi|s|\delta$ explains the experimental fact that resonances in the surface impedance can be resolved well only in a relatively small interval of weak fields, where the depth of the resonant trajectories is comparable with the thickness of the skin layer.

Summarizing the foregoing discussion of the singularities of the resonance in the surface impedance $Z(\omega, 0)$, we can draw the following conclusion. If the free path length is large enough and the inequality (3.15) is satisfied, then the resonances will be shifted somewhat relative to ω_{ns}, and additional collisionless line broadening appears.

We now proceed to derive the dispersion equation of the surface waves. As will be shown subsequently, the spectrum of the surface waves is localized near the resonant frequencies ω_{ns}. Therefore the frequency ω can be replaced by ω_{ns} everywhere except in the resonant denominator.

Using (4.2), we obtain from the boundary condition (2.5) the following dispersion equation for the surface waves in the metal:

$$\left(q^2 - \frac{\omega^2}{c^2}\right)^{-1/2} = \frac{B\omega_{ns}\alpha_{ns}^2(q)}{\omega - \omega_{ns}[1 - B\beta_{ns}(q)] + i\nu} - f(q), \tag{4.13}$$

where

$$f(q) = \frac{2}{\pi}\int_0^{\infty} \frac{dk}{k^2 + q^2 - i(\delta^3 k)^{-1}}. \tag{4.14}$$

Equation (4.13) is an implicit definition of the function $\omega = \omega(q)$. In order to obtain the explicit dependence of the frequency of the surface electromagnetic wave on the wave number q, let us consider several limiting cases.

1. We first investigate the dispersion properties of long-wave oscillations, when the wavelength is large compared with the thickness of the skin layer δ and is smaller than the wavelength in vacuum, i.e.,

$$\omega/c \leqslant q \ll 1/\delta. \tag{4.15}$$

It is obvious that in this case the surface wave attenuates in the interior of the metal at a distance on the order of δ. The functions f(q), $\alpha_{ns}(q)$, and $\beta_{ns}(q)$ can be regarded as independent of q by virtue of (4.15) and $k \sim 1/\delta$. Inasmuch as in this case $|f| \sim \delta$, the second term in (4.13) can be neglected compared with 1/q. It is then easy to write down the spectrum and the damping of the surface wave in the form

$$\omega = \omega_{ns}[1 - B\beta_{ns}(0) + B\alpha_{ns}^2(0)(q^2 - \omega_{ns}^2/c^2)^{1/2}] - i\nu. \tag{4.16}$$

The spectrum begins with $q = \omega_{ns}/c$, and the group velocity at this point becomes infinite. From (4.10) and (4.12) it follows that the imaginary part of $\beta_{ns}(0)$, which determines the collisionless damping of the surface waves, is small compared with the real part only in the limiting case (4.9), when $\rho_n \ll \delta$. The oscillation dispersion law in this case is

$$\omega(q) = \omega_{ns}\left[1 - \frac{B\rho_n}{2(\pi s)^2} + \frac{4B}{(\pi s)^4}\rho_n^2 q\right], \quad q \gg \omega_{ns}/c. \tag{4.17}$$

It should be noted that formula (4.17) is valid in order of magnitude also under the less stringent condition $\rho_n \lesssim \pi|s|\delta/2$. If it is assumed here that $q \lesssim 1/\delta$, then the dispersion of the natural frequency exceeds its damping. In the opposite limiting case, when $\rho_n > \delta$, the dispersion of the frequency turns out to be smaller than the collisionless damping due to Im $\beta_{ns}(0)$ (see (4.12)), and therefore the concept of surface waves become meaningless.

2. Let us consider now the intermediate region of wave numbers

$$1/\delta \ll q \ll 1/\rho_n. \tag{4.18}$$

Owing to the condition $q\delta \gg 1$, the surface wave is localized at distances on the order of the wavelength 1/q from the surface of the metal ($E(x) \sim e^{-qx}$). The functions f(q), $\alpha_{ns}(q)$, and $\beta_{ns}(q)$ do not depend on δ and are, in the main, real quantities, $k \sim q$, with $f(q) = 1/q$ and $\alpha_{ns}(q)$ and $\beta_{ns}(q)$ are given by formulas

$$\alpha_{ns}(q) = (-1)^s \frac{2\rho_n}{(\pi s)^2}, \quad \beta_{ns}(q) = \frac{\rho_n}{2(\pi s)^2}. \tag{4.19}$$

The spectrum of the surface wave in this case

$$\omega(q) = \omega_{ns}\left[1 - \frac{B\rho_n}{2(\pi s)^2} + \frac{2B}{(\pi s)^4}\rho_n^2 q\right] \tag{4.20}$$

differs from (4.17) in that the coefficient of the dispersion of the natural frequency turns out to be smaller by a factor of 2.

3. In the region of extremely short waves, when

$$1/\delta,\ 1/\rho_n \ll q, \tag{4.21}$$

the law of dispersion of the surface oscillations deviates from linearity because of the dependence of the function β_{ns} on q. In this case

$$\alpha_{ns}(q) = (-1)^s/2\rho_n q^2, \qquad \beta_{ns}(q) = [1/2 + \rho_n q \ln(\rho_n q)]/4q(\rho_n q)^2. \tag{4.22}$$

The dependence of the natural frequency on the wave number is given by

$$\omega(q) = \omega_{ns}\left[1 - \frac{B\ln(\rho_n q)}{4\rho_n q^2}\right]. \tag{4.23}$$

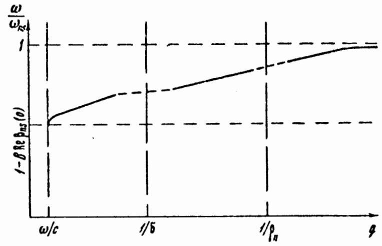

FIG. 2

In this region of wave numbers, as $q \to \infty$, the natural frequency of the surface waves tends to the limiting value ω_{ns}.

Figure 2 shows schematically the dependence of the frequency on the wave number q. Near the boundaries of the different regions, the spectrum is shown dashed; it is difficult here to obtain a simple analytic expression for the spectrum, and numerical calculations are needed. Because of the condition $\rho_n < \delta$, it can be stated that the dispersion of the natural frequency exceeds the damping on the boundaries of the intervals. Such surface oscillations exist, generally speaking, in the vicinity of each transition frequency ω_{ns}. As seen from (4.17) and (4.23), the spectrum is located in an interval of width $B\omega_{ns}\mathrm{Re}\,\beta_{ns}(0)$ below the frequency ω_{ns}. The width of this interval relative to the resonant frequency ω_{ns} is equal to $B\,\mathrm{Re}\,\beta_{ns}(0) = \hbar\rho_n/2(\pi s)^2 m\,\omega_{ns}\delta^3$. The maximum value of this quantity is reached at $\rho_n \sim \pi|s|\delta/2$ and is of the order of $\hbar/4\pi|s|m\,\omega\delta^2$. An estimate of this quantity at $s \sim 1$, $\omega \sim 10^{11}\ \mathrm{sec}^{-1}$, $\delta \sim 10^{-5}$ cm, and $m \sim 0.1m_0$ yields 0.01—0.01. It is obvious that one can speak of the spectrum of surface oscillations only in the case when the width is larger than the damping. Under real conditions the principal role is played apparently by collision damping ν. Therefore the relative width $B\,\mathrm{Re}\,\beta_{ns}(0)$ should certainly be larger than ν/ω_{ns}, corresponding to the inequality (3.16). On the basis of the foregoing numerical estimate of the width of the spectrum, ω/ν should exceed 10^2. In other words, the surface waves can exist only in very pure metals with mirror-polish surfaces in which the mean free path is $l \sim 3$—10 mm; they can exist in the millimeter wavelength band.

5. INFLUENCE OF THE FORM OF THE FERMI SURFACE

We have considered so far an idealized model of a metal with a cylindrical Fermi surface. An essential feature of this model is the fact that the transition frequencies ω_{ns} do not depend on p_z. Resonant absorption by magnetic surface levels exists also in the case of noncylindrical Fermi surfaces, for example in copper [8]. In this case the frequency of the transitions is a function of p_z, as a result of which the character of the resonant singularity and the shape of the resonance curves change. Accordingly, a change should also occur in the spectrum of the surface waves. In this section we investigate the influence of the shape of the Fermi surface on the properties of the surface oscillations.

We start with the case of a Fermi sphere (alkali metal). The first term in expression (3.15) with a current density j(k, q) has the same form as in an unbounded metal in the absence of a magnetic field

$$j_0(k, q) = \frac{3\omega_0^2\mathscr{E}(k)}{16v(k^2+q^2)^{1/2}}. \tag{5.1}$$

For lack of space we shall not present here the straightforward but rather cumbersome quantum-mechanical calculations of the resonant term in the glancing-electron current, and write down immediately the result

$$j_{\mathrm{res}}(k, q) = i\frac{3\hbar}{4\pi^2 p}\int_0^\pi d\vartheta \sin^2\vartheta \frac{\omega_0^2\psi_{ns}(k\sin^{-1/3}\vartheta)}{\omega^* - qv\cos\vartheta - \omega_{ns}(\vartheta) + i\nu} \times \int_0^\infty dk'\mathscr{E}(k')\psi_{ns}(k'\sin^{-1/3}\vartheta). \tag{5.2}$$

Here ϑ is the polar angle in p-space with polar axis parallel to the magnetic field ($v_z = v\cos\vartheta$),

$$\omega_{ns}(\vartheta) = \omega_{ns}\sin^{2/3}\vartheta, \quad \omega^* = \omega - \hbar q^2/2m. \tag{5.3}$$

The frequency ω_{ns} and the function $\psi_{ns}(x)$ are determined by the previous formulas (3.9) and (3.13). All the remaining symbols have the same meaning as before. Expression (5.2) differs from the corresponding term in (3.15) for a cylindrical Fermi surface in the presence of a Doppler frequency shift $qv\cos\vartheta$ and of a recoil frequency $\hbar q^2/2m$, and also in the dependence of the transition frequency and of the glancing angle φ_n on the polar angle ϑ, over which the averaging is now carried out. Formula (5.2) can be obtained from the classical expression by using the correspondence principle and the same reasoning as used in Sec. 3 in the derivation of (3.15).

Since the transition frequency $\omega_{ns}(\vartheta)$ is not constant, the resonance becomes smeared out even at q = 0, when there is no Doppler frequency shift and $\omega^* = \omega$. We shall show now that, nevertheless, the presence of a root singularity in j_{res} does not yield a solution of the dispersion equation for the surface wave, i.e., in the case of a spherical Fermi surface there is no surface wave. We obtain the proof in the limit as $\nu \to 0$. The main contribution to the integral with respect to ϑ is made by the small vicinity of $\vartheta = \pi/2$. Let us expand $\omega_{ns}(\vartheta)$ in powers of $(\vartheta - \pi/2)$ up to the quadratic terms inclusive, let us replace $\sin\vartheta$ in all the remaining expressions by unity, and let us extend the integration from $-\infty$ to $+\infty$. Then j_{res} can be represented in the form

$$j_{\mathrm{res}}(k, q) = i\frac{3\sqrt{3}\,\hbar\omega_0^2}{4\pi p\omega_{ns}}\psi_{ns}(k)\int_0^\infty dk'\mathscr{E}(k')\psi_{ns}(k')I. \tag{5.4}$$

The quantity I describes the resonant singularity and is equal to

$$I = \frac{1}{\pi\sqrt{3}}\int_{-\infty}^{\infty}\left(\frac{t^2}{3} - \frac{qv}{\omega_{ns}}t + \frac{\omega^* - \omega_{ns}}{\omega_{ns}} + i\gamma\right)^{-1} dt, \tag{5.5}$$

where $t = \pi/2 - \vartheta$, $\gamma = \nu/\omega_{ns} \to +0$. Direct calculation of this integral yields

$$I(\Delta) = |\Delta|^{-1/2}[\theta(\Delta) - i\theta(-\Delta)], \tag{5.6}$$

where

$$\Delta = \frac{\omega^* - \omega_{ns}}{\omega_{ns}} - \frac{3}{4}\left(\frac{qv}{\omega_{ns}}\right)^2,$$

$\theta(x) = 1$ at $x > 0$ and $\theta(x) = 0$ at $x < 0$. Solving Maxwell's equation (2.7) with the current density (5.1) and (5.4), we obtain the dispersion equation (4.13), in which the resonance denominator is proportional to the quantity

$$I^{-1}(\Delta)/\sqrt{3} + B\beta_{ns}(q). \tag{5.7}$$

The possible occurrence of surface electromagnetic waves in the case of a cylindrical Fermi surface was due to the fact that the resonance denominator in (4.13) could vanish. At positive values of B Re $\beta_{ns}(q)$, the frequency difference $\omega - \omega_{ns}$ was negative. On the other hand, in the case of a Fermi sphere, the real part of I is always positive. Therefore the resonance denominator cannot vanish and the dispersion equation has no solutions.

It must be emphasized that the absence of solutions of the dispersion equation of the surface waves is connected with the fact that the extremal resonant frequency has a local maximum in p_z. On this basis we can draw the conclusion that in the case of an arbitrary electron dispersion law there are likewise no surface electromagnetic waves near the local maxima of the transition frequencies $\omega_{ns}(p_z)$.

We proceed now to the case of a Fermi surface of arbitrary shape. We need asymptotic formulas for the screening current $j_0(k, q)$ and the resonant quantum term $j_{res}(k, q)$. In the absence of a magnetic field, $j_0(k, q)$ at an arbitrary electron dispersion law is given by the well known formula

$$j_0(k,q)=\frac{2\pi e^2}{(2\pi\hbar)^3}\int\limits_{\varepsilon=\varepsilon_F} dp_z\,d\tau\,|m_c|\,v_y{}^2\delta\left(v_x+\frac{q}{k}v_z\right)\frac{\mathscr{E}(k)}{k}. \quad (5.8)$$

Here m_c is the cyclotron mass, and τ is the dimensionless time (phase) of motion of the electron on the orbit p_z = const on the Fermi surface in p-space. A natural generalization of formula (5.2) to include the case of an arbitrary dispersion law makes it possible to represent j_{res} in the form

$$j_{res}(k,q)=i\frac{8\hbar e^2}{(2\pi\hbar)^3}\int |v_y|\,dp_z[\omega^*-qv_z-\omega_{ns}(p_z)+i\nu]^{-1}$$
$$\times\psi_{ns}(k,p_z)\int\limits_0^\infty dk'\mathscr{E}(k')\psi_{ns}(k',p_z). \quad (5.9)$$

In this expression the integration with respect to p_z is along the line $v_x = 0$ on the Fermi surface. The distance between levels $\hbar\omega_{ns}(p_z)$ is determined with the aid of the condition of quasiclassical quantization

$$S(\varepsilon,p_z,X)=2\pi\frac{\hbar eH}{c}\left(n-\frac{1}{4}\right). \quad (5.10)$$

The quantity $S(\epsilon, p_z, X)$ is an area bounded by the curve $\epsilon(\mathbf{p}) = \epsilon$, p_z = const, and the straight line $X \equiv -cp_y/eH$ = const in momentum space. From (5.10) we can find $\epsilon_n(p_z, X)$ and consequently $\epsilon_{n+s}(p_z, X) - \epsilon_n(p_z, X)$. Substituting into this difference $X_n(\epsilon, p_z)$ at $\epsilon = \epsilon_F$ for glancing electrons, we obtain the difference $\hbar\omega_{ns}(p_z)$. Here $\omega^* = \omega - \hbar q^2/2m_{\parallel}$, $v_z = \partial\epsilon_n/\partial p_z$, $1/m_{\parallel} = \partial^2\epsilon_n/\partial p_z^2$, and the derivatives with respect to p are taken at constant $X = X_n(\epsilon_F, p_z)$. The function $\psi_{ns}(k, p_z)$ in (5.9) is given by formula (3.13), in which ρ_n is a function of p_z. It should be noted that in the resonant factor in (5.9) the expansion is in powers of the wave number q. Separation of an individual resonant term is possible only when the characteristic quantity is small compared with the resonant frequency $\omega_{ns} \approx \omega$, i.e.,

$$q\ll\omega/v. \quad (5.11)$$

When this condition is satisfied, the wave number q is certainly small compared with $k \sim 1/\delta$. Consequently, the quantity q can be neglected everywhere with exception of the resonant factor in (5.9).

Let us ascertain the possibility of the existence of surface waves in the vicinity of the local minimum of the transition frequency $\omega_{ns}(p_z)$. We integrate in (5.9) in the limit as $\nu \to 0$. A simple calculation yields the result

$$j_{res}(k,q)=i\frac{q^2}{2\pi\omega}E'(0)I(\Delta)B\psi_{ns}(k)\Phi_{ns}, \quad (5.12)$$

where

$$B=\frac{2e^2|v_y|}{(\hbar c)^2}\left(\frac{2\omega_{ns}}{\omega''_{ns}}\right)^{1/2},\quad \Delta=\frac{\omega^*-\omega_{ns}}{\omega_{ns}}+\frac{q^2}{2m^2_{\parallel}\omega_{ns}''\omega_{ns}}.$$

Φ_{ns} and $\psi_{ns}(k)$ are determined by formulas (4.3) and (3.13), respectively. The values of the quantities $|v_y|$, ω_{ns}, $m_{\parallel}$, ρ_n, and $\omega''_{ns} = d^2\omega_{ns}/dp_z^2$ are taken on the section $p_z = p_z^0$, where the frequency ω_{ns} has a minimum ($\omega''_{ns} > 0$). The resonant factor is of the form

$$I(\Delta)=-|\Delta|^{-1/2}[\theta(-\Delta)+i\theta(\Delta)]. \quad (5.13)$$

Unlike the spherical Fermi surface, for which the real part of the resonant factor (5.6) was positive, in the present case Re $I(\Delta)$ is negative. Surface electromagnetic waves can therefore exist in this case in principle.

The dispersion equation for the spectrum of the surface oscillations can be readily obtained by the same method as in Sec. 4. As a result we get

$$\left(q^2-\frac{\omega^2}{c^2}\right)^{-1/2}+f(0)=\frac{B\alpha_{ns}{}^2(0)}{I^{-1}(\Delta)+B\beta_{ns}(0)}. \quad (5.14)$$

The quantities f, α_{ns}, and β_{ns} were introduced in the preceding section and differ in the present case from (4.5) and (4.14) in the definition of the depth of penetration

$$\delta=\left[\frac{e^2\omega}{\pi\hbar^3c^2}\int\limits_{\varepsilon=\varepsilon_F} dp_z\,d\tau\,|m_c|\,v_y{}^2\delta(v_x)\right]^{-1/3}. \quad (5.15)$$

The solution of the dispersion equation (5.14) exists only at $q > \omega/c$ and at negative values of Δ. Neglecting the quantity f(0) compared with 1/q, we obtain

$$\omega(q)=\omega_{ns}\left[1-B^2(\beta_{ns}(0)-\alpha_{ns}{}^2(0)q)^2-\frac{q^2}{2m_{\parallel}{}^2\omega_{ns}\omega_{ns}''}\right] \quad (5.16)$$

In this expression the principal term is the last term in the square brackets, which determines the dispersion of the surface oscillations. Unlike the case of a cylindrical Fermi surface, the dispersion of the surface waves at small q is quadratic and has an anomalous character, i.e., the signs of the group and phase velocities are opposite. This dispersion has a real meaning only in the case when it is larger than the collision damping ν. In other words, it is necessary to satisfy the inequality $q^2/2m_{\parallel}^2\omega''_{ns} \gg \nu$.

It is interesting to note that in this case the surface-wave spectrum

$$\omega(q)=\omega_{ns}-q^2/2m_{\parallel}^2\omega_{ns}'' \quad (5.17)$$

does not contain collective characteristics of the metal and is determined only by the properties of the resonant electrons on the Fermi surface. This suggests that the considered oscillations are natural surface electron waves of the type of surface zero sound in the Fermi gas of the conduction electrons of the metal. One can expect these surface electron oscillations to become manifest also in the spectrum of surface acoustic (Rayleigh) waves.

[1]M. S. Khaĭkin, Zh. Eksp. Teor. Fiz. 39, 212 (1960) [Sov. Phys.-JETP 12, 152 (1961)].
[2]T. W. Nee and R. E. Prange, Phys. Lett. 25A, 582 (1967).
[3]É. A. Kaner, N. M. Makarov, and I. M. Fuks, Zh. Eksp. Teor. Fiz. 55, 931 (1968) [Sov. Phys.-JETP 28, 483 (1969)].
[4]M. S. Khaĭkin, ZhETF Pis. Red. 4, 164 (1966) [JETP Lett. 4, 113 (1966)]. Zh. Eksp. Teor. Fiz. 55, 1696 (1968) [Sov. Phys.-JETP 28, 892 (1969)].
[5]E. Fawcett and W. M. Walsh, Phys. Rev. Lett. 8, 476 (1962).
[6]R. Herrmann, Phys. Stat. Sol. 21, 703 (1967).
[7]J. F. Koch and C. C. Kuo, Phys. Rev. 143, 470 (1966).
[8]J. F. Koch, Technical Report N 898, Univ. of Maryland, 1968.
[9]J. F. Koch and J. D. Jensen, Technical Report N922, Univ. of Maryland, 1969.
[10]V. F. Gantmakher, L. A. Fal'kovskiĭ, and V. S. Tsoĭ, ZhETF Pis. Red. 9, 246 (1969) [JETP Lett. 9, 144 (1969)].
[11]M. Ya. Azbel' and É. A. Kaner, Zh. Eksp. Teor. Fiz. 32, 896 (1957) [Sov. Phys.-JETP 5, 730 (1957)].
[12]É. A. Kaner and V. G. Skobov, Adv. in Phys. 16, N69, 1968.
[13]É. A. Kaner and V. G. Skobov, Fiz. Tverd. Tela 6, 1104 (1964) [Sov. Phys.-Solid State 6, 851 (1964)].
[14]W. M. Walsh and P. M. Platzman, Phys. Rev. Lett. 15, 784 (1965).
[15]V. P. Silin, Zh. Eksp. Teor. Fiz. 35, 1243 (1958) [Sov. Phys.-JETP 8, 870 (1959)].
[16]S. Schultz and G. Dunifer, Phys. Rev. Lett. 18, 283 (1967); P. M. Platzman and P. A. Wolff, Phys. Rev. Lett. 18, 280 (1967).
[17]A. L. McWhorter and W. G. May, IBM J. Res. Dev. 8, 285 (1964); O. V. Konstantinov and V. I. Perel', Zh. Eksp. Teor. Fiz. 53, 2034 (1967) [Sov. Phys.-JETP 26, 1151 (1968)].
[18]É. A. Kaner and V. G. Skobov, Fiz. Tekh. Poluprov. 1, 1367 (1967) [Sov. Phys.-Semicond. 1, 1138 (1968)]; Phys. Stat. Sol. 22, 333 (1967).
[19]É. A. Kaner and N. M. Makarov, ZhETF Pis. Red. 10, 253 (1969) [JETP Lett. 10, 160 (1969)].
[20]T. W. Nee, J. F. Koch, and R. E. Prange, Technical Report, N816, Univ. of Maryland, 1968; Phys. Rev. 174, 758 (1968).
[21]É. A. Kaner and N. M. Makarov, Zh. Eksp. Teor. Fiz. 57, 1435 (1969) [Sov. Phys.-JETP 30, 777 (1970)].
[22]É. A. Kaner and V. G. Skobov, Zh. Eksp. Teor. Fiz. 53, 375 (1967) [Sov. Phys.-JETP 26, 251 (1968)].
[23]R. E. Prange and T. W. Nee, Phys. Rev. 168, 779 (1968).

Translated by J. G. Adashko
243