# THEORY OF A WEAKLY IONIZED PLASMA WITH STRONG INTERACTION

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A nondegenerate plasma whose non-ideality is due to interactions between charged and neutral particles is considered. The electron state density and equation for ionization equilibrium are obtained in the case of strong nonideality. The nonideal plasma is found to acquire semiconductor properties. A larger number of localized electrons appears and the electric conductivity rises sharply. Concrete estimates are made for lithium and mercury plasmas.

IN a weakly ionized plasma, the concentration of the charged particles n is low and the nonideal character of the plasma can be due to the interaction between these particles and neutral particles, namely atoms and molecules. Vedenov[1] called attention to the increase of the electric conductivity under such conditions. He considered, however, only the weakly non-ideal case. One can expect a strongly non-ideal plasma to acquire new properties.

Let us consider the strongly nonideal conditions, when the electron interacts immediately with many atoms (for concreteness we speak of atoms), $Nv \gg 1$ (N—concentration of the atoms, v—sphere of action of the electron-atom forces). Thus, with the cross section for the scattering of an electron by a lithium atom amounting to $\sim 300\ \pi a_0^2$, some 20–30 lithium atoms can be contained in the sphere of action v of the forces simultaneously ($T \sim 4000^\circ$K). At the same time, the interatomic correlations and interaction can still be neglected. It turns out that under such conditions the plasma acquires properties characteristic of certain semiconductors. Localized electrons belonging to an assembly of randomly placed neutral particles appear in the plasma.

1. The equation for the Green's function of the electron is

$$(\omega + i\varepsilon + \hbar^2/2m\nabla_x^2 - U(x;\ldots,\mathbf{r}_i,\ldots)]G(x,x',\omega;\ldots,\mathbf{r}_i,\ldots) = \delta(\mathbf{x}-\mathbf{x}'), \tag{1}$$

$$U(x;\ldots,\mathbf{r}_i,\ldots) = \sum_i V(\mathbf{x}-\mathbf{r}_i), \tag{2}$$

Where V is the potential of the interaction between the electron and the atom, and $\mathbf{r}_i$ is the coordinate of the i-th atom. The form of the potential V is not specified. Nonetheless, the description of the interaction with the aid of (2) is a definite approximation, since not all the interaction forces can be regarded as additive.

When $Nv \gg 1$, the potential U is a slowly varying function. This can be used to solve (1), and then average G in the usual manner over the random distribution $\mathbf{r}_i$ or over the normal distribution U. Such a problem—that of the electron in the field of disordered scatterers—was solved by Bonch-Bruevich[2] in the analysis of phenomena occurring in strongly doped semiconductors. He obtained

$$G_\mathbf{p}(\omega) = i\int_0^\infty ds \exp[-\varepsilon s + is(\omega - p^2/2m - NV_0) + \alpha(s)], \tag{3}$$

$$\alpha(s) = N\int d\mathbf{r}\,\{\exp[-isV(\mathbf{r})] - 1 + isV(\mathbf{r})\}, \quad V_0 = \int V(\mathbf{r})\,dr,$$

where **p** is the electron momentum. In the derivation of (3), the second derivatives of the potential U were neglected this being justified when $nV \gg 1$.

2. We can show now that the density of states of the electrons

$$\rho(\omega) = \frac{2}{\pi}\int\frac{d\mathbf{p}}{(2\pi\hbar)^3}\,\mathrm{Im}\,G_\mathbf{p}(\omega)$$

assumes the form shown in Fig. 1. At large negative $\omega$, when $\omega^2 \gg \nu$, where $\nu \equiv (1/2)N\int V^2(\mathbf{r})d\mathbf{r}$, we have

$$\rho(\omega) = 2\pi\frac{2(2m)^{3/2}}{(2\pi\hbar)^3}\frac{4\nu}{\sqrt{2}\,\omega^2}\sqrt{|\omega|}\exp\left(-\frac{\omega^2}{4\nu}\right). \tag{4}$$

At small $\omega$, namely $\omega^2 \ll \nu$,

$$\rho(\omega) = 2\pi\frac{2(2m)^{3/2}}{(2\pi\hbar)^3}\Gamma\left(\frac{3}{4}\right)\frac{1}{\sqrt{2\pi}}\nu^{1/4}\exp\left(-\frac{\omega^2}{4\nu}\right)\left[1 + \frac{\omega}{4\sqrt{\nu}}\frac{\Gamma(1/4)}{\Gamma(3/4)}\right]. \tag{5}$$

When $\omega^2 \gg \nu$ and $\omega > 0$, the quantity $\rho(\omega)$ is close to the density of states of an ideal gas. We need only remember the overall shift of the point from which the energy is reckoned, equal to $-NV_0$ and not indicated in Fig. 1.

The obtained form of $\rho(\omega)$ is characteristic of strongly nonideal disordered systems of the type considered in the book of Mott[3]. In a strongly nonideal plasma at $\omega < 0$ there appear localized electrons belonging to the assembly of randomly disposed neutral particles.

3. Integrating the asymptotic forms of $\rho(\omega)$ in the regions of their validity, we obtain the chemical potential $\mu$ of the electrons

$$n \approx 2\left(\frac{m}{2\pi\hbar^2\beta}\right)^{3/2} e^{\beta\mu - NV_0\beta}\left[\frac{2}{\sqrt{\pi}}\int_{(\beta\sqrt{\nu})^{1/2}}^\infty e^{-x^2}dx + e^{\beta^2\nu}\right]. \tag{6}$$

Formula (6) was obtained under strongly nonideal conditions, $\beta\sqrt{\nu} \gg 1$, $\beta = 1/kT$. It has, however, interpola-

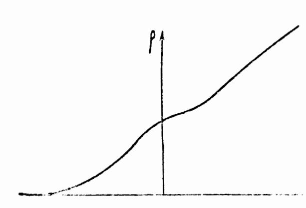


FIG. 1. Density of states of a strongly nonideal plasma.

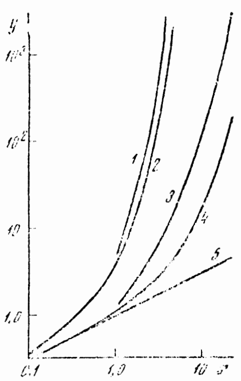

FIG. 2

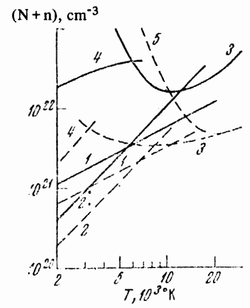


FIG. 3

FIG. 2. Isotherms of ionization equilibrium. The coordinates are $y = n(\beta^2\nu/KN)^{1/2}$ and $x = \beta^2\nu$. Curves 1 and 2 were constructed in accordance with formulas (7) and (8) with $-\beta V_0/\nu = 3$; curves 3 and 4 correspond to $-\beta V_0/\nu = 0.3$; curve 5 corresponds to the Saha equation.

FIG. 3. Characteristic regions of the parameters (density of heavy particles (N + n) and the temperature) for a mercury plasma (solid curve) and cesium plasma (dashed). Curves 1 and 2 correspond to equality in (10); 3 and 4 are the density limits due respectively to the Coulomb interactions and to the atom-atom interactions; curve 5 corresponds to the equality $e^2\beta n^{1/3} = 1$ in an ideal lithium plasma.

tion properties that permit a transition to the limit of an ideal gas.

Neglecting the influence of the atoms and of the ions on the state of the gas, we obtain in the limit when $\beta\sqrt{\nu} \gg 1$ the equations of ionization equilibrium for the localized and free electrons ($n_1$ and $n_2$)

$$n_1^2 = NK(\beta)\exp(-\beta N V_0 + \beta^2\nu), \tag{7}$$

$$n_2^2 = NK(\beta)\frac{\sqrt{2}}{\pi}\Gamma\left(\frac{3}{4}\right)(\beta\sqrt{\nu})^{1/2} \times \exp(-\beta N V_0), \quad K(\beta) = (z_e z_i / z_a)\exp(-\beta E_1), \tag{8}$$

$K(\beta)$ is the constant of the ionization equilibrium in an ideal plasma; $z_e$, $z_i$, and $z_a$ are the partition functions of the electron, ion, and atom, and $E_1$ is the ionization energy of the atom.

Figure 2 illustrates the dependence of $n_1$ and $n_2$ on the atom density N at different values of $K(\beta)$ and interaction parameters. Upon compression, the number of electrons of both types increases sharply. In this case the localized electrons prevail. In the case of weak interaction ($N\nu \ll 1$), the number of localized electrons is very small, and in place of (7) we have

$$n_1^2 = NK(\beta)\exp(-NV_0\beta). \tag{9}$$

Extrapolation of (9) under conditions when $N\nu \gg 1$ yields too low a value for the electron density.

4. Let us note certain consequences of the results. When a plasma is compressed its electric conductivity increases sharply, this being connected not only with the increase of the number of carriers but also with the change in the character of their mobility. The "conduction" electrons have at $N\nu \gg 1$ a larger mobility than given by the ideal-plasma formulas. The mobility of localized electrons has a "jumplike" character.

The negative ions, the number of which increases, lose their ideal-plasma properties under strong compressions. The electrons connected with the assembly of atoms can be regarded as a result of their radical modification. This entails changes in the corresponding optical properties of the plasma.

Worthy of analysis is the equation of state. It is not excluded that the interaction of the charges with the neutral particles can be manifest in a plasma phase transition. The possibility of the phase transition due to Coulomb interactions has been discussed in the literature (see, for example,[4,5]).

Effects brought about by electron-atom interactions should become manifest in the plasma of alkali-metal vapor, mercury, or cadmium, whose atoms have large electron-scattering cross sections. Thus, the increase of the conductivity of mercury[6] is due possibly to a considerable degree to this interaction (this is indicated in[1]), but goes outside the framework of the weak non-ideal plasma approximation.

4. Let us discuss the validity of certain assumptions, using lithium and mercury plasma as examples (Fig. 3).

The electron-atom interaction becomes strong when

$$\beta N V_0 \gtrsim 1, \quad \beta\sqrt{\nu} \gtrsim 1. \tag{10}$$

The coefficients $V_0$ and $\sqrt{\nu}$ can be estimated from the results of the scattering of electrons by atoms. In the simplest approximation $V_0 \approx -\sqrt{\pi\sigma}\,h^2/m$ and $\nu \approx 3\sqrt{\pi}NV_0^2/8\sigma\sqrt{\sigma}$. Here $\sigma$ is the cross section for the scattering of an electron by an atom ($10^{-14}$ and $3 \times 10^{-14}$ cm$^2$ for mercury and lithium, respectively). Relations (10) define lower bounds for N. However, if expressions valid under conditions of strong and weak non-ideal behavior are available, it is easy to write down interpolation formulas covering the intermediate range.

The upper bound of N in mercury is due principally to the atom-atom interaction. These values of N can be estimated with the aid of the experimental data (see, for example,[6]) and the usual Van der Waals equation. In lithium, the first to be violated with increasing N is the condition $e^2\beta n^{1/3} \ll 1$. An increase of the concentration n shifts the equation $e^2\beta n^{1/3} \approx 1$, in accordance with (7) and (8), into the region of lower densities (dashed curves 3 and 4 in Fig. 3), and the plasma becomes strongly nonideal also because of the interaction of the charged particles. An analysis of such a plasma is a complicated matter.

It follows from Fig. 3 that the proposed analysis has a larger range of applicability for mercury. The region of applicability for lithium is small. However, it is characterized by sharp changes in the properties of the plasma and is primarily accessible to experiment.

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