SOVIET PHYSICS JETP VOLUME 32, NUMBER 1 JANUARY, 1971

MOBILITY OF POSITIVE AND NEGATIVE IONS IN He^3—He^4 SUPERFLUID SOLUTIONS

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Submitted January 28, 1970

Zh. Eksp. Teor. Fiz. 59, 64—76 (July, 1970)

The mobility of positive and negative ions in He^3—He^4 solutions, containing up to 40 mol.% He^3, was measured by the time-of-flight technique at temperatures from 0.35 to 2.0°K, in fields up to 400 V/cm. In this range of temperatures, the scattering of ions by impurity excitations was predominant, and the mobility of positive ions did not depend appreciably on the temperature. The mobility of negative ions in solutions with relatively high He^3 concentrations decreased with falling temperature. The results obtained were used to estimate the energy gap in the roton part of the energy spectrum of the solutions.

IT is reported in[1-3] that the presence of He^3 impurity atoms in superfluid helium alters considerably the magnitude and nature of the mobility of ions in this liquid, primarily because of the appearance of an additional scattering mechanism.

Meyer and Reif[1] investigated He^3—He^4 solutions with molar concentrations of He^3 amounting to 1.3×10^{-5} and 5.1×10^{-5}. They found that even these small amounts of He^3 impurity atoms reduced considerably, at all temperatures below 0.8°K, the mobility of positive and negative ions in superfluid helium. However, at temperatures above 0.8°K, the mobility of ions in such solutions did not differ from their mobility in He^4.

Kovdrya and Esel'son[2,3] measured the mobility of positive ions in He^3—He^4 solutions, containing 3.9—29.0 mol. % He^3, at temperatures from 1.4 to 2.15°K. They found that, throughout this range of temperatures, the mobility of positive ions was less than that in pure He^4, and that the reduction in the mobility increased with decreasing temperature and with increasing concentration of He^3.

Neeper and Meyer[4] determined, at $T < 0.3$°K, the product of the ionic mobility and the concentration μc for solutions containing 1.5×10^{-4}—4.5×10^{-2} He^3. They found that when the temperature was lowered the mobility-concentration product for positive ions decreased by 15—20% but that for negative ions it increased strongly.

Thus, investigations of the mobility of positive and negative ions in He^3—He^4 solutions yielded certain observations which have not yet been fully explained.

The present paper reports the results of measurements of the mobility of positive and negative ions in He^3—He^4 solutions containing up to 40 mol.% He^3. These measurements were carried out at temperatures from 0.35 to 2.0°K in fields up to 400 V/cm.

We measured the mobility of ions in liquid helium by a method described in[2,3] and based on a determination of the time taken by ions to traverse a gap between two pairs of grids (A_1A_2, A_3A_4), which acted as shutters. The drift length, equal to the distance between the midpoints of these shutters, was 5.60 ± 0.05 mm and the ratio of the shutter "thickness' to the drift length was approximately 0.1. Ions were generated in liquid helium by a source consisting of titanium tritide and emitting $\sim 10^7$ electrons per second. Currents of the order of 10^{-15}—10^{-12} A were measured with an electrometer and an ÉD-05 dynamic capacitor. The pulse frequency was measured with a ChZ-3 electronic counter-type frequency meter, to which was coupled a ChZ-7 meter.

A measuring cell (MC) was placed in the cryostat (Fig. 1), in which temperatures down to 0.35°K were produced by pumping out liquid He^3 with an adsorption pump (P) located in the same Dewar flask as the cell. The adsorbent was 100 g of palladized silica gel, whose adsorption properties were investigated in[5]. A copper chamber 2, enclosing the measuring cell, was in thermal contact with a container 3 where the He^3 was condensed. The vapor above the liquid He^3 was pumped out through a thin-walled stainless-steel tube 4 of 7 mm diameter, the rate of pumping being regulated by a bellows-type value V with a 6 mm wide aperture. The connecting rod of this valve projected into the upper part of the cryo-

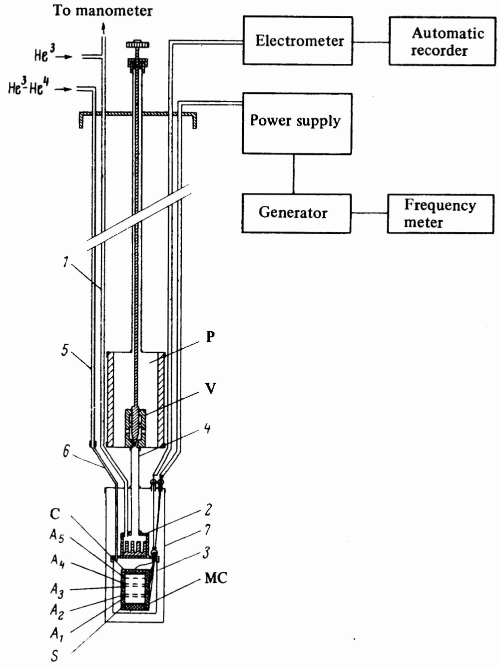

FIG. 1. Layout of the apparatus: 1) German-silver tube; 2) chamber for solution; 3) He^3 container; 4) stainless-steel tube; 5) German-silver tube; 6) capillary; 7) vacuum jacket. P—adsorption pump; V—valve; MC—measuring cell; S—ion source; C—collector; A_1-A_5—grids.

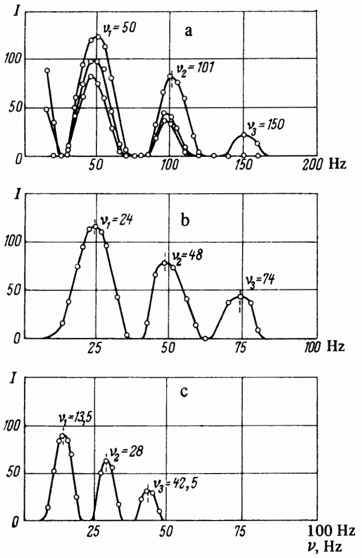

FIG. 2. Dependence of the current of negative ions in He^3-He^4 solutions on the pulse frequency, obtained at various temperatures for various concentrations of He^3: a) T = 1.537°K, c = 6.3%; b) T = 1.004°K, c = 19.7%; c) T = 0.493°K, c = 19.7%. The top part of the figure shows the gradual rise of the peak amplitudes with time, observed after a reversal of the electric field. E = 300 V/cm.

stat. The temperature in the copper chamber was maintained constant to at least ±0.003°K. The temperature was determined from the vapor pressure of the He^3 measured with a mercury manometer and a McLeod gauge connected by a tube of 3 mm diameter. A thermomolecular correction was made to the measured vapor pressure. The temperature was measured on the T_{62} scale. The value of the temperature was found to within ±0.003°K.

An He^3—He^4 mixture was condensed in the chamber 3 along a tube 5 and a capillary 6 of 0.5 mm diameter. This mixture was first passed through a carbon trap and any entrained carbon particles were then removed by a filter. He^3—He^4 solutions containing 0.75—39.9 mol.% He^3 were prepared by mixing the pure isotopes He^3 and He^4. The concentration of He^3 in solutions with up to 0.75 mol.% of this isotope was determined to within ±0.02%; in other solutions the concentration of He^3 was found to within ±0.1%.

After the completion of the condensation of a solution, the operation of the shutters was checked under dc conditions. Then the required temperature was established and the dependence of the ion current on the frequency of the pulses reaching the shutters was determined.

Typical dependences of the current on the pulse frequency are presented in Fig. 2 for negative ions. The results in this figure refer to solutions of 6.3 and 19.7% concentrations and to three different temperatures. At each of these temperatures and concentrations, the frequency dependence of the ion current had a series of maxima and each of these corresponded to the condition when the time of flight of an ion between the two shutters became equal to or a multiple of the period of the pulses reaching the shutters. These maxima were equidistant along the frequency scale to an accuracy of not less than 3%. When the electric field direction was reversed, a current of opposite charges did not appear immediately[3] but after an interval which amounted to ~10 min for low temperatures and weak solutions and which increased to one hour for concentrated solutions at high temperatures. If the frequency characteristic of the ion current was recorded during this recovery time, the maxima were not observed for some time and then the first maximum appeared, followed by the second, etc. The currents corresponding to these maxima increased continuously. The upper part of Fig. 2 shows this process of gradual growth of the maxima, which demonstrates that, in spite of the fact that the amplitudes of maxima increased by a factor of 1.5, their positions on the frequency scale did not change by more than 6%. These positions were measured when the growth of the maxima slowed down appreciably. In spite of the fact that the maxima continued to grow slowly during the measurements,[3] such growth did not affect their positions along the frequency scale.

The time of flight between the two shutters was deduced from the first maximum. The inclusion of the other maxima resulted in a correction not greater than 3%.

The scatter of the mobility values obtained in different measurements did not exceed ±5%. The absolute error could not be determined accurately because the field distribution in the direct vicinity of the grids, and particularly in the shutters, was not known. However, since the ratio of the shutter "thickness' to the drift length was 0.1 and a numerical calculation of the field inhomogeneity in the drift space gave a value of ~3%, it was concluded that the absolute error probably did not exceed 10—15%. The error was likely to be smaller because the values of the mobility of ions in He^4 obtained in the present investigation were found to agree, to within 5%, with the values obtained earlier using a measuring cell of different drift length[3], as well as with the values obtained using a measuring cell of a different configuration.[6]

1. RESULTS OF MEASUREMENTS

The mobility of charged particles in He^3—He^4 solutions was measured in fields up to 300 V/cm, with the exception of the solutions containing 11.05 and 19.7% He^3, in which measurements were carried out in fields up to 400 V/cm. Throughout this range of fields, the drift velocity of ions, v_D, was a linear function of the field E and, therefore, the mobility was independent of the field.

A weak electric field is defined by the inequality

$$v_D \ll v_T, \tag{1}$$

where v_T is the thermal velocity of ions. In weak electric fields, the drift velocity of ions is directly proportional to the field. However, measurements of the velocity of charged particles in He^4 have demonstrated that the dependence of v_D on E is linear even when the inequality (1) is violated (see, for example,[7]). This phenomenon was also observed in solutions. Thus, for example, the dependence $v_D(E)$ for positive ions in a solution containing 0.75% He^3 kept at 0.4°K remained linear up to velocities of 20 m/sec although the thermal velocity at this temperature was only ≈ 8 m/sec. Apart from the 0.75% solution, the limiting velocities of ions in the investigated solutions did not exceed 3 m/sec;

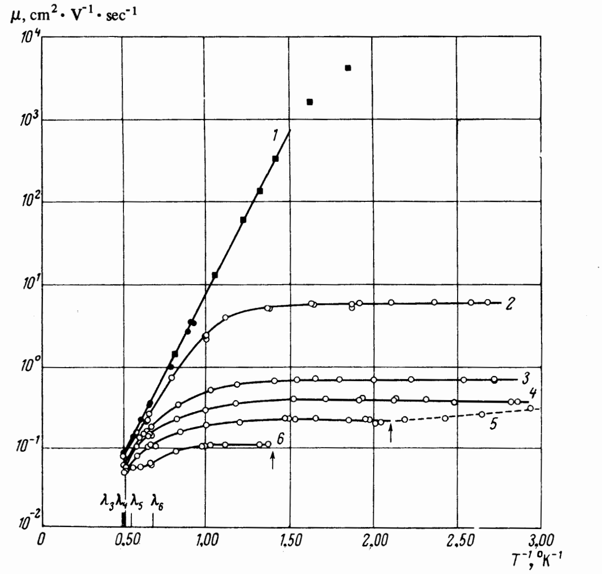

FIG. 3. Temperature dependences of the mobility of positive ions in He^4 and in He^3-He^4. Curve 1 represents the results for He^4: the continuous line and ■ are the results taken from [6]; ● are the results of the present investigation. Curves 2-6 represent He^3-He^4 solutions containing 0.75, 6.3, 11.05, 19.7, and 39.9 mol.% He^3, respectively. The arrows in this and later figures indicate the stratification temperatures of the solutions. The points λ_3-λ_6 represent the λ-transition temperatures of the solutions.

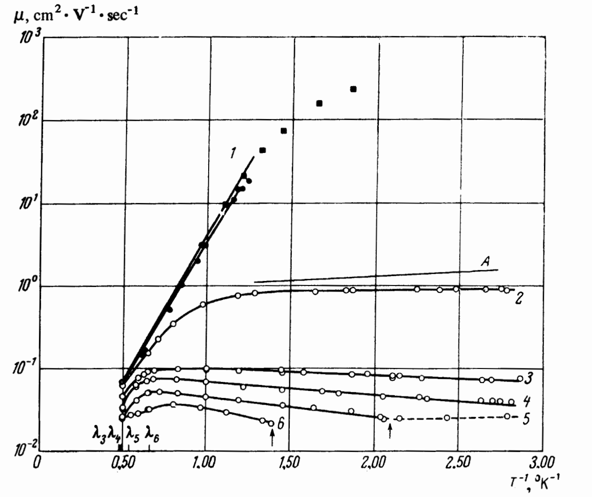

FIG. 4. Temperature dependence of the mobility of negative ions in He^4 and He^3-He^4. Curve 1 represents He^4: ■ are the results taken from [6]; ● are the results of the present investigation. Curves 2-6 represent He^3-He^4 solutions with 0.75, 6.3, 11.05, 19.7, and 39.9 mol.% He^3, respectively. Curve A represents the calculations based on Eqs. (2) and (8).

in the case of negative ions in concentrated solutions, the velocities reached only 10 cm/sec.

The results obtained are presented in Fig. 3, which gives the dependences of the mobility of positive ions on the reciprocal of temperature for pure He^4 (curve 1) and for He^3—He^4 solutions containing from 0.75 to 39.9% He^3 (curves 2—6). Our results for He^4 were in good agreement with the results of our earlier investigation[3] and with the results reported by Reif and Meyer.[6]

Throughout the investigated range of temperatures, the mobility of positive ions μ^+ in He^3—He^4 solutions was less than their mobility in He^4. When the temperature was lowered, the mobility of ions in solutions first increased and then became practically independent of temperature, beginning from some particular value of the temperature. This was observed for all solutions with the exception of that containing 11.05% He^3. The mobility μ^+ in the 11.05% He^3 solution tended to decrease slightly (within 10%) when the temperature was reduced below about 0.5°K.

The value of μ^+ for T for the He^3—He^4 solutions containing 19.7 and 39.9% He^3 could be regarded as independent of temperature only up to the temperatures of stratification of the solutions, indicated by the arrows in Fig. 3. The mobility μ^+ represented by curve 5 began to rise when the temperature was lowered below the stratification point. This was due to the fact that the measuring cell was then located in the lower phase of the stratified solution and this phase had a lower concentration of the light isotope He^3. This part of curve 5 is shown dashed.

As mentioned earlier, the presence of He^3 atoms reduced the mobility of ions in superfluid helium at all the investigated temperatures and concentrations. This reduction in the mobility was particularly strong at low temperatures. Thus, the ratio of the mobility of positive ions in He^4 to the corresponding mobility in a solution with 0.75% He^3 was $\sim 10^3$ at 0.5°K; for a solution with 19.7% He^3, this ratio reached 10^5.

Figure 4 shows the temperature dependences of the mobility of negative ions in pure He^4 (curve 1) and in the He^3—He^4 solutions (curves 2—6). The results obtained for He^4 were in satisfactory agreement with those reported by Douglass[8] but they differed somewhat from the results of Reif and Meyer.[6]

The mobility of negative ions μ^- in pure He^4 and in the solutions was lower than the mobility of positive ions. The addition of He^3 reduced the mobility of negative ions in superfluid helium.

The nature of the behavior of the mobility of negative ions in the 0.75% He^3 solution was similar to the behavior of the mobility of positive ions. When the temperature was lowered, the mobility of negative ions first increased and then, beginning from some temperature, practically ceased to depend on the temperature. The dependence $\mu^-(1/T)$ was different for some concentrated solutions. When the temperature was lowered, μ^- first increased, passed through a maximum, and then began to fall. This reduction of the mobility with decreasing temperature was exhibited more strongly by the solutions with higher concentrations of He^3. The reduction was $\sim 25\%$ for the 6.3% He^3 solution but for the 19.7% solution the value of μ^- decreased in the region of the stratification temperature to about 50% of its maximum value.

2. DISCUSSION OF RESULTS

Reif and Meyer[6] demonstrated that, above 0.7—0.9°K, the mobility of ions in superfluid He^4 is governed mainly by the scattering of ions on rotons but at lower temperatures phonons begin to contribute to the scattering. Such a contribution begins to be felt at the temperatures at which the dependences of log μ on 1/T for He^4 depart from linearity (curve 1 in Fig. 3 and curve 1 in Fig. 4). In He^3—He^4 solutions, the interaction of ions with rotons and phonons is supplemented by the interaction of ions with impurity excitations associated with the presence of He^3. The scattering by such impurity excitations is the dominant process in solutions with 0.75—40% He^3 at temperatures below 0.7—0.8°K. This follows particularly from Figs. 3 and 4, which show that, even in the weakest solution, the mobilities of positive and negative ions are at these temperatures, approximately 100 times lower than their values in pure He^4. The mobility of ions scattered by impurity excitations will be denoted by μ_3.

A. Positive Ions

Figure 5 shows the temperature dependences of the product μ_3^+c obtained in the present investigation and by Neeper and Meyer.[4] The mobility of positive ions in the He^3—He^4 solutions remains practically constant in the temperature range 0.3—0.7°K. The only exception is the solution with 11.05% He^3, in which the mobility begins to decrease slightly starting from 0.5°K. Below 0.3°K, the value of μ_3^+ of the solutions begins to fall slightly with decreasing temperature, as reported in[4].

The experimentally determined values of μ_3^+c may be compared with the values of μ_3^+c calculated using a theory given in[9,3]. According to such theoretical calculations, the formula for μ_3^+ of weak solutions in the Boltzmann range of temperatures, obtained on the assumption that impurities collide with classical hard-sphere ions, has the following form:

$$\mu_3 = \frac{3e}{8\pi(R+a_3)^2}\,\frac{\pi^{1/2}}{N_3(2m_*kT)^{1/2}} \tag{2}$$

Here, e is the electronic charge; N_3 is the number of dissolved He^3 atoms per unit volume; $m_* = 2.4m_3$[10] is the effective mass of an He^3 atom in liquid helium; a_3 is the atomic "radius" of He^3; k is the Boltzmann constant; R is the radius of an ion, which—in accordance with the electrostatic model of a positive charge[11]—is given by

$$R_+ = \left[\frac{1}{2}\,\frac{N\alpha}{V_0}\,\frac{e^2}{P_m}\right]^{1/4}, \tag{3}$$

where $N\alpha = 0.123$[12] is the molar polarizability of helium; V_0 is the molar volume; P_m is the solidification pressure of liquid helium. At 1°K, the radius R_+, calculated by means of Eq. (3), is 6.6 Å. The results of the calculations of μ_3^+c by means of Eq. (2) are given in Fig. 5 (curve A). In spite of the satisfactory agreement between the calculated and experimental values of μ_3^+ at 0.8°K, it is evident from Fig. 5 that, when the temperature is lowered, the theoretical value of μ_3^+c increases more rapidly than the experimental value.

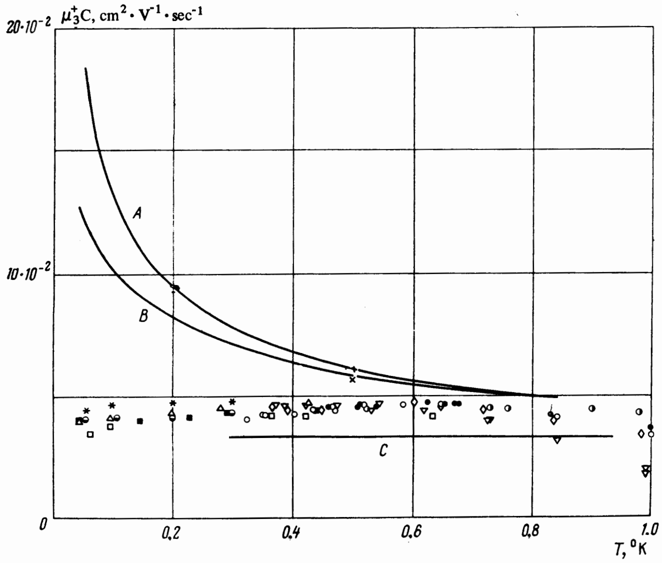

FIG. 5. Temperature dependences of the product μ_3^+c for He^3-He^4 solutions. Concentration of He^3 in the solutions: X) 1.3×10^{-5}, +) 5.1×10^{-5} taken from [1]; ◒) 1.55×10^{-4}, ■) 1.82×10^{-4}, *) 2.9×10^{-3}, △) 2.89×10^{-2}, □) 4.46×10^{-2} taken from [4]. Results of the present investigation: ▽) 0.75%; ◇) 6.3%; ○) 11.05%; ●) 19.75%; ◑) 39.9%. Curve A represents the calculations based on Eq. (2) for $R_+ = 7$ Å; curve B represents the calculations based on Eq. (2) but including quantum corrections; curve C represents the calculations based on Eqs. (6) and (7).

Retaining the model of hard spheres, we can refine the expression (2) for the mobility by making a suitable allowance for the wave properties of the impurity excitations, which begin to show in the range of temperatures of interest to us, because an increase in the characteristic thermal wavelength λ of the impurity excitations should result in an increase of the transport cross section for the scattering of impurities by hard spheres: this cross section should rise from its classical value πR_+^2, which applies in the limit $\lambda \ll R_+$, to the quantum value $4\pi R_+^2$ for $\lambda \gg R_+$. In practice, this effect can be allowed for by the use of a general formula for the transport scattering cross section expressed in terms of the scattering phases δ_l[13] (the form of the δ_l phases for the scattering by hard spheres is given, for example, in[14]). Substituting this cross section into a general expression for the mobility given in[9] and performing the necessary integration, we obtain a value of the mobility which is more accurate than that given by Eq. (2). The results of such more accurate calculations are represented by curve B in Fig. 5, which applies to $R_+ = 7$ Å. This curve exhibits a weaker temperature dependence but the agreement with the experimental data at low temperatures is still poor.

The main cause of the disagreement between the experimental and calculated mobilities is obviously the simplified representation of the nature of the ion-impurity interaction. We shall demonstrate this by calculating the mobility of positive ions in weak He^3—He^4 using a more realistic model of the interaction between positive ions and impurity excitations than the model of hard spheres.

The energy of the interaction between an ion and an impurity atom of He^3 consists of the hydrostatic and polarization components:

$$U(r) = P(r)V_3 - E\mathcal{P}. \tag{4}$$

Here, P(r) is the pressure at a distance r from a charge; V_3 is the volume occupied by an atom of He^3 in a solution; $\mathcal{P} = \alpha_3(E)$ is the polarization of such an atom, where α_3 is the atomic polarizability of He^3. The formula for U(r) given by Eq. (4) is meaningful only at distances greater than the atomic spacing. Since, in the case of sufficiently weak solutions, the pressure P(r) is described satisfactorily by the approximate expression[11]

$$P(r) = \alpha_4 e^2 / V_4 r^4, \tag{5}$$

where $\alpha_4 = \alpha_3 = \alpha$ is the atomic polarizability of helium, V_4 is the volume per one He^4 particle in liquid helium, and the field of a point charge is e/r^2, it follows that

$$U(r) = \frac{\alpha e^2}{r^4}\left(\frac{V_3}{V_4} - 1\right) \equiv \frac{\varkappa}{r^4}. \tag{6}$$

At the saturation vapor pressure, we have $V_3/V_4 - 1 = 0.28$[15] and $\kappa = 1.36 \times 10^{-44}$ erg-cm^4. Since $\kappa > 0$, the interaction between a positive ion and an impurity excitation is repulsive. This ignores the fact that the quantity $V_3/V_4 - 1$ may, in general, depend on the pressure.

Using the potential given by Eq. (6), we can estimate the ratio of the characteristic lengths in our problem: the thermal wavelength of the impurity excitations λ and the minimum approach distance r_0 between an impurity excitation and an ion (measured from the center of the ion). At $T = 1°K$, the thermal wavelength is $\lambda = 7.5 \times 10^{-8}$ cm. As far as the value of r_0 is concerned, it is obvious that, in the classical limit, a scattering center is most closely approached by those particles whose impact parameter is zero. In this case, the value of r_0 is found from the equation $1 - 2U(r)/m_* v_T^2 = 0$,[16] where $U(r) = \kappa/r^4$ and v_T is the thermal velocity of He^3 impurities. At $T = 1°K$, we obtain the value $r_0 = 9 \times 10^{-8}$ cm. Comparing $2\pi r_0$ with λ, we find, on the basis of the estimates just given, that $2\pi r_0/\lambda = 7.5 > 1$. Thus, in the vicinity of $T \sim 1°K$, the scattering of impurities by positive ions may be regarded as quasiclassical. Similar estimates of r_0 show that at temperatures $T \approx 0.5-2°K$ the impurity excitations in liquid helium are scattered by a simple power-law potential without reaching the cores of ions of 6–7 Å radius (such cores are usually assumed to be hard spheres).

The properties of systems with an interatomic interaction of the κ/r^4 type with $\kappa > 0$ have been investigated thoroughly. Thus, we can show easily that the transport scattering cross section is, in this case, inversely proportional to the velocity of an impurity particle[16] and, therefore, that the mobility of ions is independent of the temperature. This result differs basically from the hard-sphere model, which predicts a temperature dependence of the mobility which is of the $T^{-1/2}$ type.

A quantitative description of the motion of ions in He^3-He^4 solutions can be given using the results of Maxwell[17], who investigated the mutual diffusion of two gases assuming that the interaction between atoms is described by a repulsive potential of the κ/r^4 type. The expression for the mobility of ions in a gas of impurity excitations can be written in the following form:

$$\mu_3 = e / 5.32 N_3 \sqrt{m_* \varkappa}. \tag{7}$$

Equation (7) is valid in the classical limit when $2\pi r_0/\lambda \gg 1$. As demonstrated earlier, this ratio is 7.5 at 1°K; at 0.1°K it drops to 4.0. Thus, the inequality $2\pi r_0/\lambda \gg 1$ is satisfied reasonably well at temperatures $T \sim 1°K$. In the $T \le 0.1°K$ range, the mobility should be calculated by the quantum method. The values of $\mu_3^+ c$, calculated in accordance with Eq. (7), are represented by curve C in Fig. 5. The calculated value $\mu_3^+ c = 3.3 \times 10^{-2}$ cm^2 V^{-1}sec^{-1} is in satisfactory agreement with the experimental values of $\mu_3^+ c$. This can be regarded as supporting evidence that the interaction between positive ions and impurity excitations in He^3-He^4 solutions is indeed described approximately by a potential of the type given by Eq. (6).

It must be stressed that the condition $T \gg T_F$ (T_F is the Fermi degeneracy temperature of solutions) is violated by the solutions containing 19.7 and 39.9% He^3 in that range of temperatures in which the mobility of positive ions is independent of the temperature. Moreover, beginning from about 10% He^3, the condition $l^+ > R^+$ is also violated (l^+ is the characteristic range of positive ions in a gas of excitations). Nevertheless, the value of $\mu_3^+ c$ found experimentally for the concentrated solutions is the same as that for the weak solutions.

B. Negative Ions

It is evident from Fig. 4 that the mobility of negative ions in the 0.75% He^3 solution is practically independent of the temperature in the 0.35–0.7°K range (curve 1). In this range, the value of μ_3^- is approximately one-tenth of the value of μ_3^+, and the value of $\mu_3^- c$ for this solution is $(7.0 \pm 0.4) \times 10^{-3}$ cm^{2}V^{-1}sec^{-1}. This value should be compared with the results given by Meyer and Reif,[1] who report $\mu_3^- c = 7.3 \times 10^{-3}$ and 9.2×10^{-3} cm^{2}V^{-1}sec^{-1} at 0.5°K for solutions with, respectively, 1.3×10^{-5} and 5.1×10^{-5} of He^3. It should also be compared with the results of Neeper and Meyer,[4] who give $\mu_3^- c = (6-9) \times 10^{-3}$ cm^{2}V^{-1}sec^{-1} for solutions containing $1.5 \times 10^{-4}-4.5 \times 10^{-2}$ of He^3.

The radius of a negative ion is given, in the bubble model, by the following expression:[18]

$$R_- = (\pi\hbar^2 / 8m\gamma)^{1/4} \approx 20\ \text{Å}, \tag{8}$$

where m is the mass of an electron; γ is the surface tension of liquid helium; $\hbar$ is the Planck constant. The mean free path of a negative ion in a gas of impurity excitations is $l_- = 5$ Å for the 0.75% He^3 solution. Thus, in this case, $l^- < R_-$. Nevertheless, if we use Eqs. (2) and (8) to find μ_3^- in this solution, we find that the value of μ_3^- calculated for 0.7°K is approximately 20% larger than the experimental value, and that the discrepancy between the calculated and experimental values increases with decreasing temperature, exactly as in the case of positive ions.

The values of μ_3^- for the more concentrated solutions (5–6% He^3) decrease with falling temperature in the range 0.35–0.7°K.

We may assume that this effect is due to the existence of impurity excitations on the surfaces of negative-ion bubbles and that the dispersion law of these excitations is different from the law which they obey in the bulk of a solution.[19] These excitations reduce the surface tension of solutions when the temperature is lowered.[20] Using the results of Zinov'eva and Boldarev,[20] we find that the lowering of the temperature from 1.5 to 0.5°K increases by ~ 7% the radius R_- found by applying Eq. (8) to the 19.7% He^3 solution; μ_3^- decreases correspondingly by ~ 15%. On the other hand, the experimentally observed decrease in μ_3^- is ~ 80% in the same range of temperatures. Thus, the influence of the impurity excitations on the surface tension cannot, by itself, explain the dependence $\mu_3^-(T)$ at low temperatures. A correct interpretation of the experimental results requires not only an allowance for the changes which occur in the structure of a negative-ion bubble in solutions but also the knowledge of the nature of the interaction between the negative ions and impurity excitations.

3. INFLUENCE OF He^3 ON THE PARAMETERS OF THE ENERGY SPECTRUM OF SUPERFLUID HELIUM

The ion mobilities obtained in the present investigation can be used to estimate the energy gap in the roton part of the spectrum of the solutions on the assumption that the boson branch of this spectrum is analogous to the spectrum of He^4. As in the case of He^4, it is assumed that the roton part of the spectrum of the solutions has a roton minimum but that the parameters of this minimum depend on the concentration of He^3.

In pure He^4 at sufficiently high temperatures, the scattering of ions is dominated by the interaction with rotons. In this case, the ion mobility is given by the following expression:[21]

$$\mu_r = \frac{3e}{4\pi R^2}\frac{1}{p_0 N_r}, \tag{9}$$

where p_0 is the zero-point momentum of a roton and N_r is the number of rotons per unit volume. Since[22]

$$N_r = \frac{2(\mu_0 kT)^{1/2} p_0^2}{(2\pi)^{3/2}\hbar^3} e^{-\Delta/T}, \tag{10}$$

where μ_0 is the effective mass of a roton and Δ is the amplitude of the roton minimum, it follows that

$$\mu_r = \frac{3e}{4\sigma_{ir}}\frac{(2\pi)^{3/2}\hbar^3}{2(\mu_0 kT)^{1/2} p_0^3} e^{\Delta/T}. \tag{11}$$

Here, $\sigma_{ir} = \pi R^2$ is the cross section for the scattering of an ion by a roton.

In practice, the dependence of log μ_r on $1/T$ is linear and the slope of this dependence gives the value of Δ. The parameter Δ, determined from the results of the present investigation, is 8.8 deg K for positive ions and 7.8 deg K for negative ions, in satisfactory agreement (in both cases) with the results reported in[6,8]. It is worth noting a considerable difference between the values of Δ for positive and negative ions, which cannot be explained within the framework of the current theories.

The values of μ_r^+ for positive ions, calculated using Eq. (11), are several times larger or smaller than the experimental values but the temperature dependence is predicted correctly. As mentioned in[3], this may be regarded as an indication that the hard-sphere model does not describe accurately the interaction of ions with rotons.

A comparison of the corresponding quantities in the case of negative ions results in better agreement between the numerical values of μ_r^- but the theoretical and experimental temperature dependences differ considerably.

The value of Δ for the He^3—He^4 solutions can be estimated only if we can find μ_r from the ion mobility μ in these solutions. Above 0.8°K, the mobility of ions in the solutions is governed by their scattering on rotons and impurity excitations. If the scattering centers are independent of one another, the mobility can be represented in the form

$$\mu^{-1} = \mu_r^{-1} + \mu_3^{-1}. \tag{12}$$

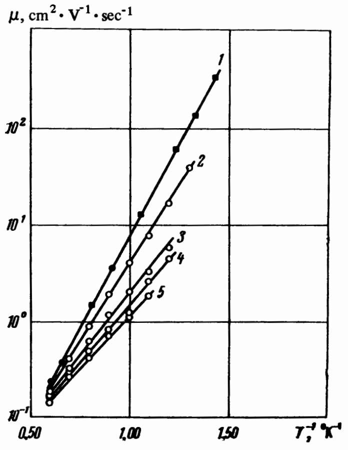

FIG. 6. Temperature dependences of the mobility of positive ions scattered by rotons in He^4 and He^3-He^4. Curve 1 represents He^4. Curves 2-5 represent He^3-He^4 solutions with 0.75, 6.3, 11.05, and 19.7 mol.% He^3, respectively.

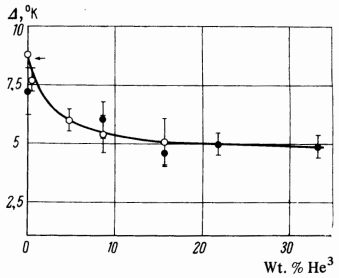

FIG. 7. Dependence of the energy gap in the roton part of the spectrum of the solutions on the weight concentration of He^3: O) results of the present investigation; O) results reported in [24]. The arrow indicates the value of Δ found from the inelastic scattering of neutrons in liquid helium. [23]

Assuming that the contribution of impurity excitations to the scattering of positive ions remains the same at higher temperatures, we can find the values of $1/\mu_r^+$ from the difference $1/\mu - 1/\mu_3^+$. The results of such a calculation of the quantity μ_r^+ for the investigated solutions are given in Fig. 6. It is evident that the dependences log $\mu_r^+(1/T)$ are linear for pure He^4 (curve 1) as well as for the solutions, but the values of μ_r^+ for the solutions are smaller than the corresponding values for pure He^4. The influence of He^3 on μ_r^+ is fairly strong. Thus, for example, the value of the mobility in the 6.3% He^3 solution at 0.8°K is approximately one-tenth of that in He^4.

The dependences log $\mu_r^+(1/T)$ can be used to calculate the values of Δ for the He^3—He^4 solutions. The results of such a calculation for positive ions are presented in Fig. 7. We can see that Δ does indeed depend on the composition and that it decreases with increasing He^3 concentration. The value of Δ for pure He^4 is in good agreement with the energy gap in the roton part of the spectrum deduced in experiments involving inelastic scattering of neutrons in liquid helium.[23]

The obtained dependence $\Delta(c)$ is in satisfactory agreement with the dependence deduced from measurements of the velocity of the fourth sound in He^3—He^4 solutions[24] (Fig. 7).

A similar analysis of the results obtained for negative ions demonstrates a concentration dependence $\Delta^-(c)$ although it is difficult to obtain accurate values of Δ^- because of the more complex dependence $\mu_3^-(T)$.

Slyusarev and Strzhemechnyĭ[25] considered theoretically the characteristic features of the energy spectrum of He^3—He^4 solutions and also concluded that the presence of He^3 affects the value of Δ. They did not deduce the exact form of the dependence $\Delta(c)$ but they predicted $\partial\Delta/\partial c < 0$ for $c \to 0$ and $\partial\Delta/\partial c \to 0$ at sufficiently high concentrations, i.e., that the curve $\Delta(c)$ should exhibit saturation. These conclusions are in qualitative agreement with the results of the present investigation.

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Translated by A. Tybulewicz
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