THERMAL CONDUCTIVITY OF A FERROMAGNETIC DIELECTRIC WITH ALLOWANCE FOR PHONONS IN THE HYDRODYNAMIC APPROXIMATION

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The thermal conductivity of ferromagnetic dielectrics is considered in the hydrodynamic approximation for two limiting cases, taking into account the interaction of magnons with phonons. In the case $T \ll \Theta_D^2/\Theta_C$ (where Θ_D is the Debye temperature, Θ_C the Curie temperature) it is shown that, although the heat transfer under the conditions considered is mainly due to magnons, the viscosity of the magnon-phonon gas may be determined by the phonons. If $T \gg \Theta_D^2/\Theta_C$, the thermal conductivity is of a phonon nature and the viscosity of a magnon nature. In both cases, allowance for magnon-phonon interaction modifies the temperature dependence of the thermal conductivity coefficient.

IN sufficiently pure ferromagnetic dielectrics and at low temperatures, when the normal collisions between quasiparticles are more probable than collisions accompanied by losses of quasimomentum, the gas of quasiparticles behaves like a viscous liquid and is described by equations similar to the equations of ordinary hydrodynamics. In this case, as Gurzhi has shown,[1] the thermal conductivity of the bounded samples can be regarded as the Poiseuille flow of the magnon gas. Recently, Tsarev[2] evidently observed experimentally the hydrodynamic thermal conductivity in thin specimens of the ferromagnet $CrBr_3$.

Phonons are not usually taken into account in the consideration of the thermal conductivity of ferromagnets in the hydrodynamic approximation in the low temperature region, since the heat is transferred chiefly by the magnons.[1] However, under definite conditions, as will be shown below, the viscosity of the magnon-phonon gas can be determined by the phonons, and the heat capacity by the magnons. In the other limiting case, when the heat capacity is connected with the phonons, the viscosity is determined by the magnons. In both cases, account of the magnon-phonon interaction leads to a change in the curve of the temperature dependence of the coefficient of thermal conductivity. As Gurzhi and Konotorovich[3] have shown, a similar situation is possible for electrons and phonons in metals with different numbers of electrons and holes.

In the following, we shall limit ourselves, for simplicity, to the isotropic dispersion law for magnons and phonons:

$$\varepsilon(\mathbf{p}) = \Theta_C (ap/\hbar)^2 + \varepsilon_0, \quad \varepsilon_0 = \beta\mu M_0,$$
$$\Omega(\mathbf{q}) = sq, \quad s = \Theta_D a/\hbar, \tag{1}$$

where $\epsilon(\mathbf{p})$, $\mathbf{p}$ and $\Omega(\mathbf{q})$, $\mathbf{q}$ are the dispersion laws and the quasimomenta of the magnons and the corresponding phonons, Θ_C and Θ_D are the Curie and Debye temperatures, s the sound velocity, a the lattice constant, μ the Bohr magneton, β the constants of anisotropicity, M_0 the equilibrium magnetization.

In the spin subsystem, the normal collisions between magnons can be connected both with four-particle exchange and with three-particle relativistic interactions. Normal collisions in the phonon subsystem are due to three-particle interactions between phonons. Theoretical estimates lead to the following expressions for the probabilities of the processes enumerated above (see, for example,[1,4]):

$$W_{ss}^{(4)} = \frac{1}{\tau_{ss}^{(4)}} \sim \frac{\Theta_C}{\hbar}\left(\frac{T}{\Theta_C}\right)^4, \quad W_{ss}^{(3)} = \frac{1}{\tau_{ss}^{(3)}} \sim \frac{(\mu M_0)^2}{\hbar\Theta_C}\left(\frac{T}{\Theta_C}\right)^{1/2}, \tag{2}$$
$$\varepsilon_0 \ll T,$$
$$W_{pp}^{\text{theor}} = \frac{1}{\tau_{pp}} \sim \frac{\Theta_D}{\hbar}\frac{\Theta_D}{Ms^2}\left(\frac{T}{\Theta_D}\right)^5,$$

where M is the atomic mass.

Interaction of the magnons with the phonons takes place as the result of Cerenkov radiation or the absorption of a phonon by a magnon and is possible only if $v \geq s$ or in other units, $\epsilon(\mathbf{p}) \geq \Theta_D^2/4\Theta_C$ ($\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity of the magnon). The corresponding probabilities have the form:[4]

a) for longitudinal phonons

$$W_{sp} = \tau_{sp}^{-1} \sim \frac{\Theta_D}{\hbar}\frac{\Theta_D}{Ms^2}\left(\frac{T}{\Theta_C}\right)^{5/2}\exp\left(-\frac{\Theta_D^2}{4T\Theta_C}\right), \quad \text{if} \quad \varepsilon_0 \ll T \ll \frac{\Theta_D^2}{\Theta_C}; \tag{2a}$$

b) for transverse phonons

$$W_{sp} \sim \frac{\Theta_D}{\hbar}\frac{\Theta_D}{Ms^2}\left(\frac{T}{\Theta_C}\right)^{5/2}, \quad \text{if} \quad \frac{\Theta_D^2}{\Theta_C} \ll T. \tag{2b}$$

(In both cases, $W_{ps} = W_{sp}(\Theta_D^2/T\Theta_C)^{3/2}$.)

However, better agreement with experiment is obtained for the phonon-phonon interaction with the dependence T^4, i.e.,[5]

$$W_{pp}^{\exp} \sim (T/\Theta_D)^4. \tag{3}$$

1. Let the magnon heat capacity exceed the phonon one, i.e., $C_s/C_p \approx (\Theta_D^2/T\Theta_C)^{3/2} \gg 1$, and let the free path lengths of the quasiparticles satisfy the inequalities $l_{ss}^N \ll l_{ps}^N \ll d \ll l_s^V, l_p^V$, where the index N denotes normal collisions and V collisions accompanied by nonconservation of the quasimomentum; d is the transverse dimension of the sample. The region of hydrodynamic thermal conductivity generally consists of two temperature intervals: in one of them the normal collisions are determined by exchange interactions, and in the other by relativistic interactions between magnons.[1]

If $T \gg T_0$, where $T_0 = \Theta\ (\mu M_0/\Theta_C)^{4/7}$, and exchange interactions predominate, then, to obtain the hydrodynamic equations with account of viscosity, one must substitute the solution of the set of kinetic equations for magnons and phonons, obtained with accuracy up to first order, inclusive, in the relations

$$\operatorname{div}(\langle\langle p_i \mathbf{v} f\rangle\rangle + \langle\langle q_i \mathbf{s} N\rangle\rangle) = \langle\langle p_i \hat{I}_s{}^V\{f\}\rangle\rangle + \langle\langle q_i \hat{I}_p{}^V\{N\}\rangle\rangle,$$
$$\operatorname{div}(\langle\langle \varepsilon \mathbf{v} f\rangle\rangle + \langle\langle \Omega \mathbf{s} N\rangle\rangle) = 0, \quad \operatorname{div}\langle\langle \mathbf{v} f\rangle\rangle = \langle\langle \hat{I}_{ss}^{(3)}\{f\}\rangle\rangle, \tag{4}$$

where

$$\langle\langle \Phi(\mathbf{p})\rangle\rangle = \frac{1}{h^3}\int d\mathbf{p}\,\Phi(\mathbf{p}), \quad \langle\langle \Psi(\mathbf{q})\rangle\rangle = \frac{1}{h^3}\int d\mathbf{q}\,\Psi(\mathbf{q}),$$

$\hat{I}_s^V$ and $\hat{I}_p^V$ are the magnon and phonon collision integrals, which are accompanied by losses in quasimomentum, but which conserve the energy and the number of quasiparticles, $\hat{I}_{ss}^{(3)}$ is the integral of magnon-magnon relativistic interactions. The relations (4) express the laws of conservation of quasimomentum densities, energy and number of magnons in the stationary case.

In what follows, we restrict ourselves to the simplest case of a plate, the thickness of which is small in comparison with its length and width, since the results in the hydrodynamic region are but little sensitive to the shape of the transverse cross section of the sample.[1] We choose the x axis in the direction of the constant temperature gradient and the z axis perpendicular to the surface of the plate. For an approximate solution of the set of kinetic equations, we make use of the Chapman-Enskog method.[3,6] As is known, the zeroth approximation leads to the drift solutions:

$$f^{(0)} = \left[\exp\left(\frac{\varepsilon - \mathbf{pu} - \delta\mu}{T(1+\vartheta)}\right) - 1\right]^{-1},$$
$$N^{(0)} = \left[\exp\left(\frac{\Omega - \mathbf{qu}}{T(1+\vartheta)}\right) - 1\right]^{-1}, \tag{5}$$

where $\mathbf{u} = (u(z), 0, 0)$ is the drift velocity, $\vartheta = \vartheta(x)$ is the relative temperature change, and $\delta\mu$ the nonequilibrium chemical potential of the magnons. According to the Chapman-Enskog method, the energy densities of the quasimomentum, energy and magnons are completely determined by the functions of the zeroth approximation, i.e.,

$$\mathbf{P}\delta_{i0} = \langle\langle \mathbf{p} f^{(i)}\rangle\rangle + \langle\langle \mathbf{q} N^{(i)}\rangle\rangle, \quad E\delta_{i0} = \langle\langle \varepsilon f^{(i)}\rangle\rangle + \langle\langle \Omega N^{(i)}\rangle\rangle, \quad N_s\delta_{i0} = \langle\langle f^{(i)}\rangle\rangle. \tag{6}$$

In the case of a quadratic dispersion law for the magnons, it follows from (4) and (6) that[1)]

$$\langle\langle \hat{I}_{ss}^{(3)}\{f^{(0)}\}\rangle\rangle = -\frac{\langle 1\rangle\,\delta\mu}{\tau_{ss}^{(3)}} = 0, \quad \delta\mu = 0,$$

where

$$\langle \Phi(\mathbf{p})\rangle = -\frac{1}{h^3}\int d\mathbf{p}\,\Phi(\mathbf{p})\frac{\partial f_0}{\partial\varepsilon}.$$

The set of kinetic equations has the following form in first approximation:

$$-\frac{\partial f_0}{\partial\varepsilon} p_x v_z \frac{\partial u}{\partial z} = \hat{I}_{ss}^N\{f^{(1)}\} + \hat{I}_{sp}^N\{f^{(1)}; N^{(1)}\},$$
$$-\frac{\partial N_0}{\partial\Omega} q_x s_z \frac{\partial u}{\partial z} = \hat{I}_{pp}^N\{N^{(1)}\} + \hat{I}_{ps}^N\{N^{(1)}, f^{(1)}\}. \tag{7}$$

Here $\hat{I}_{ss}^N = \hat{I}_{ss}^{(4)} + \hat{I}_{ss}^{(3)}$, $\hat{I}_{sp}^N$, $\hat{I}_{ps}^N$, $\hat{I}_{pp}^N$ denote the linearized integrals of normalized collisions between quasiparticles, the explicit form of which, and also the corresponding matrix elements, can be found, for example, in the monograph[7]. Terms which do not depend on the coordinates, and which therefore are unimportant for the subsequent considerations, are omitted from these equations.

The set (7) forms a set of linear inhomogeneous integral equations. The general solution of the corresponding homogeneous set is known: $f^{(0)}$ and $N^{(0)}$; therefore, we must find only the particular solution $f^{(1)}$ and $N^{(1)}$, which is conveniently sought in the form

$$f^{(1)} = -\tau_{ss}^{(4)}\frac{\partial f_0}{\partial\varepsilon} p_x v_z \frac{\partial u}{\partial z}\varphi\left(\frac{\varepsilon}{T}\right),$$
$$N^{(1)} = -\tau_{ps}\frac{\partial N_0}{\partial\Omega} q_x s_z \frac{\partial u}{\partial z}\psi\left(\frac{\Omega}{T}\right). \tag{8}$$

For the unknown functions $\varphi(\epsilon/T)$ and $\psi(\Omega/T)$, we obtain the following set of integral equations, with accuracy to within numerical coefficients of the order of unity:

$$-[1+f_0(y)]\,y\psi(y) - \xi^{-4}e^{\xi/4}\hat{K}_{pp}\{\psi\} + \frac{\Theta_D}{Ms^2}\left(\frac{\Theta_C}{\Theta_D}\right)^2\xi^3 e^{-\xi/4}\hat{K}_{ps}^{\mathrm{II}}\{\varphi\} = ye^y f_0^2(y),$$

$$-\hat{K}_{ss}^{(4)}\{\varphi\} - \frac{\Theta_D}{Ms^2}\left(\frac{\Theta_C}{\Theta_D}\right)^2\xi e^{-\xi/4}\hat{K}_{sp}^{\mathrm{I}}\{\varphi\} + \xi^{-2}\hat{K}_{sp}^{\mathrm{II}}\{\psi\} = xe^x f_0^2(x). \tag{9}$$

In the following, we shall not need the explicit form of the remaining operators.

$$\hat{K}_{ss}^{(4)}\{\varphi\} = \int_0^\infty dx'\,dx''\sqrt{\frac{x''}{x}}(\sqrt{xx'} + \sqrt{x''|x+x'-x''|})^2 \cdot$$
$$\times [1+f_0(x)][1+f_0(x')]f_0(x'')f_0(x+x'-x'')\{x\varphi(x) + x'\varphi(x') - x''\varphi(x'') - (x+x'-x'')\varphi(x+x'-x'')\}.$$

We set $\Theta_D \approx 3\times10^2$ °K, $\Theta_C \approx 5\times10^2$ °K, $\mu M_0 \sim 1$°K, $M \sim 10^{-22}$ g, $s^2 \approx 10^{11}$ cm²/sec². (The values of the parameters have been chosen with the condition that the mean velocity of the magnons, $v_T \approx ah^{-1}\sqrt{T\Theta_C}$ was close to the velocity of sound.) Then the coefficients in front of the operators $\hat{K}_{ps}^{\mathrm{II}}$, $\hat{K}_{sp}^{\mathrm{I}}$ and $\hat{K}_{sp}^{\mathrm{II}}$ are small for $\xi > 3$. So far as the coefficient $\xi^{-4}e^{\xi/4}$, which appears in front of $\hat{K}_{pp}$, is concerned, it is small if $3 \le \xi \le 50$. In the case $\xi \ge 50$ for $d \le 0.1$ cm, l_{ps}^N, $l_{pp}^N > d$ and the hydrodynamic approximation for the phonons is less valid, although, generally speaking, it remains valid for the magnons. From the system (9), with account of the properties of the $\hat{K}$ operators, it follows that the functions $\varphi(x)$ and $\psi(y)$ are of the order of unity in their norms.

When $T \le T_0$ and the normal collisions between the magnons are determined by three-particle relativistic interactions, we seek a solution in the form

$$f^{(1)} = -\tau_{ss}^{(3)}\frac{\partial f_0}{\partial\varepsilon} p_x v_z \frac{\partial u}{\partial z}\varphi_1\left(\frac{\varepsilon}{T}\right),$$
$$N^{(1)} = -\tau_{ps}\frac{\partial N_0}{\partial\Omega} q_x s_z \frac{\partial u}{\partial z}\psi_1\left(\frac{\Omega}{T}\right). \tag{10}$$

For the unknown functions $\varphi_1(x)$ and $\psi_1(y)$, we obtain the set of integral equations

$$-[1+f_0(y)]\,y\psi_1(y) - \xi^{-4}e^{\xi/4}\hat{K}_{pp}\{\psi_1\}$$
$$+\frac{\Theta_D}{Ms^2}\left(\frac{\Theta_C}{\mu M_0}\right)^2\left(\frac{\Theta_D}{\Theta_C}\right)^5\frac{1}{\sqrt{\xi e^{\xi/4}}}\hat{K}_{ps}^{\mathrm{II}}\{\varphi_1\} = ye^y f_0^2(y),$$

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1) In the case of an arbitrary dispersion law for the magnons, the equation for $\delta\mu$ will be: $\langle v_i v_j\rangle\,\delta^2\delta\mu/\delta x_i\delta x_j = \langle 1\rangle\delta\mu/\tau_{ss}^{(3)} \times \tau_{ss}^{(4)}$, whence it follows that the relaxation of $\delta\mu$ takes place over distances $l_\mu^V = \sqrt{l_{ss}^{(3)}l_{ss}^{(4)}}$.

$$-\hat{K}_{ss}^{(3)}\{\varphi_1\} - \frac{\Theta_D}{Ms^2}\left(\frac{\Theta_C}{\mu M_0}\right)^2\left(\frac{\Theta_D}{\Theta_C}\right)^5 \frac{1}{\xi^{5/2}e^{\xi/4}}\hat{K}_{sp}^{\mathrm{I}}\{\varphi_1\} + \xi^{-2}\hat{K}_{sp}^{\mathrm{II}}\{\psi_1\} = xe^x f_0^2(x), \tag{11}$$

where

$$\hat{K}_{ss}^{(3)} = \int_0^\infty dx'\, x^{-1/2}[1+f_0(x)][1+f_0(x')]f_0(x+x')\{x\varphi(x)+x'\varphi(x')$$

$$-(x+x')\varphi(x+x')\} + \int_0^\infty dx'\, x^{-1/2}[1+f_0(x)]f_0(x')f_0(x-x')\{x\varphi(x)$$

$$-x'\varphi(x')-(x-x')\varphi(x-x')\}.$$

Since the set (11) is valid for the condition $\xi \gg \xi_0$, where $\xi_0 = (\Theta_D/\Theta_C)^2(\Theta_C/\mu M_0)^{4/7}$, all the coefficients in front of the K operators are small if only $\xi_0 \ll \xi \le 50$, and the parameters of the material have the numerical values given above. Therefore the functions $\varphi_1(x)$ and $\psi_1(y)$ are also of the order of unity in their norms.

Substituting $f = f^{(0)} + f^{(1)}$ and $N^{(0)} + N^{(1)}$, where $N^{(1)}$ and $f^{(1)}$ are determined by the formulas (8) and (10), in the first of the relations (4), which expresses the law of conservation of quasimomentum, we get the hydrodynamical equation

$$a\frac{\partial\vartheta}{\partial x} = \nu^{\mathrm{eff}}\frac{\partial^2 u}{\partial z^2} - \frac{u}{\tau_s^V}, \tag{12}$$

where

$$a = \frac{\langle pv\varepsilon\rangle + \langle qs\Omega\rangle}{\langle p^2\rangle}, \quad \langle q^2\rangle \ll \langle p^2\rangle,$$

$$\nu^{\mathrm{eff}} = \frac{\tau_{ss}^*\langle p^2v^2\rangle + \tau_{ps}\langle q^2s^2\rangle}{\langle p^2\rangle} = v_T l_{ss}^* + s l_{ps}^N\left(\frac{T\Theta_C}{\Theta_D^2}\right)^{5/2},$$

$$\tau_{ss}^* = \begin{cases}\tau_{ss}^{(4)}, & T \gg T_0\\ \tau_{ss}^{(3)}, & T \ll T_0\end{cases}.$$

The second and third of the relations (4), which express the laws of conservation of energy density and of magnons, vanish identically.

The coefficient of thermal conductivity is defined by the relation

$$\varkappa = -Q\Big/T\frac{\partial\vartheta}{\partial x},$$

where $Q = \langle\langle v\varepsilon f^{(0)}\rangle\rangle = +\langle\langle s\Omega N^{(0)}\rangle\rangle \approx TC_s u$ is the heat flux density. As a result of the solution of Eq. (12), we have

$$\varkappa = C_s v_T l^{\mathrm{eff}}, \tag{13}$$

where

$$l^{\mathrm{eff}} = l^V(1 - w^{-1}\,\mathrm{th}\, w), \quad l^V = v_T\tau_s^V, \quad w = {}^1/_2 d(\nu^{\mathrm{eff}}\tau_s^V)^{-1/2}.$$

The magnons can interact with one another both directly and through the phonons. Both mechanisms make a contribution to the kinematic viscosity and, therefore, to the coefficient of thermal conductivity. We consider the limiting cases.

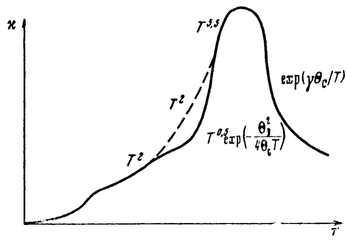
If $w \ll 1$ and $T \gg T_0$, then, for the numerical values of the parameters given above, the viscosity of the magnon-phonon gas is determined by the magnons:

$$\varkappa = C_s v_T\frac{d^2}{l_{ss}^{(4)}} \sim d^2T^{5,5}, \quad l^{\mathrm{eff}} = \frac{d^2}{l_{ss}^{(4)}}, \quad T_0 \ll T \ll T_2, \tag{14}$$

where T_2 is found from the condition $l^{\mathrm{eff}}(T_2) = l_S^V(T_2)$.

When $w \ll 1$ and $T_1 \ll T \ll T_0$, where I_1 satisfies the condition $l^{\mathrm{eff}}(T_1) = d$, the viscosity is associated with the phonons:

$$l^{\mathrm{eff}} = \frac{d^2}{l_{ps}}\left(\frac{\Theta_D^2}{T\Theta_C}\right)^2, \quad \varkappa \sim d^2T^{1/2}\exp\left\{-\frac{\Theta_D^2}{4T\Theta_C}\right\}. \tag{15}$$



Dependence of the thermal conductivity of ferromagnets on the temperature in the hydrodynamic region, with account of the interaction of magnons with phonons. The dashed curve describes the similar dependence without account of the interaction with phonons. (Case $T \ll \Theta_D^2/\Theta_C$, where Θ_D is the Debye temperature and Θ_C the Curie temperature).

For $T \ll T_1$, a Knudsen situation exists for phonons and the hydrodynamic approximation remains valid only for magnons, since $l_{SS}^{(3)} = a(\Theta_C/\mu M_0)^2 \ll d$. Therefore,

$$l^{\mathrm{eff}} = d^2/l_{ss}^{(3)}, \quad \varkappa \sim d^2T^2, \quad \varepsilon_0 \ll T \ll T_1. \tag{16}$$

Finally, if $T \ll \epsilon_0$, the thermal conductivity is determined by the phonons, since that of the magnons is exponentially small.

The results (14)—(16) have a simple physical meaning. Under the influence of the normal collisions, the quasiparticle undergoes a random walk, which increases the effective free path length between two collisions with losses of quasimomentum. According to the formulas of Brownian motion, the path followed by the particles between two collisions with the boundaries amounts to d^2/l^N, where l^N is the greatest of the lengths associated with normal collisions in each temperature range. The factor $(\Theta_D^2/T\Theta_C)^2$ takes into account the contribution of the phonons in comparison with the magnons.

When $w \gg 1$, we have $l^{\mathrm{eff}} - l_S^V$ and the momentum loss is due to processes of transport and scattering by impurities and lattice defects. In this case, the thermal conductivity falls off with increase in T by an exponential law if the transport processes predominate,[7] and in nonmonotonic fashion when the principal role is played by scattering from defects.[8]

The qualitative path of the coefficient of thermal conductivity in the considered temperature range is drawn in the figure (continuous curve). For comparison, the dashed curve shows a similar dependence, without consideration of the phonons.

It should be noted that the results given above remain in force if the experimental value is used for the probability of phonon-phonon collisions (3).

2. Now let $T \gg \Theta_D^2/\Theta_C$ and, consequently, the heat capacity is determined by phonons, while the free path lengths satisfy the inequalities $l_{pp}^N \ll l_{sp}^N \ll d \ll l_p^V, l_S^V$. The expression for the kinematic viscosities in this case can be shown to have the form

$$\nu_l^{\mathrm{eff}} = s l_{pp}^N + v_T l_{sp}^N(\Theta_D^2/T\Theta_C)^{5/2}. \tag{17}$$

It follows from (17) that the viscosity associated with magnons exceeds the phonon viscosity by a factor $T\Theta_C/\Theta_D^2$. The coefficient of thermal conductivity in the hydrodynamic region is equal to

$$\varkappa = C_p s l^{\mathrm{eff}} \sim d^2T^7, \quad l^{\mathrm{eff}} = (d^2/l_{sp}^N)(T\Theta_C/\Theta_D^2)^2. \tag{18}$$

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