SOVIET PHYSICS JETP VOLUME 31, NUMBER 6 DECEMBER, 1970

SUBSONIC PROPAGATION OF A LIGHT SPARK AND THRESHOLD CONDITIONS FOR THE MAINTENANCE OF PLASMA BY RADIATION

Yu. P. RAĬZER

Institute for Problems in Mechanics, USSR Academy of Sciences

Submitted December 26, 1969

Zh. Eksp. Teor. Fiz. 58, 2127–2138 (June, 1970)

Subsonic conditions for the propagation of a laser spark, similar to those of a slow combustion, are considered. The steady state problem is formulated and approximately solved. Plasma temperature, wave velocity, and threshold power requirements of a nondecaying wave are determined. The results are in good agreement with published experimental data obtained with a neodymium glass laser. The feasibility of an optical plasmotron based on a CO_2 cw laser is discussed. The maintenance of plasma in ambient air requires ~ 2 kW of well focused light. The temperature of the resulting plasma is 18,000°K.

1. INTRODUCTION. COMBUSTION ANALOGY OF DISCHARGE PROPAGATION

THERE is a profound analogy between the processes of discharge (spark) propagation due to the liberation of electromagnetic energy from plasma and combustion. The chemical reactions that are practically absent in combustible mixtures at ordinary temperatures sharply accelerate on heating. If a mixture does not burn instantaneously throughout its volume, the energy transfer from heated layers to the neighboring cold and uninvolved layers causes heating and ignition of the latter so that combustion propagates through the material. There are two possible propagation mechanisms: supersonic, or detonation, in which the mixture is heated up to the ignition temperature of the shock wave, and subsonic, in which combustion is slow and energy is transferred by thermal conduction.

Similarly cold gas does not conduct electrical current and does not absorb electromagnetic waves in many broad frequency bands. The field energy is emitted in the gas as Joule heat or as a result of radiation absorption only if the gas is ionized. The stronger the ionization the larger the emission of heat, and the former, while initially small, increases with temperature according to the same type of law, $e^{-E/kT}$, as the rate of chemical reaction. Once a discharge occurred at some point, the transfer of energy from plasma to the neighboring cold layers ionizes them and enables the propagation of discharge to regions occupied by the field.

Understanding of discharge phenomena was greatly aided by recourse to the physical models and methods of the well developed theory of combustion and detonation of gases[1,2]. The analogy was used for the explanation of the rapid propagation of a light spark observed in gas breakdown by a giant laser pulse. Ramsden and Savic[3] advanced the idea of the optical detonation wave and using the detonation theory formula computed the velocity of the plasma front that was in agreement with experimental data. A general hydrodynamic theory of supersonic light absorption wave and gas heating was developed in[4] where the experimental wave temperature was computed and other rapid spark propagation mechanisms based on radiation and breakdown were discussed. Later it was shown[5] that optical detonation, just as ordinary detonation, has its limits determined by losses. The minimum light intensity capable of sustaining a nondecaying regime turned out to be much lower than that required for gas breakdown. This means that with an external plasma source one can start a discharge in a light channel with a sub-breakdown light intensity.

The concepts of combustion theory were also used to help understand and explain the laws of a high-frequency discharge in a gas flow[6,7], the basis of the electrodeless plasmotron. In this device the cold gas is blown in a tube through a solenoid containing a stationary high-frequency discharge. A continuous plasma jet at atmospheric pressure issues from the tube. The theory shows that the plasma torch is in many ways similar to the ordinary torch. The velocity of the normal discharge propagation along the cold gas, just as the velocity of the chemical flame, can be completely determined and depends on the flux of electromagnetic energy from the inductor and on thermal conductivity. Its equation is similar to the Zel'dovich formula for normal flame velocity[1]. Recently Frank-Kamenetskiĭ published some ideas on the stability of high-frequency discharge that were close in spirit to the principle of combustion theory[8].

Superhigh-frequency plasmotrons have been designed and are in operation[9,10]. Their stationary discharge is also similar to combustion[11].

The combustion analogy was also applied to the theory of new phenomena involving the laser spark. Bunkin and others[12] observed slow propagation of a spark in ambient air that was ignited by special means at light intensities below the breakdown threshold. The beam of a high power free running neodymium laser was weakly focused by a long-focus lens (diameter of focal spot—3 mm; light intensity—of the order of 10 MW/cm^2). The intensity was insufficient to produce optical detonation whose threshold under these conditions is ~ 100 MW/cm^2 according to[5]. However when the pulse energy exceeded 730 J (intensities over 8–15 MW/cm^2) the laser spark did appear and propagate symmetrically in both directions of the light channel at velocities of $\lesssim 50$ m/sec. The authors assumed that the propagation mechanism was the ordinary thermal conduction and that the front propagated in the moving gas as in the case of combustion in a tube with a closed end. Their computation of velocity derived from the Zel'dovich formula for a flame was found to be in agreement with experiment. The "combustion" temperature was determined experimentally using the measurements of laser light absorption by the spark which was strongly transparent.

This paper presents a more detailed discussion of the subsonic propagation of the discharge (spark) at optical frequencies. The problem includes the computation of plasma temperature as well as plasma velocity. In particular it is of interest to find the threshold conditions of such a regime, i.e., to determine the minimum power and intensity of light that can still maintain plasma by radiation and to clarify the conditions that facilitate the "combustion." All this will also permit us the estimate the feasibility of maintaining plasma with cw lasers (using CO_2) and thus to judge the practicality of an "optical plasmotron" (the "light torch"), a device of considerable appeal. A brief note on this subject was published in[13].

2. FORMULATION OF THE PROBLEM AND SIMPLIFICATIONS

Let a parallel axially symmetric light beam of radius R pass through the gas and encounter somewhere a light-absorbing plasma. The intensity of the light is too low to cause optical detonation. The shock wave formed by the initial ignition is now far gone beyond the plasma region and the pressure is equalized. The heat liberated in plasma inside the light channel slowly diffuses by conduction (and by radiation transfer) in all directions including that along the channel. The heated gas slowly expands, all velocities are subsonic, and the hydrodynamic process occurs at an almost constant pressure p that is close to the pressure of unperturbed gas.

We consider the self-maintaining process in which the thermal wave propagating along the optical channel toward the beam does not decay because of continuous emission of energy. From now on we use the term "wave" to designate only the layer of plasma that is adjacent to the leading front of the heating process and that mainly affects the motion of the front and is responsible for the maintenance of the regime. The axial extent (width) of the wave is of the order of R[1)] even if plasma is transparent to the light beam when the absorption length $l_\nu \gg R$ and heat is liberated in a long column. On the other hand if the beam is absorbed in a thin layer $l_\nu \ll R$, the width of the wave can only be smaller than R.

Assuming that the beam intensity varies little in the time it takes for a gas particle to pass through the wave, we consider as usual the stationary regime in a system of coordinates in which the wave is at rest. Cold gas of density ρ_0 enters the wave in the direction of the beam at a velocity u equal in magnitude to the velocity of propagation of the wave through matter.

1)The waves propagate here in both directions along the channel as observed in the experiments [12].

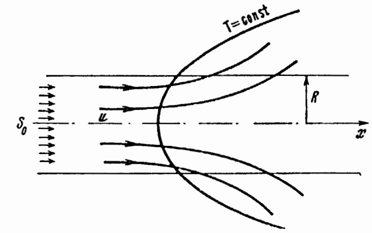


FIG. 1. Diagram of heat leakage and gas expansion. Flow lines and isotherm are shown.

The gas is heated and expands in the direction of its initial motion and to the sides. Once heated, the gas ionizes, absorbs light energy, and the liberated heat spreads to meet the flow of cold gas and to escape radially from the channel.

The picture of the expanding material and heat is shown diagrammatically in Fig. 1. It is described by the energy and continuity equations:

$$\rho v_x c_p \frac{\partial T}{\partial x} + \rho v_r c_p \frac{\partial T}{\partial r} = \frac{\partial}{\partial x}\lambda \frac{\partial T}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r} r\lambda \frac{\partial T}{\partial r} + S\varkappa_\nu - \Phi, \tag{1}$$

$$\frac{\partial}{\partial x}\rho v_x + \frac{1}{r}\frac{\partial}{\partial r} r\rho v_r = 0. \tag{2}$$

Here ρ is density and T is temperature related to each other by the approximate condition of constant pressure (roughly speaking $\rho \sim 1/T$), while v_x and v_r are velocity components, $c_p(T)$ is specific heat capacity, $\lambda(T)$ is the coefficient of thermal conductivity, S is light intensity (flux density), $\kappa_\nu(T) = l_\nu^{-1}$ is light absorption coefficient, and Φ is the divergence of the thermal radiation flux equal to the difference between the light emission by plasma and its absorption by 1 cm^3 per sec. The kinetic energy of slow motion is not accounted for in (1).

The two-dimensionality of the process renders the solution much more complex and we now consider a simplified one-dimensional problem. At an axial distance of the order of a wavelength, i.e., $\lesssim$R, the heat propagates radially beyond the channel to a distance of also $\lesssim$R. Furthermore in this region $v_r \lesssim v_x$. Therefore the orders of magnitude of the quantities involved do not change when we neglect the radial expansion of gas; the light channel is enclosed in a tube, as it were, and we may consider axial velocities independent of radius and the wave front flat. Then $v_r = 0$, and $\rho v_x = \text{const} = \rho_0 u$.

We average (1) over the channel cross section. The radial portion of the thermal flux divergence gives an average volumetric energy loss due to heat leakage through the lateral surface of the channel: $(2/R)(\lambda\,\partial T/\partial r)_{r=R}$. This quantity can be represented in the form

$$-A\Theta/R^2, \quad \Theta = \int_0^T \lambda(T)\,dT$$

is the thermal flux potential corresponding to the mean temperature T of the channel and A is a numerical coefficient determined by the radial profile of the potential. The remaining terms of (1) can be left without changes, approximately considering T, S, and the other quantities as averaged over the cross section.

For example if the profile $\Theta(r)$ is considered the same as in a cylinder with strongly cooled walls without axial gradients and with heat sources falling off along the radius according to the Bessel function $J_0(\beta r/R)$, where $\beta = 2.4$ is the first root, then $\Theta(r) \sim J_0(\beta r/R)$, $\Theta = 0.43\,\Theta(0)$, and $A = \beta^2 = 5.8$. Under our conditions the effective cross section A is two or three times smaller because the temperature at the channel boundary is also high (see Fig. 1).

These simplifications yield the equation

$$\rho_0 u c_p \frac{dT}{dx} = \frac{d}{dx}\lambda\frac{dT}{dx} + F, \tag{3}$$

$$F = S\varkappa_\nu(T) - A\Theta/R^2 - \Phi, \quad \Theta = \int_0^T \lambda\, dT.$$

The light intensity in this equation obeys the relation

$$dS/dx = -S\varkappa_\nu. \tag{4}$$

Any rigorous accounting of the radiant heat exchange represents an extremely complex problem due to the insertion of the spectral radiation transfer equation into the system. Considerable simplification may be required depending upon specific conditions. In the case of moderate pressures, such as the atmospheric pressure which is of the greatest interest, the plasma is transparent to thermal radiation and Φ is determined mainly by radiative capacity. Reabsorption is significant in some spectral regions (spectral lines) but the loss function $\Phi(T)$ can be approximately determined for the plasma one way or another.

The ultraviolet portion of plasma emission is completely absorbed in the cold region of the wave and participates in gas heating; here $\Phi < 0$ and is not related to temperature. Even a purely radiative mechanism of wave propagation of this type is possible in principle.[2)] However as our analysis shows below, at atmospheric pressure the role of radiative heating does not exceed that of conduction and thus we now consider only the heat conduction mechanism of propagation, assuming that $\Phi > 0$.[3)]

The introduction of radiation losses in the form of a temperature function $\Phi(T)$ closes the system (3) and (4). The order of the system can be decreased by introducing the thermal flux

$$J = -\lambda dT/dx \tag{5}$$

and then eliminating the x coordinate

$$\frac{dJ}{dT} = -\frac{\lambda F(T,S)}{J} - \rho_0 u c_p, \tag{6}$$

$$dS/dT = S\varkappa_\nu/J. \tag{7}$$

We now formulate the boundary conditions. In a cold gas in front of the wave the thermal flux vanishes, there is no heat emission or losses (F = 0), and light intensity is determined by the initial power of the beam $P = \pi R^2 S_0$. The stationary case is possible only if the heat source function F also turns to zero at some high temperature (final wave temperature T_f). Otherwise heating does not stop and leads up to something like a heat explosion. The final temperature is at the maximum and the thermal flux at T_f also vanishes. Consequently the following conditions obtain:

$$\text{for } T = 0 \; J = 0,\; S = S_0,\; x = -\infty, \tag{8}$$

where

$$F(T, S) = 0,\; J = 0,\; x = +\infty,\; T = T_f. \tag{9}$$

As we see the problem is predetermined, i.e., the solution is possible only for the selected value of the u parameter, the velocity of wave propagation.

The stationary temperature T_f can be reached by virtue of two causes corresponding to two extreme cases of the problem. If the plasma is transparent to light, $S \approx \text{const} = S_0$ in the wave and the final temperature is determined by the condition of loss compensation of thermal emission:

$$F(T_f, S_0) = 0. \tag{10}$$

We readily see that such a regime is stable, i.e., it is realizable if in the region of the stationary point T_f losses rise faster than thermal emission as temperature increases.

On the other hand if the beam is strongly absorbed in plasma, the stationary condition is reached even in the complete absence of losses because of the attenuation of the light flux. In this case F = 0 when S = 0 and the final temperature can be determined only by solving the equations of the regime. This was analyzed in[6]. The regime is achievable at optical frequencies only under high pressures and is not considered here.

3. OPTICAL PROPERTIES OF PLASMA

We now consider the data on optical properties of plasma necessary for further discussion. The absorption coefficient of laser light with frequency ν corrected for stimulated emission can be computed from the continuous spectrum formula[14]. The formula is conveniently represented in the following computational form (for the primary ionization region):

$$\varkappa_\nu = \frac{0.14\xi(\nu)\, p_{\text{atm}}^2\, x_e^2\, e^{h\Delta\nu/kT}\left(e^{h\nu/kT} - 1\right)}{(T°/10^4)^{1/2}(h\nu_{eV})^3}\ \text{cm}^{-1} \tag{11}$$

where $\xi(\nu)$ is a function characteristic of each atomic species[14], $\Delta\nu$ is the depression of the continuous spectrum boundary in plasma, and $x_e = p_e/p$ is the molar fraction of electrons determined by the Saha equation

$$\frac{x_e^2}{1 - 2x_e} = 6.7\cdot 10^3\,\frac{g_+}{g_a}\,\frac{(T°/10^4)^{5/2}}{p_{\text{atm}}}\,e^{-I/kT}. \tag{12}$$

From the Ingliss-Teller equation for neutral atoms

$$h\Delta\nu/kT = 0.68\, p_{e\,\text{atm}}^{0.27}(T°/10^4)^{-1.27}.$$

Figure 2 shows the results of computing neodymium laser light absorption ($h\nu = 1.17$ eV) in air. The following mixture-averaged values were adopted: ionization potential I = 14.4 eV, $g_+/g_a = 1.9$, and $\xi \approx 0.7$. The factor $e^{h\Delta\nu/kT} \approx 1.2$–$1.5$. The contribution from secondary ionization was also approximately accounted for. Figure 3 shows the absorption of light from a CO_2 gas laser ($h\nu = 0.124$ eV) which offers some promise with respect to the continuous maintenance of plasma. In this

---

2)This was discussed in [4] relative to the case of high light intensities where the wave is supersonic.

3)At very high pressures when plasma is completely opaque another form of heat exchange, radiant heat conduction, can assume primary importance. In this case instead of inserting Φ into (1) and (3) we simply determine $\lambda(T)$ in a corresponding manner.

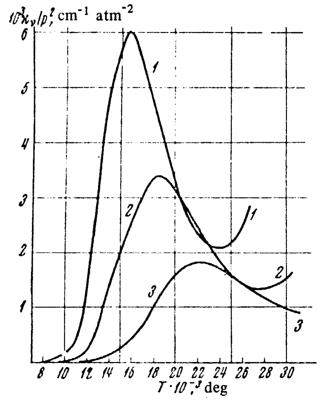


FIG. 2

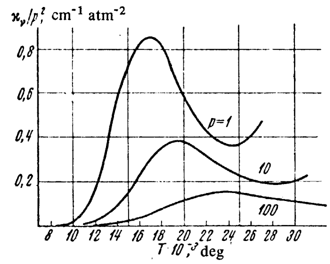


FIG. 3

FIG. 2. Coefficients of neodymium laser light absorption in air. Curve 1—p = 1 atm, κ_ν max = 6 × 10^{-3} cm^{-1}; curve 2—p = 10 atm, $\kappa_{\nu\,max}$ = 0.34 cm^{-1}; curve 3—p = 100 atm, $\kappa_{\nu\,max}$ = 18 cm^{-1}.

FIG. 3. Coefficients of CO_2 laser light absorption in air.

case $h\nu \ll kT$ and absorption is mainly of brehmsstrahlung nature. Therefore we drop the factor $e^{h\Delta\nu/kT}$ from (11) and replace $\xi(\nu)$ by the Gaunt factor g (g ≈ 2.5):

$$\varkappa_{\nu(CO_2)} = \frac{10{,}4\, p_{atm}^2 x_e^2}{(T^\circ/10^4)^{7/2}} g\ \mathrm{cm}^{-1} \qquad (13)$$
$$g = 0{,}55 \ln[27 (T^\circ/10^4)^{4/3} p_{e\,atm}^{-1/3}].$$

Under atmospheric pressure the absorption of neodymium laser light by air is fairly weak: $\kappa_{\nu\,max} \approx 6 \times 10^{-3}$ cm^{-1} according to measurements[12] showing that $\kappa_\nu < 7 \times 10^{-3}$ cm^{-1}. The absorption of infrared emission from a gas laser is two orders larger: $\kappa_{\nu\,max} \approx 0.85$ cm^{-1}.

The first-order representation of radiation losses can be obtained from the formula for continuous spectrum emissivity of hydrogen plasma:

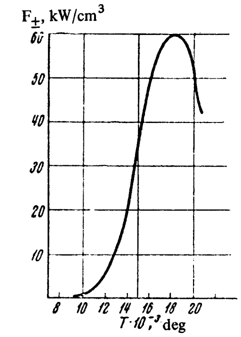
$$\varphi = \frac{280\, p_{atm}^2 x_e^2}{(T^\circ/10^4)^{5/2}} (1 + 0.027 T^\circ/10^4)\ \mathrm{kW/cm^3}. \qquad (14)$$

The main role is assumed here by recombination emission, 84% of which is due to the capture of an electron at the lower atomic level. The small correction to the unity in parentheses corresponds to brehmsstrahlung radiation (the effect of secondary ionization can also be easily accounted for).

Real volume losses differ from (14) because complex ions are not hydrogen-like and due to spectral line emission. Spectral lines produce a tremendous effect in the case of an absolutely transparent plasma. However when the hot column diameter is of the order of several millimeters, as is the case under the present discussion, the lines are strongly reabsorbed and their contribution to losses is approximately the same as that of continuous spectrum.

Computations for air taking all these effects into account were performed in[15,16] giving optical density tables for plane layers and hemispherical volumes with thickness (radius) d, for d ≥ 1 cm and T ≤ 20,000°K. The average volumetric radiation losses computed in steps of optical density for p = 1 atm and the smallest size d = 1 cm closest to the diameters of interest are not more than twice the figure obtained from the compact formula (14). There are also computations and experiments for nitrogen arcs at atmospheric pressure, T ≈ 13,000—15,000°K and channel diameter of 3 and 5 mm[17]. Radiation losses are in this case approximately twice the figures computed in optical density steps for d = 1 cm.

FIG. 4. Radiation losses in air at atmospheric pressure for cylinders several mm in diameter.



Combining all these data we plotted an approximate function of average volumetric radiation losses $\varphi(T)$ in air plasma for p = 1 atm, T ≈ 10,000—21,000°K, and channels several millimeters in diameter (Fig. 4). This function was used in our computations. We note that the accuracy of radiation losses is not a basic problem here because for small diameters these losses in general hardly exceed thermal conduction losses (which incidentally is the case with arcs[17]).

4. THE SOLUTION FOR THE CASE OF TRANSPARENT PLASMA

We begin with a model problem capable of an exact analytical solution that clearly demonstrates all the relationships of this regime for the case of transparent plasma.

We assume that for $T < T_0$ no thermal emission exists, $\kappa_\nu = 0$, and for $T > T_0$ thermal emission is constant, i.e., κ_ν = const as well as S = const = S_0. The possible range in such a substitution of the real curve $\kappa_\nu(T)$ is fairly small. For example, in air at p = 1 atm and $h\nu$ = 1.17 eV the "ignition" temperature T_0 lies in a narrow interval 12,000 = 14,000°K and the "constant" $\kappa_\nu \approx 4 \times 10^{-3}$ cm^{-1} (see Fig. 2). Furthermore we neglect radiation losses, an acceptable procedure in the case of channel diameters smaller than a few millimeters (for p = 1 atm), and assume $c_p(T)/\lambda(T)$ = const which in general is also acceptable.

These simplifications linearize (3)

$$\rho_0 u \frac{c_p}{\lambda}\frac{d\Theta}{dx} = \frac{d^2\Theta}{dx^2} - \frac{A\Theta}{R^2} + S_0\varkappa_\nu\delta, \qquad \delta = \begin{cases} 0, & \Theta < \Theta_0 \\ 1, & \Theta > \Theta_0 \end{cases}. \qquad (15)$$

We place the origin of coordinates x = 0 at the point where $\Theta = \Theta_0$ and find a solution that satisfies boundary conditions (8) and (9) and continuity conditions for temperature and flow for x = 0:

$$\Theta = \Theta_0 e^{-\alpha_1|x|} \text{ for } x < 0,$$
$$\Theta = \Theta_f(1 - e^{-\alpha_2 x}\alpha_1/(\alpha_1 + \alpha_2)) \text{ for } x > 0,$$
$$\alpha_{1,2} = \frac{1}{2a}[\sqrt{1 + 4Aa^2/R^2} \pm 1], \qquad a = \frac{\lambda}{\rho_0 u c_p},$$
$$\Theta_f = S_0\varkappa_\nu R^2/A,\ \Theta_f \doteq (1 + \alpha_1/\alpha_2)\Theta_0.$$

Solving the last equation for a and expressing Θ_f in terms of beam power P according to the formula $P = \pi R^2 S_0 = \pi A \Theta_f / \kappa_\nu$ we find the propagation velocity

$$u = \sqrt{2A}\frac{\lambda}{\rho_0 c_p R}\frac{1-P_t/P}{\sqrt{1-P_t/2P}}\sqrt{\frac{P}{P_t}}$$

where

$$P_t = 2\pi A\Theta_0/\varkappa_\nu,\ \Theta_0 = \Theta(T_0). \tag{16}$$

Since $u \geq 0$ the regime exists (the wave does not decay) only under the condition that beam power exceeds a threshold value P_t. At threshold the wave does not propagate through the material; the emitted heat is sufficient only to compensate for the "idle" leakage of heat across the lateral surface of the channel. Here $\Theta_f = 2\Theta_0$ and the widths of the heating region and that part of the thermal emission region giving rise to the "working" axial thermal flux are of the order of channel radius ($\alpha_1 = \alpha_2 = \sqrt{A}/R$). The effective width of the wave is $2R/\sqrt{A} \approx R$.

As the power increases the final temperature and wave velocity increase, the heating region narrows down (the counter flow of gas "squeezes" the thermal heating wave), and the working part of the thermal emission region expands. With moderate power excess over threshold the propagation velocity is

$$u \approx \frac{2\sqrt{A}\,\lambda}{\rho_0 c_p R}\frac{P-P_t}{P_t}, \tag{17}$$

and the wave velocity in heated gas is

$$D \approx \frac{\rho_0 u}{\rho_f} \approx 2\sqrt{A}\frac{\chi_f}{R}\frac{P-P_t}{P_t},$$

where ρ_f is density and $\chi_f = \lambda/\rho_f c_p$ is thermal conductivity at final temperature.

With large power excess over threshold $u \approx (\lambda/\rho_0 c_p)\sqrt{S_0 \kappa_\nu/\Theta_0}$ which to some extent corresponds to the Zel'dovich formula for the flame.

It follows from (16) that the stronger the light absorption, the lower the "ignition" temperature (lower ionization potential) and thermal conductivity, the lower the threshold power. It does not depend on the channel radius (provided the radiation losses are small relative to conduction losses).

We now consider the general problem, taking the real function $\kappa_\nu(T)$ and radiation losses into account. We represent sources F in the form $F = F_+ - F_-$ in the general equation (6) and plot the curves of thermal emission $F_+ = S\kappa_\nu$ and losses $F_- = A\Theta/R^2 + \Phi$ as functions of potential Θ (Fig. 5). The terminal state determined by (10) corresponds to the upper point of intersection of curves F_+ and F_-.

We integrate (6) over the entire temperature interval of the wave. Considering (8) and (9) we obtain

$$\rho_0 u \int_0^{w_f} |J|\,dw = \int_0^{\Theta_f} F(\Theta)\,d\Theta, \tag{18}$$

where $w = \int_0^T c_p dT$ is the specific enthalpy of the wave. The threshold intensity S_t corresponding to zero velocity of the wave is determined by

$$\int_0^{\Theta_f} F(\Theta)\,d\Theta = 0 \tag{19}$$

This equation expresses the condition of compensation of thermal emission and losses; its geometrical interpretation is the equality of the upper (σ_+) and lower (σ_-) areas between curves F_+ and F_- in Fig. 5. If $S < S_t$ the upper area is smaller and the regime does not occur; if $S > S_t$ the upper area is greater, energy emission exceeds losses and the larger the difference between the areas the faster the wave propagates.

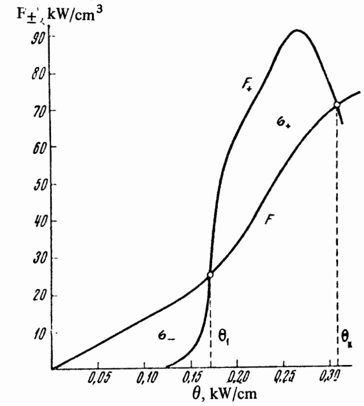


FIG. 5. Thermal emission F_+ and thermal loss F_- curves. Air, p = 1 atm, neodymium laser light, $S = 1.5 \times 10^4$ kW/cm², A = 2.9, $\Phi = \varphi/2$, R = 0.15 cm.

We find approximately the propagation velocity u[4)]. The heat flow J vanishes for $\Theta = 0$ and $\Theta = \Theta_f$. The point Θ_m of maximum heat flow can be regarded as a boundary between the working region of the wave and the heating region. The maximum value $|J_m|$ is the rate of heat transfer from one region to the other and satisfies the equation

$$\rho_0 u \int_{w_m}^{w_f} |J|\,dw + \frac{J_m^2}{2} = \int_{\Theta_m}^{\Theta_f} F(\Theta)\,d\Theta, \tag{20}$$

obtained by integrating (6) over the interval $T_m < T < T_f$.

We see from (6) that for $S = S_t$ and $u = 0$ the flow is maximum at a point where $F = 0$, i.e., at the lower point Θ_1 of intersection of curves F_+ and F_-; here $J_m^2/2 = \sigma_+$ according to (20).

It can be shown (the verification is based on the evaluation of neglected terms after the solution is obtained) that the above remains valid with sufficient accuracy for any S and u especially near the threshold. Thus

$$|J_m| \approx \left(2\int_{\Theta_1}^{\Theta_f} F\,d\Theta\right)^{1/2} = \sqrt{2\sigma_+}. \tag{21}$$

To compute u we approximate function J(w) by a triangle; such was the relationship of $J(\Theta)$ and J(w) in the model problem discussed above. We then obtain from (18) and (21)

$$\rho_0 u = \frac{\sqrt{2}}{w_f}\int_0^{\Theta_f} F\,d\Theta \Big/ \left(\int_{\Theta_1}^{\Theta_f} F\,d\Theta\right)^{1/2} = \frac{\sqrt{2}}{w_f}\frac{\sigma_+ - \sigma_-}{\sqrt{\sigma_+}}. \tag{22}$$

For $S \gg S_t$ and $\sigma_+ \gg \sigma_-$ (22) coincides with[18] and the Zel'dovich formula for the flame. For $S \approx S_t$ and $\sigma_+ \approx \sigma_-$ the value of (22) is twice that of[18].

5. COMPUTATION RESULTS. FOCUSED BEAM.

Numerical computations were performed for air at atmospheric pressure. Thermal conductivity data were

4)It is interesting that (3) describes the transformation of phases with different temperatures and densities in interstellar gas heated by cosmic rays and cooled by radiation [18]. Condition (19) determines pressure in steady state. The phase transformation rate $\rho_0 u$ in an unstable state is computed in [18] somewhat differently than here.

S, kW/cm²	Θ_f, kW/cm²	T_f, 10^4 deg	w_f, kT/g	Θ_1, kW/cm	T_1, 10^4 deg	σ_+, (kW/cm²)²	σ_-, (kW/cm²)²	u, m/sec
Neodymium glass laser								
$1.26 \cdot 10^4$	0.29	1.7	8.6	0.17	1.2	1.69	1.69	0
$1.5 \cdot 10^4$	0.31	1.8	9.4	0.17	1.2	3.1	1.6	1.0
$2.0 \cdot 10^4$	0.34	1.9	9.8	0.17	1.1	6.8	1.5	9.8
CO_2 gas laser								
$0.94 \cdot 10^2$	0.31	1.8	9.4	0.18	1.2	1.65	1.65	0
$1.0 \cdot 10^2$	0.32	1.8	9.4	0.18	1.2	2.2	1.6	0.48
$1.2 \cdot 10^2$	0.32	1.8	9.4	0.18	1.2	3.9	1.6	1.3
$1.5 \cdot 10^2$	0.39	2.1	11	0.17	1.2	8.4	1.4	2.4

*Air, p = 1 atm, A = 2.9, $\Phi = \varphi/2$.

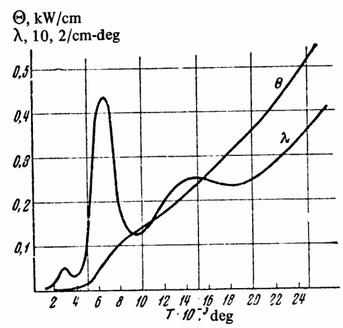


FIG. 6. Coefficient of thermal conductivity λ and potential of thermal flux Θ in air at p = 1 atm.

taken from[19] and used to plot the $\Theta(T)$ curve (Fig. 6). The coefficient A characterizing conduction losses was assumed equal to one-half of the maximum possible $A = 0.5 \times 5.8 = 2.9$ in accordance with the statement made in Section 2. Radiation losses were also reduced by half: $\Phi = 0.5\varphi$. As we said at the end of Sec. 2, the portion of heat radiation which propagates in the direction of the light channel against the beam is almost completely absorbed in the heating region (since the radiation is mainly ultraviolet) and consequently it is not "lost." Since the effective thickness of the wave $\lesssim R$, the radiating wave volume is a relatively thin disc so that a noticeable portion of the total radiation is emitted in the indicated direction. It should be noted that in the final analysis the remaining radiation also is not completely "lost." The radiation absorbed near the wave beyond the channel boundary helps to heat the gas in the vicinity of the lateral channel surface and to reduce conduction losses (decreases A). Computations were performed for R = 0.15 cm, as in the experiments reported in[12], for frequencies of neodymium and CO_2 lasers. The computation results are given in the Table; see also Fig. 5.

The computed threshold for neodymium laser $S_t \approx 1.3 \times 10^4$ kw/cm², $P_t \approx 920$ kw is in excellent agreement with the experimental value $S_t \approx (0.8-1.5) \times 10^4$ kw/cm² [12]. Since both the conduction and radiation losses were reduced to one-half of the maximum possible, the upper limit of the threshold value S_t was only twice the computed value. Radiation losses for R = 0.15 cm are approximately the same as conduction losses. For the same R = 0.15 cm the gas laser threshold $S_t \approx 10^2$ kw/cm², i.e., at the power $P_t \approx 7$ kw we must consider the fact that the output frequency of a gas laser is an order lower than that of solid state lasers so that light absorption is two orders stronger and the regime threshold is two orders lower. The plasma temperature is approximately 18,000°K.

We note that the elementary formula (16) for threshold power yields good results. In fact it is clear that the "ignition" temperature T_0 must be represented by T_1 which according to the Table is fairly stable: $T_0 \approx T_1 \approx 12{,}000$°K, $\Theta_0 \approx 0.17$ kw/cm. For neodymium lasers the "constant" $\kappa_\nu \approx 4 \times 10^{-3}$ cm^{-1} and from (16) $P_t \approx 800$ kw that practically coincides with the result of a more rigorous computation (this also applies to the CO_2 laser). A comparative evaluation of radiation flow which is of the order of $\varphi(T_k)R/4$ with the conduction flow $|J|$ shows that they are more or less the same. This indicates an approximately the same role played by the conduction and radiation mechanisms of wave propagation (the effect of the latter was accounted for to some extent by the fact that we reduced radiation losses in half).

The rates of wave propagation in heated gas in the one-dimensional case of $D \approx \rho_0 u/\rho_f$ turn out to be of the order of 200 m/sec that is a few times higher than experimental values[12]. This is due to the fact that the actual combustion does not quite follow the process in a "tube with a closed end." The heated gas shows a strong lateral expansion and does not remain at rest as in the tube with the closed end. Therefore the situation resembles more an intermediate state between "closed" and "open" "tubes" and the actual wave velocity in the laboratory system of coordinates is a few times smaller than D.

In the case of small $R \lesssim 0.05$ cm radiation losses are significantly lower than conduction losses, $S_t \sim R^2$, and the threshold is minimal. For a CO_2 laser with A = 2.9 we have $P_{min} \approx 4$ kw. The most favorable conditions for the maintenance of plasma are best realized by a sharp focusing of the beam. The discharge is then localized in the focal region and the stabilized combustion region is static. The consideration of the spherical model more relevant to our case enables us to improve the accuracy of coefficients A and P_{min}.

Let the light be absorbed only within the sphere R in which heat emission F_+ is homogeneous and radiation losses small. The total energy $Q = 4\pi F_+ R^3/3$ is removed to infinity by heat conduction. Solving the conduction equation we find $Q = 10\pi\Theta R/3$, where Θ is averaged over the volume of the sphere. We set $Q = P_t\kappa(\theta)l$ and $4\pi R^3/3 = \pi r_{foc}^2 l$, where r_{foc} is focal radius, $l = 2r_{foc}f/r_1$ is the length of the caustic, f is the focal length, and r_1 is the radius of the initial beam. For threshold power we obtain $P_t = \pi A'\theta/\kappa$ with $A' = 1.9(r_1/f)^{2/3}$. For example for $f/r_1 = 2$ we have $A' = 1.2$ instead of A = 2.9 assumed above. Consequently it is possible that ~2 kw is sufficient to maintain plasma in atmospheric air by a well-focused CO_2 laser beam. The conditions of maintaining plasma are liberalized at increased pressures (roughly speaking $P_t \sim \kappa_{max}^{-1} \sim p^{-2}$) and in gases with low thermal conductivity (such as xenon). It is significant that plasma can be continuously translated in space by moving the beam together with the focal region at a velocity not exceeding the order of u (see Table).

The author is indebted to G. I. Barenblatt for interest in this work.

[1] Ya. B. Zel'dovich. Teoriya goreniya i detonatsiĭ gazov (Theory of Combustion and Detonation in Gases), AN SSSR, 1944.

[2] L. D. Landau and E. M. Lifshitz, Mekhanika sploshnykh sred (Fluid of Mechanics), Gostekhizdat, 1954 [Addison-Wesley, 1958].

[3] S. A. Ramsden and P. Savic, Nature N4951, 1217 (1964).

[4] Yu. P. Raĭzer, Zh. Teor. Eksp. Fiz. **48**, 1508 (1965) [Sov. Phys.-JETP **21**, 1009 (1965)].

[5] Yu. P. Raĭzer, ZhETF Pis. Red. **7**, 73 (1968) [JETP Lett. **7**, 55 (1968)].

[6] Yu. P. Raĭzer, Zh. Prikl. Mat. Tekh. Fiz. **3**, 3 (1968).

[7] Yu. P. Raĭzer, Usp. Fiz. Nauk **99**, 687 (1969) [Sov. Phys.-Uspekhi **12**, 777 (1970)].

[8] D. A. Frank-Kamenetskiĭ, Vtoroĭ Vsesoyuznyĭ simpozium po goreniyu i vzryvu (The Second All-Union Symposium on Combustion and Explosion), October, 1969, Erevan, Abstracts of reports.

[9] V. P. Aksenov, L. M. Blinov, V. P. Marin, L. S. Polak, and V. S. Shchipachev, Sb. Nizkotemperaturnaya plasma (Coll. Low Temperature Plasma), Proc. Intern. Symposium in Moscow, 1965, Mir, 1967.

[10] L. M. Blinov and G. B. Lysov, Sb. Ocherki fiziki i khimii nizkotemperaturnoĭ plazmy (Coll. Essays on Physics and Chemistry of Low Temperature Plasma). L. S. Polak, Ed. Nauka, 1970.

[11] Yu. P. Raĭzer, Sb. Ocherki fiziki i khimii nizkotemperaturnoĭ plazmy (Coll. Essays on Physics and Chemistry of Low Temperature Plasma). L. S. Polak, Ed. Nauka, 1970.

[12] F. V. Bunkin, V. I. Konov, A. M. Prokhorov, and V. B. Fedorov, ZhETF Pis. Red. **9**, 609 (1969) [JETP Lett. **9**, 371 (1961)].

[13] Yu. P. Raĭzer, ibid. **11**, 195 (1970) [**11**, 120 (1970)].

[14] L. M. Biberman and G. E. Norman, Usp. Fiz. Nauk **91**, 193 (1967) [Sov. Phys.-Uspekhi **10**, 52 (1967)].

[15] L. M. Biberman, V. S. Vorob'ev, G. E. Norman, and I. T. Yakubov, Kosmich. Issled. **2**, 441 (1964).

[16] I. V. Avilova, L. M. Biberman, V. S. Vorob'ev, V. M. Zamalin, G. A. Kobzev, A. N. Lagar'kov, A. Kh. Mnatsakanyan, and G. E. Norman, J. Quant. Spectr. Rad. Transfer 9, 1969, p. 89, 113, 1285.

[17] E. I. Asinovskiĭ, E. V. Drokhanova, A. V. Kirillin, and A. N. Lagar'kov, Teplofiz. Vys. Temp. **5**, 739 (1967).

[18] Ya. B. Zel'dovich and S. B. Pikel'ner, Zh. Eksp. Teor. Fiz. **56**, 310 (1969) [Sov. Phys.-JETP **29**, 170 (1969)].

[19] V. Penski, Proc. Fourth Symposium on Thermophysical Properties. April, 1968, p. 189.

Translated by J. G. Adashko
259