POLARIZATION OF THE RADIATION FROM A GAS LASER

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An expression for the depolarization coefficient of the radiation from a gas laser is obtained as a function of the radiation energy, cavity anisotropy and type of operating transition. When the circular polarization regime is classically stable, the depolarization coefficient may be of the order of unity; in this case, however, the coherence time is exponentially large. If the steady state with linear polarization is classically stable, the degree of polarization varies between 1 and $1/\langle n\rangle$ ($\langle n\rangle$ is the mean number of photons), depending on the anisotropy of the cavity.

1. In classical theory of quantum generators it is usually assumed that the degree of depolarization of the radiation is small[1-3]. Such an approximation is justified only in the case of sufficiently strong anisotropy of the resonator, when the polarized part of the radiation is large compared with the unpolarized one, the presence of which is due to the amplitude and phase fluctuations with the electromagnetic field.

It is of interest to solve the problem of polarization of radiation of a gas laser in the case when the ratio of the polarized part of the radiation to the natural one can be arbitrary. By way of a model we consider a system of N atoms that interact resonantly with an arbitrarily polarized electromagnetic field of frequency ω. The Hamiltonian of the interaction is ($\hbar = 1$)

$$H_{int}=\sum_{i=1}^{N}\sum_{q=\pm}\left(\frac{2\pi\omega}{V}\right)^{1/2}\hat{d}_i^q(a_q+a_q^+)\sin kr_i. \quad (1)$$

Here $\hat{d}_i^q$ are the circular component of the dipole-moment operator of the i-th atom, a_q^+ and a_q are the photon creation and annihilation operators, $r_i = r_i^0 + v_i t$ is the coordinate of the i-th atom, v_i is its velocity, V is the volume of its system, and $k = \omega/c$.

Since we shall consider henceforth the near-threshold generation regime (low radiation energies), it suffices to obtain the equation for the density matrix of the photons with accuracy to fourth order in the interaction energy. Without dwelling on the calculations (they are performed in analogy with[4-6]), we present the equation for the photon distribution function in the coherent-state representation (the P-representation of Sudarshan and Glauber[7,8]), $a_\pm|z_\pm\rangle = z_\pm|z_\pm\rangle$, where $z_\pm$ are the complex amplitudes of the circular components of the field:

$$\frac{2\tilde{Q}}{\omega}\frac{\partial P}{\partial t}+\sum_q(\nabla_q J_q+\text{c.c.})=0,\quad \nabla_q=\frac{1}{2}\left(\frac{\partial}{\partial x_q}-i\frac{\partial}{\partial y_q}\right), \quad (2)$$

$$J_q=P\left[\xi z_q-\alpha z_{-q}-2\beta\left(1+\frac{i\delta}{2+\delta^2}\right)(|z_q|^2+A|z_{-q}|^2)\right]-\mathring{\nabla}_q P. \quad (3)$$

The following notation is used: ξ—relative excess of the pump over threshold ($\xi \ll 1$), β—saturation parameter, δ—dimensionless detuning. These quantities are given by

$$\xi=\frac{4\pi^{3/2}N\tilde{Q}}{Vks}\sum_{l,\lambda}|d_{l,\lambda}^+|^2-1,$$

$$\beta=\frac{2\pi\omega}{V\gamma^2}\frac{\sum_{l,\lambda}|d_{l,\lambda}^+|^4}{\sum_{l,\lambda}|d_{l,\lambda}|^2}\frac{2+\delta^2}{1+\delta^2},\quad \delta=\frac{\omega-\omega_0}{\gamma}. \quad (4)$$

The indices l and λ number the Zeeman sublevels of the upper and lower states with total angular momentum j_1 and j_0, respectively; s is the average thermal velocity of the atoms, $\tilde{Q}$ is the average Q of the resonator[4], $1/\gamma$ is the characteristic lifetime of the excited atom[5], ω_0 is the transition frequency, $\alpha = (Q_y - Q_x)/(Q_y + Q_x)$ is the relative difference of the Q's of the resonator, due to the anisotropy of the mirrors (for concreteness we assume $a > 0$), and A is the coefficient of nonlinear coupling of the circular components of the field[1], equal to

$$A=\left(\sum_{l,\lambda}|d_{l,\lambda}^+|^4\right)^{-1}\sum_{l,\lambda}|d_{l,\lambda}^+|^2(|d_{l+2,\lambda}^-|^2+|d_{l,\lambda+2}^-|^2). \quad (5)$$

In the derivation of (2) and (3) we employed the approximations customarily used in the theory of gas lasers, namely $\omega/2\tilde{Q}\gamma$, $\gamma|\delta|/ks$ and $\gamma/ks \ll 1$, and assume for simplicity that all the atoms are excited in the absence of the field.

The properties of the partly polarized radiation are characterized by a polarization tensor $\rho_{\alpha,\beta}$ ($\alpha, \beta = x, y$)[9], whose components, expressed in terms of $z_\pm$, are

$$\rho_{xx,yy}=\frac{1}{2}\pm\frac{\langle\operatorname{Re} z_+z_-^*\rangle}{\langle|z_+|^2+|z_-|^2\rangle},\quad \rho_{xy}=-\frac{i}{2}\frac{\langle|z_-|^2-|z_+|^2\rangle}{\langle|z_+|^2+|z_-|^2\rangle}+\frac{\langle\operatorname{Im} z_+z_-^*\rangle}{\langle|z_+|^2+|z_-|^2\rangle}.$$

The averaging here is taken in the sense of integration with a stationary distribution function P_s (the limit of P as $t \to \infty$). As follows from (6), in solving Eqs. (2) and (3) it is convenient to change over to a new coordinate system:

$$z_\pm=\sqrt{\frac{n\mp z}{2}}\exp\left[\frac{i}{2}(\psi\pm\varphi)\right], \quad (7)$$

where $n = \sqrt{r^2 + z^2}$ determines the total number of photons, z is the energy difference ($z = |z_-|^2 - |z_+|^2$), r is double the product of the moduli of the amplitudes, while the angles φ and ψ represent the difference and the sum of the phases of the circular components of the field. Equation (2) assumes in terms of these variables the form

$$\frac{Q}{\omega n}\frac{\partial P}{\partial t}+\frac{1}{r}\frac{\partial}{\partial r}r\left[P\left(\frac{\xi r}{n}-\alpha\cos\varphi-\beta(1+A)r\right)-\frac{\partial P}{\partial r}\right]+\frac{\partial}{\partial z}\left[P\left(\frac{\xi}{n}-2\beta\right)z-\frac{\partial P}{\partial z}\right]$$
$$+\frac{1}{r^2}\frac{\partial}{\partial\varphi}\left[Pr\left(\alpha\sin\varphi+\frac{1-A}{2+\delta^2}\delta\beta\frac{rz}{n}\right)-\left(\frac{\partial P}{\partial\varphi}+\frac{z}{n}\frac{\partial P}{\partial\psi}\right)\right]$$
$$+\frac{1}{r^2}\frac{\partial}{\partial\psi}\left[Pr\left(\frac{\alpha z}{n}\sin\varphi-\frac{1+A}{2+\delta^2}\delta\beta r\right)-\left(\frac{\partial P}{\partial\psi}+\frac{z}{n}\frac{\partial P}{\partial\varphi}\right)\right]=0. \quad (8)$$

The greatest interest attaches to an investigation of the statistical properties of the radiation in the generation regime $\xi-|\alpha|\gg\sqrt{\beta}$, when the system is not in thermodynamic equilibrium.

2. Let us consider the stationary solution of Eq. (8). The distribution function P_S is in this case independent of the angle ψ, since stationarity means that $\langle z_+\rangle=\langle z_-\rangle=0$. In order not to clutter up the analysis with superfluous details, let us investigate in greater detail the case of resonance $\delta=0$, in which Eq. (8) can be solved exactly:

$$P_s=P_0\exp\left(\xi n-\alpha r\cos\varphi-\frac{1+A}{2}\beta r^2-\beta z^2\right). \quad (9)$$

The change occurring in the solution when the detuning is not equal to zero will be discussed separately where it is of importance.

The problem of the polarization and statistical characteristics of the radiation reduces, in principle, to a determination of the normalization constant P_0, which depends upon the parameters α, ξ, and A, since all the mean values, for example of the type (6), can be obtained by differentiating $\ln P_0$ with respect to these parameters.

Since the argument of the exponential in (9) is large, $\langle n\rangle\xi\gg1$, it is possible to use the saddle-point method in the calculation of P_0. The saddle points z_0r_0 and φ_0, which determine the position of the absolute maximum of P_S, are connected with each other by the classical stationary equations:

$$z_0\left(\frac{\xi}{n_0}-2\beta\right)=0,\quad \varphi_0=\pi\quad \frac{\xi r_0}{n_0}+\alpha-(1+A)\beta r_0=0. \quad (10)$$

The solutions of these equations greatly depend on the coefficient of the nonlinear connection of polarizations A. There are two distinct characteristic regions, depending on whether this coefficient is larger than or smaller than unity. The value of $A>1+2\alpha/\xi$ corresponds to radiation polarized circularly, and $A<1+2\alpha/\xi$ corresponds in classical theory to the position of indifferent equilibrium.

The phase difference φ_0 in (10) is assumed to be equal to π, for at this value the distribution function P_S has a maximum ($\alpha>0$). When $\alpha=0$ the phase difference is not completely defined and the radiation will be completely unpolarized, regardless of the value of A. It is possible to deduce from (9) the value of the anisotropy α needed in order that the phase difference φ be well defined (that the relative fluctuations of φ be small); to this end it is necessary to satisfy the condition $\alpha\langle r\rangle=2\alpha\sqrt{n_+n_-}\gg1$. In the opposite case, $\alpha\langle r\rangle\lesssim1$, the definition of the stationary phase difference becomes meaningless, and therefore the equations for the saddle points z_0 and r_0 at $\alpha\langle r\rangle\lesssim1$ must be sought by using a distribution function averaged beforehand with respect to φ. We note that Eqs. (10) are obtained from the distribution function (9) averaged over φ, using the asymptotic expansion of the modified Bessel function $I_0(\alpha r)$ with $\alpha r\gg1$. It will be shown below that the physical meaning of the condition $\langle r\rangle\alpha\gg1$ is as follows: the frequency difference of the circular components of the field, due to the anisotropy of the resonator, must be larger than the emission line width.

3. Let us consider first the case $2\alpha/\xi(A-1)<1$, which is apparently realized for transitions with a total angular momentum change $1\rightleftarrows0$ in the presence of depolarizing collisions[10,11].

Assuming that $\langle r\rangle\alpha\gg1$, and putting $2\alpha/\xi(A-1)=\Delta$, we obtain from (10) the three stationary points:

$$z_0^{\pm}=\mp\langle n\rangle\sqrt{1-\Delta^2},\quad r_0=\langle n\rangle\Delta,\quad \langle n\rangle=\xi/2\beta, \quad (11)$$
$$z_0=0,\quad r_0=\langle n\rangle\left[1-\frac{A-1}{A+1}(1-\Delta)\right]. \quad (12)$$

The first two solutions, which exist when $\Delta<1$, determine the position of two maxima of the function P_S that are symmetrical with respect to the r axis. Located between them is the minimax point, the coordinates of which are given by the solution (12). As $\Delta\to1$, all three solutions merge into one and the stationary point becomes degenerate. The solution (11) is valid if

$$|1-\Delta|\sqrt{\alpha\langle n\rangle}\gg1. \quad (13)$$

Noting that the polarization tensor (6) is diagonal by virtue of the parity of P_S with respect to z and φ, and taking (13) into account, we find an expression for the degree of depolarization of the radiation:

$$d=(1-\Delta)/(1+\Delta). \quad (14)$$

This result has a simple physical meaning. As is well known from classical theory, when $\Delta<1$ the steady-state radiation is elliptically polarized, since both directions of rotation are equally probable. The equal probability follows also from the fact that P_S is even in z. Naturally, averaging over the directions takes place after a sufficiently long time, as a result of which the radiation will be a superposition of two incoherent oscillations polarized in mutually orthogonal directions. The depolarization coefficient will then equal the ratio of their intensities $\langle n_x\rangle/\langle n_y\rangle$[9]. The relative dispersion of each of the linearly polarized components of the field is small:

$$\frac{\langle(\Delta n_{x,y})^2\rangle}{\langle n_{x,y}\rangle^2}=\frac{1}{\xi\langle n\rangle}\frac{1+A}{(-1+A)}\frac{1}{(1\mp\Delta)^2},\quad \langle n_{x,y}\rangle=\frac{\langle n\rangle}{2}(1\mp\Delta). \quad (15)$$

The incoherence of these waves is manifest in the fact that the fluctuations of the circularly polarized oscillations, produced upon their superposition, are large:

$$\langle(\Delta n_\pm)^2\rangle=\frac{\langle n\rangle^2}{4}(1-\Delta^2),\quad \langle n_\pm\rangle=\frac{\langle n\rangle}{2}. \quad (16)$$

The coherence time τ_0 is obviously connected with transitions between states (11) resulting from the fluctuations. These classical stationary stable solutions are metastable from the statistical point of view. To determine τ_0 it is necessary to find the probability of repetition of the state (12). Calculations similar to those performed in[6,12] yield

$$\tau_0=\frac{4Q}{\omega}\sqrt{2\pi}\sqrt{\frac{\Delta}{\langle n\rangle\xi}}\sqrt{\frac{[2+(A-1)\Delta]}{(1-\Delta^2)(A-1+2\Delta^2)}}\exp\left[\frac{\xi\langle n\rangle}{2}\frac{A-1}{A+1}(1-\Delta)^2\right]. \quad (17)$$

Since τ_0 is exponentially large, it is possible to average in the expression for the polarization tensor (6) over times that are small compared with τ_0 but large compared with the time of the phase collapse τ. The latter determines the width of the emission line $\Delta\nu = 1/\tau$, for which, following[5,6], and starting from the temporal equation (8), we can obtain

$$\Delta\nu=\frac{\omega}{4Q}\frac{1}{\Delta^2\langle n\rangle}. \quad (18)$$

If the radiation produced at the initial instant of time is polarized elliptically with a specified direction of rotation, then it remains stable when $\tau_0 \gg t \gg \tau$. This means that at a chosen sign of $z_0^\pm$ in (11), for example z_0^-, the light will have left-hand elliptic polarization with a semiaxis ratio $1/\sqrt{1+\Delta}$ and with a direction along the y axis. The angle between the major axis of the ellipse and the x axis depends on the magnitude of the detuning δ and equals $\pi/2$ at $\delta = 0$. We present expressions for the semiaxis ratio ϵ and the angle of inclination θ at $\delta \neq 0$:

$$\varepsilon=\frac{\langle n\rangle-r_0}{|z_0|},\quad \operatorname{tg}2\theta=\mu\frac{z_0}{\langle n\rangle},\quad \mu=\frac{\delta}{2+\delta^2}, \quad (19)$$

$$\langle n\rangle=\sqrt{r_0^2+r_0^2},\quad r_0=\sqrt{2}\langle n\rangle\Delta[(1+\mu^2)+\sqrt{(1+\mu^2)^2-4\Delta^2\mu^2}]^{-1/2}, \quad (20)$$

where the average number of photons $\langle n \rangle$ is given as before by formula (11) and depends on δ via the saturation parameter β [Eq. (4)]. The quantity μ^2 as a function of δ varies in the range $0 \le \mu^2 \le 1/8$, and as a result all the characteristics of the radiation depend little on the detuning. Thus, for example, when μ changes from zero to the maximum value we have $2\sqrt{2/3} < r_0/\langle n \rangle\Delta \le 1$.

We emphasize once more that all the calculations presented above are valid when the condition (13) is satisfied. In the opposite limiting case, i.e., when $|1-\Delta|\sqrt{\alpha\langle n\rangle} \lesssim 1$, we fall in the region of classically indifferent equilibrium position, where both circular and linear polarizations can exist. In this region, the presence of an unpolarized part of the radiation is due to fluctuations of the energy difference between the circular components of the field. Taking into account the fluctuations, the depolarization coefficient does not vanish, as would follow from (14), but remains finite

$$d=\frac{1}{2}\frac{\Gamma(3/4)}{\Gamma(1/4)}\sqrt{\frac{1+A}{\alpha\langle n\rangle}} \quad (21)$$

($\Gamma(x)$ is the Euler Gamma function). The radiation can be regarded, as before, as a superposition of incoherent linearly-polarized waves with different intensities:

$$\langle n_x\rangle=\sqrt{\frac{\langle n\rangle}{\alpha}}\frac{\sqrt{1+A}}{2}\frac{\Gamma(3/4)}{\Gamma(1/4)},\quad \langle n_y\rangle=\langle n\rangle-\langle n_x\rangle. \quad (22)$$

However, the coherence time is determined now by the emission line width (18), since τ_0 at $\Delta = 1$ is the relaxation time of the stationary distribution function.

4. In this section we investigate the case $A < 1 + 2\alpha/\xi$, which is realized, for example, in a helium-neon laser. We assume also that condition (13) is satisfied. In this case the distribution function (9) has one maximum, whose coordinates are given by the solution (12) describing the stationary generation of radiation that is linearly polarized along the axis of maximum Q (the y axis).

Let us consider some limiting cases. If the anisotropy of the resonator is sufficiently large $\sqrt{\langle(\Delta n)^2\rangle}\alpha \gg 1$, then the depolarization of the light will be caused mainly by the x-component of the radiation:

$$d=\frac{1}{4\alpha\langle n\rangle}\left[1+\frac{\alpha(1+A)}{2\alpha+(1-A)\xi}\right],\quad \langle n\rangle=\frac{\xi+\alpha}{\beta(1+A)}, \quad (23)$$
$$\langle n_x\rangle=\langle n\rangle d,\quad \langle n_y\rangle=\langle n\rangle(1-d),$$

whose fluctuations are large:

$$\frac{\langle(\Delta n_x)^2\rangle}{\langle n_x\rangle^2}=2\frac{[2\alpha+\xi(1-A)]^2+\alpha^2(1+A)^2}{[2\alpha+\xi(1-A)+\alpha(1+A)]^2}. \quad (24)$$

As seen for the foregoing results, the relative variance of n_z is minimal and is equal to unity when $A = 1$ (transition $j_1 = 1$, $j_0 = 0$). In all the remaining cases it is larger than unity. The variance of the total number of photons consists of the variances of the linearly polarized components

$$\langle(\Delta n)^2\rangle=\frac{1}{2}[\langle(\Delta n_x)^2\rangle+\langle(\Delta n_y)^2\rangle]=\frac{\langle n\rangle}{\xi+\alpha}. \quad (25)$$

Since the polarization direction is sufficiently well determined, the circular components of the field are likewise, naturally, well determined:

$$\langle(\Delta n_\pm)^2\rangle=\frac{\langle(\Delta n)^2\rangle}{4}\frac{2\xi+\alpha(3+A)}{2\alpha+(1-A)\xi},\quad \langle n_\pm\rangle=\frac{\langle n\rangle}{2}. \quad (26)$$

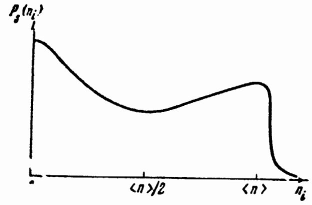
The next limiting case considered is $A < 1$. If the binding energy of the circular components of the field is large ($\xi\langle n\rangle(1-A) \gg 1$), and the anisotropy of the resonator is sufficiently small ($\sqrt{\langle(\Delta n)^2\rangle}\alpha \ll 1$), then the depolarization coefficient is defined as follows:

$$d=\frac{I_0(\alpha\langle n\rangle)-I_1(\alpha\langle n\rangle)}{I_0(\alpha\langle n\rangle)+I_1(\alpha\langle n\rangle)},\quad \langle n\rangle=\frac{\xi}{\beta(1+A)}. \quad (27)$$

We note that when $\langle n \rangle\alpha \gg 1$ the obtained expression coincides with (23).

As expected, d increases monotonically with increasing α and tends to unity in the limit of small α ($\langle n \rangle\alpha \ll 1$). The radiation is in this case almost completely unpolarized. The classical theory in the case of $a = 0$ and $A < 1$ shows that linearly polarized radiation is stable; it is concluded at the same time that the direction of polarization depends only on the initial conditions. In other words, when radiation polarized in some direction is produced at the initial instant, it will conserve its direction in all the subsequent instants of time. From the statistical point of view, this means that the distribution function $P_s(n_i)$, where i = x or y, has two extremal points $n_i = 0$ and $n_i = \langle n \rangle$, which determine either the absence of the generation of radiation with linear polarization. Indeed, changing over in (9) from circular components to linear components and integrating the resultant expression with respect to one of the n_i at $\alpha = 0$, we find that P_s has two maxima at $n_j = 0$ and $n_j = \langle n \rangle$ (see the figure).

Further, it is easy to find the distribution function at the minimum $n_j = \langle n \rangle/2$, which corresponds to a classically unstable state. It turns out that the probability of repeating the state $n_j = \langle n \rangle/2$ is low, but not exponentially, as in the case $A > 1$ [Eq. (17)] (P_s(min)



$\sim 1/\langle n \rangle$). Consequently, the time of transition between the states $0 \longleftrightarrow n$ coincides with the relaxation time of P_S, i.e., with the lifetime of the photon and the resonator. For this reason, the depolarization of an initially polarized radiation will occur after a time τ determined by the emission line width:

$$\Delta\nu = \frac{1}{\tau} = \frac{\omega}{4\tilde{Q}} \frac{1}{\langle n \rangle}.$$

The fluctuations of the linearly polarized components

$$\langle n_{x,y} \rangle = \frac{\langle n \rangle}{2} \left(1 \mp \frac{I_1(\alpha\langle n \rangle)}{I_0(\alpha\langle n \rangle)}\right) \quad (28)$$

will be large because of the large variance of the distribution function shown in the figure

$$\langle(\Delta n_{x,y})^2\rangle = \frac{\langle n \rangle^2}{4} \frac{d}{d\eta}\left[\frac{I_1(\eta)}{I_0(\eta)}\right], \quad \eta = \alpha\langle n \rangle. \quad (29)$$

In this expression we have neglected the difference between the variances of n_x and n_y by an amount equal to the variance of the total energy

$$\langle(\Delta n)^2\rangle = 4[\langle(\Delta n_y)^2\rangle - \langle(\Delta n_x)^2\rangle] = \langle n \rangle / \xi,$$

since $\langle(\Delta n_{x,y})^2\rangle \sim 1/\alpha^2$ even at large anisotropy ($\langle n \rangle\alpha \gg 1$).

We note further that the intensities of the circular components of the field are better defined than the intensities of the linear components, since their variance is small compared with $\langle(\Delta n_{x,y})^2\rangle$:

$$\langle(\Delta n_\pm)^2\rangle = \frac{\langle n \rangle}{2\xi} \frac{1}{1-A}, \quad \langle n_\pm \rangle = \frac{\langle n \rangle}{2}. \quad (30)$$

Thus, we can conclude from the foregoing results that in the limiting case $\sqrt{\langle(\Delta n)^2\rangle}\alpha \ll 1$ under consideration the partially polarized radiation is a superposition of two circularly polarized waves, the coherence time of which is determined by the emission line width.

At nonzero detuning, the formulas of this section should be modified, replacing everywhere the parameter α by $\alpha/(1+\mu^2)$.

We now explain the physical meaning of the condition $\langle n \rangle\alpha \gg 1$. As is well known, in a passive resonator the anisotropy causes a splitting of the frequency[3]. The magnitude of this splitting is $\Delta\omega = (\omega/\tilde{Q})\alpha$. Comparing $\Delta\alpha$ with the line width $\Delta\nu$, we see that the condition $\langle n \rangle\alpha \gg 1$ is equivalent to the condition $\Delta\omega \gg \Delta\nu$. In this connection, it is of interest to note that in a helium-neon laser ($A = 11/23$) far from the generation threshold $\xi \gg \alpha$ ($\alpha \sim 10^{-3}$) the depolarization coefficient turns out to be directly proportional to the emission line width:

$$d = \Delta\nu / \Delta\omega. \quad (31)$$

5. Below the generation threshold $\xi < 0$, when saturation can be neglected $(|\xi| - |\alpha|) \gg \sqrt{\beta}$ the radiation comprises partly polarized "quantum" noise made up of a superposition of two waves that are linearly polarized in mutually perpendicular directions:

$$d = \frac{|\xi| - |\alpha|}{|\xi| + |\alpha|}, \quad \langle n_{x,y} \rangle = \frac{1}{|\xi| \pm \alpha}. \quad (32)$$

The relative variance of n_x and n_y is equal to unity in this case.

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