PROPERTIES OF THIN SUPERCONDUCTING FILMS IN HIGH FREQUENCY FIELDS

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The linear response to a weak rapidly alternating field in thin superconducting films ($d \ll \xi$) is found for arbitrary static magnetic field and current. The dependence of the critical current on the magnetic field strength is also obtained. The results are valid for a mean free path $l \ll (\xi d)^{1/2}$.

1. INTRODUCTION

A rapidly alternating field causes an alternating current to flow in a superconductor; the amplitude of this current greatly depends on the value of the static magnetic field and on the current. Direct current and a strong magnetic field give rise to a correction to the ordering parameter even in a weak rapidly-alternating field. This change of Δ introduces a contribution to the current. We shall determine the connection between the current and the vector potential for thin superconducting films $d \ll \xi$, where d is the thickness of the film and $\xi = v/\pi\Delta$ is the ordering parameter. When $l \ll (\xi d)^{1/2}$ (l—mean free path), this connection holds for arbitrary fields and currents. When $\xi_0 \gg l \gtrsim (\xi d)^{1/2}$, there is an upper bound on the current and on the field. For strongly contaminated superconductors ($l \ll d$), the results do not depend on the character of reflection of the electrons from the surface. When $l \gtrsim d$, there arises a dependence on the law governing the reflection of the electrons from the surface. Nevertheless, it is possible to obtain a general answer for an arbitrary law of reflection of electrons from the surface. Both in the static case and in the presence of an alternating field, the current averaged over the cross section of the film is expressed in terms of two constants. The same constants determine the conductivity of the film in the normal state and the value of the critical field. They can be determined experimentally. The case of diffuse and specular reflection of electrons from the film boundaries has been analyzed in detail.

We also obtain formulas that make it possible to determine the dependence of the critical current on the magnetic field at $l \ll (d\xi)^{1/2}$.

2. DEPENDENCE OF THE CRITICAL CURRENT ON THE MAGNETIC FIELD

To calculate the linear response to a rapidly alternating field, it is necessary to know the Green's function of the superconductor in an arbitrary constant magnetic field and in the presence of current in the superconductor. To find this function, it is convenient to use the Gor'kov equations for the Green's functions, integrated over the energies ξ[1,2]:

$$\left(\mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)\hat{G}_{\mathbf{p}}(\mathbf{r}) + \hat{\omega}\,\hat{G}_{\mathbf{p}}(\mathbf{r}) - \hat{G}_{\mathbf{p}}(\mathbf{r})\,\hat{\omega} = 0,$$

$$\hat{\omega} = \omega\tau_z - ie(\mathbf{vA})\tau_z - i\hat{\Delta} + in\Sigma_{\mathbf{pp}}(\mathbf{r}),$$

$$\Sigma_{\mathbf{pp'}}(\mathbf{r}) = \chi_{\mathbf{pp'}} - \frac{i\nu}{4}\int \chi_{\mathbf{pp}_1}\hat{G}_{\mathbf{p}_1}(\mathbf{r})\Sigma_{\mathbf{p}_1\mathbf{p'}}(\mathbf{r})\,d\Omega_{\mathbf{p}_1},$$

$$\hat{\Delta} = \begin{pmatrix} 0 & \Delta \\ -\Delta^* & 0 \end{pmatrix}, \quad \hat{G}_{\mathbf{p}}^2(\mathbf{r}) = 1, \quad \mathrm{Sp}\,\hat{G}_{\mathbf{p}}(\mathbf{r}) = 0, \tag{1}$$

where n is the impurity concentration, $\mathbf{A} = \mathbf{A}(\mathbf{r})$ is the vector potential, v is the velocity of the electron on the Fermi surface, $\nu = mp_0/2\pi^2$ is the density of states on the Fermi surface, and $\chi_{pp'}$ is expressed in terms of the scattering amplitude

$$f_{\mathbf{pp'}} = \chi_{\mathbf{pp'}} - \frac{i\nu}{4}\int \chi_{\mathbf{pp}_1} f_{\mathbf{p}_1\mathbf{p'}}\,d\Omega_{\mathbf{p}_1}.$$

We note that the system of equations (1) differs from those given in[2] in the immaterial substitutions $\mathbf{p} \to -\mathbf{p}$ and $\mathbf{A} \to -\mathbf{A}$, in order to obtain agreement with the standard notation.

We choose a coordinate system with the z axis across the film and with $z = \pm d/2$ on the boundaries of the film, where d is the film thickness. We confine ourselves to the case

$$eAl \ll 1, \quad eAd \ll 1. \tag{2}$$

In a wide range of free paths

$$d^2/\xi \ll l \ll (\xi d)^{1/2}$$

the condition (2) is satisfied for all fields, up to critical. Under condition (2), the Green's function can be written in the form of an expansion in powers of small parameters eAl and eAd[2,3]. In the principal approximation in the parameters l/ξ_0 and eAl, the Green's function takes the form

$$\hat{G}_{\mathbf{p}}(\mathbf{r}) = (\mathbf{n}_\omega^0 \boldsymbol{\tau}),$$

where the unit vector $\mathbf{n}_\omega^0$ does not depend on the coordinates $\mathbf{r}$ and the angles of the vector $\mathbf{p}$. In this approximation, the system of equations (1) is satisfied for an arbitrary vector $\mathbf{n}_\omega^0$, and can be obtained from a consideration of the next higher terms in the expansion of $\hat{G}_p(\mathbf{r})$ in powers of the parameters eAl and eAd:

$$\hat{G}_{\mathbf{p}}(\mathbf{r}) = (\mathbf{n}_\omega^0\boldsymbol{\tau}) + [i(\mathbf{n}_1\boldsymbol{\tau}) - (\mathbf{n}_2\boldsymbol{\tau})]\left[\frac{1}{v}(\mathbf{v}\chi_1(x,z)) + \chi_0(x,z)\right]$$

$$- [i(\mathbf{n}_1\boldsymbol{\tau}) + (\mathbf{n}_2\boldsymbol{\tau})]\left[\frac{1}{v}(\mathbf{v}\psi_1(x,z)) + \psi_0(x,z)\right] + \ldots, \tag{3}$$

where the unit vectors $\mathbf{n}_i$ depend only on ω; $x = v^{-1}(\mathbf{e}_3 \cdot \mathbf{v})$; $\mathbf{e}_3$ is a unit vector along the z axis; $\chi_1, \psi_1 \sim \mathbf{A}$; $\chi_0, \psi_0, \sim A^2$. In the general case, the vector $\mathbf{n}_\omega^0$ depends on the free path and on the wall of reflection of the electrons from the film boundaries. When $l \ll d$, the dependence on the reflecting properties of the surface disappears. Without loss of generality, we can choose the vectors n_i in the form

$$\mathbf{n}_\omega^0 = (\cos\tilde\theta,\ \beta\sin\tilde\theta,\ \alpha\sin\tilde\theta),\quad \mathbf{n}_1 = (\sin\tilde\theta,\ -\beta\cos\tilde\theta,\ -\alpha\cos\tilde\theta),\quad \mathbf{n}_2 = (0, \alpha, -\beta),\quad \alpha^2+\beta^2=1. \tag{4}$$

The ordering parameter Δ does not depend on the coordinates in the principal approximation and can therefore be chosen to be real. Substituting the expression (3) for the Green's function $\hat{G}_p(\mathbf{r})$ in the system (1), we get

$$\frac{\partial}{\partial z}\chi_1 + \frac{1}{x}\Big[\frac{1}{l}\chi_1 - \frac{3}{4l_1}\int_{-1}^{1}(1-x_1^2)\chi_1(x_1)\,dx_1\Big] = -\frac{ie}{x}[\beta + i\alpha\cos\tilde\theta]\mathbf{A}(z),$$

$$\frac{\partial}{\partial z}\psi_1 - \frac{1}{x}\Big[\frac{1}{l}\psi_1 - \frac{3}{4l_1}\int_{-1}^{1}(1-x_1^2)\psi_1(x_1)\,dx_1\Big] = \frac{ie}{x}[\beta - i\alpha\cos\tilde\theta]\mathbf{A}(z), \tag{5}$$

where the mean free paths l and l_1 are expressed in the usual manner in terms of the cross section

$$n\sigma = l^{-1},\ n\sigma_1 = l_1^{-1},\quad \sigma_{\mathbf{pp}'} = \frac{1}{4\pi}[\sigma + 3\sigma_1\cos\widehat{\mathbf{pp}'} + \ldots].$$

We shall not need the functions χ_1 and ψ_1 themselves, but only their integrals with respect to the angles with the function $(1 - x^2)$. We therefore introduce the notation

$$\tilde\chi_1(\omega) = \int_{-1}^{1}(1-x^2)\chi_1(x,\omega)\,dx,\qquad \tilde\psi_1 = \int_{-1}^{1}(1-x^2)\psi_1(x,\omega)\,dx. \tag{6}$$

At an arbitrary law of reflection of the electrons from the film boundaries, the solution of (5) for the functions $\tilde\chi_1$ and $\tilde\psi_1$ is

$$\tilde\chi_1(\omega) = -\frac{4iel_1}{3}(\beta + i\alpha\cos\tilde\theta)(\hat{L}^{-1}-1)\mathbf{A}(z),\qquad \tilde\psi_1(\omega) = -\frac{4iel_1}{3}(\beta - i\alpha\cos\tilde\theta)(\hat{L}^{-1}-1)\mathbf{A}(z), \tag{7}$$

where the integral operator $\hat{L}$ is determined only by the properties of the film in the normal state, since the left side of the system (5) and the boundary conditions do not contain Δ, and can be obtained if one knows the law of deflection of the electrons through the surface of the film. We present below an explicit form of the operator $\hat{L}$ of both diffuse and specular reflection of the electrons from the film boundaries.

In diffuse reflection, the functions χ_1 and ψ_1 satisfy the boundary conditions[3]

$$\chi_1(d/2, x<0) = \chi_1(-d/2, x>0) = \psi_1(d/2, x>0) = \psi_1(-d/2, x<0) = 0. \tag{8}$$

Integrating Eqs. (5) under boundary conditions (8), we obtain the explicit form of the operator $\hat{L}$:

$$\hat{L}f(z) = f(z) - \frac{3}{4l_1}\int_0^1\frac{dx(1-x^2)}{x}\int_{-d/2}^{d/2}\exp\Big(-\frac{|z-z_1|}{lx}\Big)f(z_1)\,dz_1. \tag{9}$$

Analogously, the operator $\hat{L}$ can be found also for specular reflection. In this case it turns out to be somewhat more complicated:

$$\hat{L}f = f - \frac{3}{4l_1}\int_0^1\frac{dx(1-x^2)}{x}\int_{-d/2}^{d/2}\Big\{\exp\Big(-\frac{|z-z_1|}{lx}\Big) + \Big[\mathrm{ch}\Big(\frac{z+z_1}{lx}\Big) + \mathrm{ch}\Big(\frac{z-z_1}{lx}\Big)\exp\Big(-\frac{d}{lx}\Big)\Big]\Big/\mathrm{sh}\Big(\frac{d}{lx}\Big)\Big\}f(z_1)\,dz_1. \tag{10}$$

For the case of a very highly contaminated superconductor ($l \ll d$), the operator $\hat{L}$ does not depend on the boundary conditions, and in this case we have

$$\hat{L} = l/l_{tr},\quad \hat{L}^{-1} - 1 = l_{tr}/l_1. \tag{11}$$

In isotropic scattering $l_1 \to \infty$ and the operator $l_1(\hat{L}^{-1} - 1)$ can be readily obtained for an arbitrary ratio l/d. For example, in diffuse reflection

$$l_1(\hat{L}^{-1}-1)f(z) = \frac{3}{4}\int_0^1\frac{dx(1-x^2)}{x}\int_{-d/2}^{d/2}\exp\Big(-\frac{|z-z_1|}{xl}\Big)f(z_1)\,dz_1.$$

We now integrate Eq. (1) with the Green's function with respect to the angles of the vector pn with respect to the coordinate z. Then the unknown functions χ_0 and ψ_0 drop out, and we obtain two additional equations for the determination of the angle $\tilde\theta$ and the constants α and β:

$$(\omega\tau_z + \Delta\tau_y)(\mathbf{n}_\omega^0\boldsymbol{\tau}) - (\mathbf{n}_\omega^0\boldsymbol{\tau})(\omega\tau_z + \Delta\tau_y) = \frac{2e^2vl_1}{3d}\int_{-d/2}^{d/2}(\mathbf{A}(\hat{L}^{-1}-1)\mathbf{A})\,dz\,\{[\beta(\mathbf{n}_2\boldsymbol{\tau}) + \alpha\cos\tilde\theta(\mathbf{n}_1\boldsymbol{\tau})]\tau_z - \tau_z[\beta(\mathbf{n}_2\boldsymbol{\tau}) + \alpha\cos\tilde\theta(\mathbf{n}_1\boldsymbol{\tau})]\}. \tag{12}$$

In the derivation of (12) we used expression (7) for the functions $\tilde\chi_1$ and $\tilde\psi_1$. From (4) and (12) we get

$$\cos\tilde\theta = 0,\quad \alpha\Delta - \beta\omega = \alpha\beta\Gamma, \tag{13}$$

where

$$\Gamma = \frac{2e^2vl_1}{3d}\int_{-d/2}^{d/2}(\mathbf{A}(z)(\hat{L}^{-1}-1)\mathbf{A}(z))\,dz.$$

For a strongly contaminated superconductor ($l \ll d$) and for $d \ll \delta$, formula (13) goes over into the well known Maki expression[4].

The ordering parameters Δ and the current density $\mathbf{j}(\mathbf{r})$ are expressed in terms of the Green's function $G_p(\mathbf{r})$. Using formulas (3), (4), (7), and (13) we obtain

$$\Delta = -\frac{i|\lambda|mp_0}{8\pi^2}T\sum_\omega \mathrm{Sp}\begin{pmatrix}0&1\\0&0\end{pmatrix}\int\hat{G}_p(\mathbf{r})\,d\Omega_p = \frac{|\lambda|mp_0}{2\pi}T\sum_\omega\beta(\omega),$$

$$\mathbf{j}(r) = -\frac{iep_0}{4\pi^2}T\sum_\omega\mathrm{Sp}\begin{pmatrix}1&0\\0&0\end{pmatrix}\int\mathbf{p}\hat{G}_p(\mathbf{r})\,d\Omega_p = -\frac{2e^2p_0^2l_1}{3\pi}(\hat{L}^{-1}-1)\mathbf{A}T\sum_\omega\beta^2(\omega). \tag{14}$$

Formulas (13) and (14) in conjunction with Maxwell's equations make it possible to obtain the dependence of the critical current on the temperature. In the simpler case of a thin film $d \ll \delta$ (δ—depth of penetration) it is possible to neglect the magnetic field due to the presence of the current. In this case the vector potential $\mathbf{A}(z)$ can be chosen in the form*

$$\mathbf{A}(z) = \mathbf{A}_0 + z[\mathbf{He}_3],$$

where $\mathbf{A}_0$ does not depend on the coordinate z and produces the average current, while $\mathbf{H}$ is the magnetic

*$[\mathbf{He}_3] \equiv \mathbf{H}\times\mathbf{e}_3$.

field. Substituting this expression for $\mathbf{A}(z)$ in formulas (13) and (14), we obtain

$$\alpha\Delta - \beta\omega = \alpha\beta\Gamma, \quad \Gamma = \frac{2e^2v}{3}[b_1A_0^2 + b_2d^2H^2],$$

$$\mathbf{J} = \int_{-d/2}^{d/2} \mathbf{j}(z)\,dz = -\frac{2e^2p_0^2}{3\pi}\mathbf{A}_0 d b_1 T \sum_\omega \beta^2(\omega), \qquad (15)$$

where

$$b_1 = \frac{l_1}{d}\int_{-d/2}^{d/2} dz\cdot(\hat{L}^{-1}-1), \quad b_2 = \frac{l_1}{d^3}\int_{-d/2}^{d/2} dz\cdot z(\hat{L}^{-1}-1)z. \qquad (16)$$

For strongly contaminated superconductors ($l \ll d$) we have

$$b_1 = l_{tr}, \quad b_2 = l_{tr}/12.$$

At an arbitrary mean free path, the constant b_1 is expressed in terms of the resistance of the film in the normal state, and the constant b_2 can be obtained, for example, from the value of the critical magnetic field near the critical temperature:

$$\sigma_N = \frac{p_0^2e^2}{3\pi^2}b_1, \quad T_c - T = \frac{\pi e^2 v}{6}b_2H_c^2d^2.$$

The critical current is attained at a value H_0 for which $\partial J/\partial A_0 = 0$. Equation (14) for Δ and formula (15) make it possible to obtain the value of the critical current for arbitrary temperatures. However, the system of Eqs. (14) and (15) is complicated and to obtain the value of J_c in the general case it is apparently necessary to carry out a numerical calculation. Near T_c we get for the critical current

$$J_c(H) = \frac{8\sqrt{2\pi}\,ep_0^2d}{63\zeta(3)}\left(\frac{b_1}{v}\right)^{1/2}\left[T_c - T - \frac{\pi e^2v}{6}b_2H^2d^2\right]^{3/2}, \quad l \ll \xi_0. \qquad (16a)$$

Expression (16a) for the critical current can also be written in the form

$$J_c(H) = J_c(0)\,[1-(H/H_c)^2]^{3/2},$$

where $J_c(0)$ is the critical current in the absence of a magnetic field, and H_c is the critical magnetic field in the absence of current. From formula (16a) we can readily obtain

$$J_c(0) = \frac{8\sqrt{2\pi}\,ep_0^2d}{63\zeta(3)}\left(\frac{b_1}{v}\right)^{1/2}(T_c-T)^{3/2}, \quad H_c = \frac{1}{d}\left[\frac{6(T_c-T)}{\pi e^2vb_2}\right]^{1/2}.$$

For thin films ($d \ll \delta$) the dependence of the critical current on the value of the magnetic field, the temperature, the mean free path, and the character of the reflection of the electrons from the surface of the film is described with the aid of two constants b_1 and b_2, which can be determined experimentally.

3. LINEAR RESPONSE TO A RAPIDLY ALTERNATING FIELD

To calculate the linear response of a superconductor to a rapidly alternating field, it is sufficient to find the corresponding response in the temperature technique to a field in the form[5]

$$\mathbf{A}_1(\mathbf{r},\tau) = \mathbf{A}_1(\mathbf{r})\exp(-i\omega_0\tau). \qquad (17)$$

At an arbitrary vector potential $\mathbf{A}(\mathbf{r},\tau)$, the system of equations for the Green's function can be written in the form

$$\left[-\xi - \tau_z\frac{\partial}{\partial\tau} + i\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right) + e(\mathbf{v}\mathbf{A})\tau_z + \hat{\Delta}(\mathbf{r},\tau)\right]\hat{G}(\tau,\tau',\mathbf{p},\mathbf{r})$$
$$- n\int_0^{1/T} d\tau_1\Sigma_{\mathbf{pp}}(\tau,\tau_1,\mathbf{r})\hat{G}(\tau_1,\tau',\mathbf{p},\mathbf{r}) = \delta(\tau-\tau'), \qquad (18)$$

$$\Sigma_{\mathbf{pp}'}(\tau,\tau',\mathbf{r}) = \chi_{\mathbf{pp}'}\delta(\tau-\tau') - \frac{iv}{4}\int\chi_{\mathbf{pp}_1}\hat{G}_{\mathbf{p}_1}(\tau,\tau_1,\mathbf{r})\,\Sigma_{\mathbf{p}_1\mathbf{p}'}(\tau_1,\tau',\mathbf{r})\,d\Omega_{\mathbf{p}_1}d\tau_1,$$

where

$$\hat{G}(\tau,\tau_1,\mathbf{r},\mathbf{r}_1) = \frac{1}{(2\pi)^3}\int\hat{G}(\tau,\tau_1,\mathbf{p},\mathbf{r})\exp(i\mathbf{p}(\mathbf{r}-\mathbf{r}_1))\,d\mathbf{p},$$

$$\hat{G}_{\mathbf{p}}(\tau,\tau',\mathbf{r}) = \frac{i}{\pi}\int\hat{G}(\tau,\tau_1,\mathbf{p},\mathbf{r})\,d\xi, \qquad (19)$$

$$(\hat{\Delta}(\mathbf{r},\tau))_{12} = -|\lambda|(\hat{G}(\tau,\tau,\mathbf{r},\mathbf{r}))_{12}, \quad (\hat{\Delta}(\mathbf{r},\tau))_{21} = -|\lambda|(\hat{G}(\tau,\tau,\mathbf{r},\mathbf{r}))_{21}.$$

Just as was done earlier[2], it turns out to be possible to write an equation for the function $\hat{G}_p(\tau,\tau',\mathbf{r})$ at arbitrary $\mathbf{A}(\mathbf{r},\tau)$. In our case

$$\mathbf{A}(\mathbf{r},\tau) = \mathbf{A}(\mathbf{r}) + \mathbf{A}_1(\mathbf{r})\exp(-i\omega_0\tau) \qquad (20)$$

and it is necessary to find a response linear in the amplitude $\mathbf{A}_1$. We therefore write down the equation for $\hat{G}_p(\tau,\tau',\mathbf{r})$ in linearized form. We put

$$\hat{G}_{\mathbf{p}}(\tau,\tau_1,\mathbf{r}) = \hat{G}_{\mathbf{p}}^0(\tau-\tau_1,\mathbf{r}) + \hat{G}_{\mathbf{p}}^1(\tau-\tau_1,\mathbf{r})\exp(-i\omega_0\tau),$$
$$\hat{\Delta}(\mathbf{r},\tau) = \hat{\Delta}(\mathbf{r}) + \hat{\Delta}_1(\mathbf{r})\exp(-i\omega_0\tau). \qquad (21)$$

For the Green's function $G_p^1(\omega,\mathbf{r})$ we obtain the system of equations

$$\left(\mathbf{v}\frac{\partial}{\partial\mathbf{r}}\right)\hat{G}_{\mathbf{p}}^1(\omega,\mathbf{r}) + \hat{\omega}(\omega+\omega_0)\hat{G}_{\mathbf{p}}^1(\omega,\mathbf{r}) - \hat{G}_{\mathbf{p}}^1(\omega,\mathbf{r})\hat{\omega}(\omega)$$
$$+ in[\Sigma_{\mathbf{pp}}^1(\omega,\mathbf{r})\hat{G}_{\mathbf{p}}^0(\omega,\mathbf{r}) - \hat{G}_{\mathbf{p}}^0(\omega+\omega_0,\mathbf{r})\Sigma_{\mathbf{pp}}^1(\omega,\mathbf{r})]$$
$$= ie(\mathbf{v}\mathbf{A}_1)[\tau_z\hat{G}_{\mathbf{p}}^0(\omega,\mathbf{r}) - \hat{G}_{\mathbf{p}}^0(\omega+\omega_0,\mathbf{r})\tau_z]$$
$$+ i[\hat{\Delta}_1\hat{G}_{\mathbf{p}}^0(\omega,\mathbf{r}) - \hat{G}_{\mathbf{p}}^0(\omega+\omega_0,\mathbf{r})\hat{\Delta}_1],$$
$$\Sigma_{\mathbf{pp}'}^1(\mathbf{r}) = -\frac{iv}{4}\int\Sigma_{\mathbf{pp}_1}^0(\omega+\omega_0,\mathbf{r})\hat{G}_{\mathbf{p}_1}^1(\omega,\mathbf{r})\Sigma_{\mathbf{p}_1\mathbf{p}'}^0(\omega,\mathbf{r})\,d\Omega_{\mathbf{p}_1}, \qquad (22)$$

where $\hat{G}_p^0(\omega,\mathbf{r})$ and $\Sigma_{pp'}^0(\omega,\mathbf{r})$ are the solutions of the system (1) in the field of the vector potential $\mathbf{A}(\mathbf{r})$.

In the diffuse-reflection model proposed by the authors[6], it is possible to obtain the boundary conditions for the system (22). To write down these conditions, it is convenient to introduce four matrices $\hat{B}_k$, which are determined from the equations

$$(\mathbf{n}_{\omega+\omega_0}^0\tau)\hat{B}_k - \hat{B}_k(\mathbf{n}_\omega^0\tau) = \rho_k\hat{B}_k, \quad k = 1,2,3,4,$$
$$\rho_1 = \rho_2 = 0, \quad \rho_3 = -\rho_4 = 2,$$
$$(\mathbf{n}_\omega^0\tau) = \frac{1}{\pi p_0}\int_{(\mathbf{pn})>0}(\mathbf{pn})\hat{G}_{\mathbf{p}}^0(\omega,\mathbf{r}_{\lim})\,d\Omega_{\mathbf{p}}, \qquad (23)$$

where the vector $\mathbf{n}$ is directed along the inward normal to the surface. Expanding the Green's function in terms of the matrices $\hat{B}_k$

$$\hat{G}_{\mathbf{p}}^1(\omega,\mathbf{r}_{\text{rp}}) = \sum_{k=1}^4 C_k(\mathbf{p},\mathbf{r}_{\lim})\hat{B}_k,$$

we find that the expansion coefficients satisfy the condition

$$C_1(\mathbf{p}) = C_1(\mathbf{p}-\mathbf{n}(\mathbf{pn})), \quad C_2(\mathbf{p}) = C_2(\mathbf{p}-\mathbf{n}(\mathbf{pn})),$$
$$p_0C_3((\mathbf{pn})>0) = \frac{1}{\pi}\int_{(\mathbf{pn})<0}C_3(\mathbf{p})\,|(\mathbf{pn})|\,d\Omega_{\mathbf{p}},$$

$$p_0C_4((\mathbf{pn})<0)=\frac{1}{\pi}\int_{(\mathbf{pn})>0}C_4(\mathbf{p})(\mathbf{pn})\,d\Omega_\mathbf{p}. \qquad (24)$$

For thin superconducting films ($d \ll \xi$) under condition (2) we shall obtain below an expression for $\hat{\Delta}_1$—corrections to the ordering parameter, and the current density j_1 at an arbitrary law of reflection of the electrons from the boundary.

In the principal approximation in the parameters l/ξ_0 and eAl, the function $\hat{G}^1_\mathbf{p}(\omega, \mathbf{r})$ is of the form

$$\hat{G}_\mathbf{p}^1(\omega,\mathbf{r})=\hat{G}_0^1,$$

where the matrix $\hat{G}^1_0$ does not depend on the coordinate $\mathbf{r}$ and the angles of the vector $\mathbf{p}$. Linearizing the system (18) and using the expression for the Green's function in the absence of an alternating field, in the principal approximation in the parameters eAl and l/ξ_0

$$\hat{G}^0(\mathbf{p},\omega,\mathbf{r})=-\left[(\xi+nf_{\mathbf{pp}}^1)+\frac{i}{2\tau}(\mathbf{n}_\omega{}^0\boldsymbol{\tau})\right]\left[(\xi+nf_{\mathbf{pp}}^1)^2+\left(\frac{1}{2\tau}\right)^2\right]^{-1},$$

we find that the matrix $\hat{G}^1_0$ satisfies the condition

$$(\mathbf{n}^0_{\omega+\omega_0}\boldsymbol{\tau})\hat{G}_0{}^1(\mathbf{n}_\omega{}^0\boldsymbol{\tau})=-\hat{G}_0{}^1, \qquad (25)$$

where the matrix $(\mathbf{n}^0_\omega\tau)$ is determined by formulas (4) and (13). Eq. (25) does not contain completely the matrix $\hat{G}^1_0$, and to find it it is necessary to consider the next higher terms in the expansion of the function $\hat{G}^1_\mathbf{p}(\omega, \mathbf{r})$ in powers of the parameters eAl and eAd:

$$\hat{G}_\mathbf{p}{}^1(\omega,\mathbf{r})=\hat{G}_0{}^1+\frac{1}{v}(\mathbf{v}\hat{\mathbf{G}}_1{}^1(x,\omega,z))+\hat{G}_3{}^1(x,\omega,z)+\ldots,$$

where

$$x=(\mathbf{e}_3\mathbf{v})/v, \qquad \mathbf{G}_1{}^1\sim\mathbf{A}_1A^2.$$

In order to find the matrix $\hat{G}^1_0$, let us integrate both sides of Eq. (2) over the angles of the vector $\mathbf{v}$ and the coordinate z. As a result we obtain

$$\int_{-d/2}^{d/2}dz\int d\Omega_\mathbf{p}\{[(\omega+\omega_0)\tau_z+\Delta\tau_y]\hat{G}_0{}^1-\hat{G}_0{}^1[\omega\tau_z+\Delta\tau_y] - ie(\mathbf{vA})(\tau_z\hat{G}_\mathbf{p}{}^1-\hat{G}_\mathbf{p}{}^1\tau_z)\}=$$
$$=\int_{-d/2}^{d/2}dz\int d\Omega_\mathbf{p}\{ie(\mathbf{vA}_1)[\tau_z\hat{G}_\mathbf{p}{}^0(\omega,z)-\hat{G}_\mathbf{p}{}^0(\omega+\omega_0,z)\tau_z] + i[\hat{\Delta}_1(\mathbf{n}_\omega{}^0\boldsymbol{\tau})-(\mathbf{n}^0_{\omega+\omega_0}\boldsymbol{\tau})\hat{\Delta}_1]\}. \qquad (26)$$

Multiplying both sides of (26) by τ_z and calculating the trace, we obtain with allowance for (25)

$$\hat{\Delta}_1=i\Delta_1\tau_y,\quad \hat{G}_0{}^1=g(\hat{B}_3-\hat{B}_4), \qquad (27)$$

where the matrices $\hat{B}_k$ satisfy Eqs. (23) and are chosen in the form

$$\begin{aligned}\hat{B}_1&=\varkappa(\mathbf{n}_\omega{}^0\boldsymbol{\tau})+(1-\gamma)(\mathbf{n}_2\boldsymbol{\tau}),\quad \hat{B}_2=\varkappa+i(1-\gamma)(\mathbf{n}_1\boldsymbol{\tau})\\ \hat{B}_3&=-\varkappa(\mathbf{n}_\omega{}^0\boldsymbol{\tau})-i(1+\gamma)(\mathbf{n}_1\boldsymbol{\tau})+(1+\gamma)(\mathbf{n}_2\boldsymbol{\tau})+\varkappa,\\ \hat{B}_4&=\varkappa(\mathbf{n}_\omega{}^0\boldsymbol{\tau})-i(1+\gamma)(\mathbf{n}_1\boldsymbol{\tau})-(1+\gamma)(\mathbf{n}_2\boldsymbol{\tau})+\varkappa,\end{aligned} \qquad (28)$$

where

$$\gamma=\alpha\alpha(+)+\beta\beta(+),\quad \varkappa=\alpha\beta(+)-\beta\alpha(+),$$
$$\alpha\equiv\alpha(\omega),\ \alpha(+)\equiv\alpha(\omega+\omega_0),\ \beta\equiv\beta(\omega),\quad \beta(+)\equiv\beta(\omega+\omega_0). \qquad (29)$$

Calculating the traces of both sides of (26), we get

$$g=\frac{e^2vl_1}{3}\frac{\beta(+)-\beta}{\omega_0}\mathscr{A}_1-\frac{\Delta_1}{2\omega_0}\frac{\beta(+)-\beta}{\beta(+)+\beta}, \qquad (30)$$

$$\mathscr{A}_1=\frac{1}{d}\int_{-d/2}^{d/2}dz(\mathbf{A}_1(z)(\hat{L}^{-1}-1)\mathbf{A}(z)). \qquad (30')$$

Substituting (27) and (30) in the equation for Δ_1 (19), we obtain

$$\Delta_1=\frac{2e^2vl_1}{3}\mathscr{A}_1\mathscr{F}(\omega_0),$$
$$g=\frac{e^2vl_1}{3}\frac{\beta(+)-\beta}{\omega_0}\left(1-\frac{\mathscr{F}(\omega_0)}{\beta(+)+\beta}\right)\mathscr{A}_1, \qquad (31)$$

where

$$\mathscr{F}(\omega_0)=\left[T\sum_\omega(\alpha(+)+\alpha)\frac{\beta(+)-\beta}{\omega_0}\right] \times\left[T\sum_\omega\left(\frac{\beta}{\Delta}+\frac{(\alpha(+)+\alpha)(\beta(+)-\beta)}{\omega_0(\beta(+)+\beta)}\right)\right]^{-1}. \qquad (32)$$

The current $j_1\exp(-i\omega_0\tau)$, which is connected with the presence of the rapidly-alternating field $\mathbf{A}_1(z)\exp(-i\omega_0\tau)$, is expressed in terms of the Green's function $\hat{G}^1_\mathbf{p}(\omega, z)$ and is equal to

$$\mathbf{j}_1=-\frac{iem}{4\pi^2}T\sum_\omega \mathrm{Sp}\begin{pmatrix}1&0\\0&0\end{pmatrix}\int p(\mathbf{v}\hat{\mathbf{G}}_1{}^1(x,\omega,z))\,d\Omega_\mathbf{p} =-\frac{iep_0{}^2}{4\pi}T\sum_\omega\sum_{k=1}^4\mathrm{Sp}\begin{pmatrix}1&0\\0&0\end{pmatrix}\tilde{\mathbf{D}}_k\hat{B}_k, \qquad (33)$$

where the vector matrix $\hat{\mathbf{G}}^1_1(x, \omega, z)$ is expanded in terms of the matrices $\hat{B}_k$, and where we introduced the notation

$$\tilde{\mathbf{D}}_k=\int_{-1}^{1}(1-x^2)\mathbf{D}_k(x,\omega,z)\,dx,\quad \hat{\mathbf{G}}_1{}^1(x,\omega,z)=\sum_{k=1}^4\mathbf{D}_k\hat{B}_k. \qquad (34)$$

Substituting the expression (34) for the function $\hat{\mathbf{G}}^1_1$ in the system (22) with allowance for (27), we obtain

$$\begin{aligned}\frac{\partial}{\partial z}\mathbf{D}_3+\frac{1}{x}\left[\frac{1}{l}\mathbf{D}_3-\frac{3}{4l_1}\tilde{\mathbf{D}}_3\right]&=\frac{ie}{x}\mathbf{A}(z)g(\alpha+\alpha(+)) +\frac{ie}{2x}\left(\beta+\frac{\alpha(1-\gamma)}{\varkappa}\right)\mathbf{A}_1(z),\\ \frac{\partial}{\partial z}\mathbf{D}_4-\frac{1}{x}\left[\frac{1}{l}\mathbf{D}_4-\frac{3}{4l_1}\tilde{\mathbf{D}}_4\right]&=\frac{ie}{x}\mathbf{A}(z)g(\alpha+\alpha(+)) +\frac{ie}{2x}\left(\beta+\frac{\alpha(1-\gamma)}{\varkappa}\right)\mathbf{A}_1(z).\end{aligned} \qquad (35)$$

It is impossible to obtain analogous equations for the coefficients $\mathbf{D}_1$ and $\mathbf{D}_2$, since it is necessary for this purpose to know the Green's function in the absence of the alternating field with larger accuracy than is given by formula (3). To obtain expressions for the currents it is necessary to know not the functions $\mathbf{D}_k$ themselves, but only their integrals with respect to the angles with the function $(1 - x^2)$. The corresponding values $\tilde{\mathbf{D}}_1$ and $\tilde{\mathbf{D}}_2$ are obtained from Eq. (26). The system (35) has the same form as the system (5). Using the previously introduced operator $\hat{L}$, we get from (35)

$$\tilde{\mathbf{D}}_3=-\tilde{\mathbf{D}}_4=\frac{2iel_1}{3}\left(\beta+\frac{\alpha(1-\gamma)}{\varkappa}\right)(\hat{L}^{-1}-1)\mathbf{A}_1(z) +\frac{4iel_1}{3}g(\alpha+\alpha(+))(\hat{L}^{-1}-1)\mathbf{A}(z). \qquad (36)$$

Substituting formulas (27), (34), and (36) in formula (26), we obtain two equations for the functions $\tilde{\mathbf{D}}_1$ and $\tilde{\mathbf{D}}_2$:

$$\frac{1}{d}\int_{-d/2}^{d/2}(\mathbf{A}\tilde{\mathbf{D}}_1)\,dz=-\frac{4iel_1}{3}\frac{\Gamma}{\omega_0}(\beta+\beta(+))$$

$$\times(\alpha+\alpha(+))\left(1-\frac{\mathscr{F}(\omega_0)}{\beta(+)+\beta}\right)\mathscr{A}_1, \qquad (37)$$
$$\widetilde{D}_2=0,$$

where the expression for Γ is given by formula (13). Inasmuch as the integral relation is satisfied for arbitrary $A_1(z)$, we can obtain from it $D_1(z)$:

$$\widetilde{D}_1(z)=-\frac{4iel_1}{3}\frac{\Gamma}{\omega_0}(\beta(+)+\beta)(\alpha(+)+\alpha)\cdot$$
$$\times\left(1-\frac{\mathscr{F}(\omega_0)}{\beta(+)+\beta}\right)\mathscr{A}_1/\mathscr{A}(\hat{L}^{-1}-1)A(z),$$
$$\mathscr{A}=\frac{1}{d}\int_{-d/2}^{d/2}(A(z)(\hat{L}^{-1}-1)A(z))\,dz. \qquad (38)$$

Substituting in (33) the expression (28) for the matrices $\hat{B}_k$ and the expressions (36), (37), and (38) for the coefficients $\widetilde{D}_k$, we get

$$j_1=-\frac{e^2p_0{}^2l_1}{3\pi}T\sum_\omega(1+\beta\beta(+)-\alpha\alpha(+))(\hat{L}^{-1}-1)A_1(z)$$
$$-\frac{2e^2p_0{}^2l_1}{3\pi}\frac{\Gamma}{\omega_0}T\sum_\omega\left(1-\frac{\mathscr{F}(\omega_0)}{\beta(+)+\beta}\right)(\alpha+\alpha(+))$$
$$\times(\beta^2(+)-\beta^2)\mathscr{A}_1/\mathscr{A}(\hat{L}^{-1}-1)A(z). \qquad (39)$$

To obtain the current produced in a superconductor under the influence of an alternating field $A_1(z)\exp(-i\omega t)$ it is necessary to carry out an analytic continuation of the right side of (39) with respect to the frequency ω_0, from the values $\omega_0=-i\omega=2\pi Tn$ to the real axis ω[5].

Carrying out the analytic continuation, we obtain

$$j_1(\omega)=-\frac{Ne^2}{m}\tau_1(\hat{L}^{-1}-1)\{A_1(z)Q^{(1)}(\omega)+A_2(z)Q^{(2)}(\omega)\mathscr{A}_1/\mathscr{A}\}, \qquad (40)$$

where $N=p_0^3/3\pi^2$, and the kernels $Q^{(1)}(\omega)$ and $Q^{(2)}(\omega)$ are given by

$$Q^{(1)}(\omega)=-i\omega+\frac{i}{2}\int_{-\infty}^{\infty}dt\,\mathrm{th}\frac{t}{2T}(1+\beta_t{}^+\beta_{t-\omega}^+-\alpha_t{}^+\alpha_{t-\omega})$$
$$+\frac{i}{4}\int_{-\infty}^{\infty}dt\left(\mathrm{th}\frac{t}{2T}-\mathrm{th}\frac{t-\omega}{2T}\right)(1-\beta_{t-\omega}^+\beta_t{}^-+\alpha_{t-\omega}^+\alpha_t{}^-),$$
$$Q^{(2)}(\omega)=\frac{\Gamma}{\omega}\left\{\int_{-\infty}^{\infty}dt\,\mathrm{th}\frac{t}{2T}[\alpha_{t-\omega}((\beta_t{}^+)^2-(\beta_t{}^-)^2)-(\beta_{t-\omega}^+)^2(\alpha_t{}^+-\alpha_t{}^-)]\right.$$
$$\left.+\mathscr{F}(\omega)\int_{-\infty}^{\infty}dt\,\mathrm{th}\frac{t}{2T}[\beta_{t-\omega}^+(\alpha_t{}^+-\alpha_t{}^-)-\alpha_{t-\omega}^+(\beta_t{}^+-\beta_t{}^-)]\right\}. \qquad (41)$$

The function $\mathscr{F}(\omega)$ is obtained by analytic continuation of expression (32):

$$\mathscr{F}(\omega)=\frac{1}{\omega}\left\{\int_{-\infty}^{\infty}dt\left(\mathrm{th}\frac{t}{2T}-\mathrm{th}\frac{t-\omega}{2T}\right)\alpha_{t-\omega}^+\beta_t^-+\int_{-\infty}^{\infty}dt\,\mathrm{th}\frac{t}{2T}(\alpha_t^+\beta_{t-\omega}^+\right.$$
$$\left.-\alpha_{t-\omega}^+\beta_t^+)\right\}\left\{\int_{-\infty}^{\infty}dt\,\mathrm{th}\frac{t}{2T}\left[-\frac{i\beta_t^+}{\Delta}+\frac{(\alpha_t^++\alpha_{t-\omega}^+)(\beta_{t-\omega}^+-\beta_t^+)}{\omega(\beta_{t-\omega}^++\beta_t^+)}\right.\right.$$
$$\left.\left.-\frac{(\alpha_{t-\omega}^++\alpha_t^-)(\beta_{t-\omega}^+-\beta_t^-)}{\omega(\beta_{t-\omega}^++\beta_t^-)}\right]\right\}^{-1}, \qquad (42)$$

where the functions $\alpha_t^\pm$ and $\beta_t^\pm$ are defined as follows:

$$\alpha_t{}^\pm=\alpha(it\pm\delta),\quad \beta_t{}^\pm=\beta(it\pm\delta),\quad \delta\to+0. \qquad (43)$$

The functions $\alpha_t^\pm$ and $\beta_t^\pm$ satisfy the relations

$$\alpha_t{}^+=-\alpha^-(-t),\quad \beta_t{}^+=\beta^-(-t),\quad \lim_{t\to\pm\infty}\alpha_t{}^+=1.$$

Formulas (13), (40), (41), and (42) in conjunction with Maxwell's equations enables us to find the current produced in a thin superconducting film under the influence of the alternating electromagnetic field at arbitrary temperatures and values of the static magnetic field and current. By way of an example, let us consider the passage of a plane wave through a thin film $d\ll\delta$, δ_{skin}. Writing down the incident, reflected, and transmitted waves respectively in the form

$$A_1\exp[-i\omega t-ik(z-d/2)],\quad A_2\exp[-i\omega t+ik(z-d/2)],$$
$$A_3\exp[-i\omega t-ik(z+d/2)], \qquad (44)$$

we obtain from (40) and from Maxwell's equation

$$\left(\omega^2+\frac{\partial^2}{\partial z^2}\right)A=-4\pi j$$

a connection between the amplitudes A_1, A_2, and A_3:

$$A_3=\frac{1}{1+\rho^{(1)}}\left[A_1-A_0\frac{\rho^{(2)}(A_0A_1)}{\rho^{(2)}A_0{}^2+(1+\rho^{(1)})(A_0{}^2+b_2/b_1H^2d^2)}\right],$$
$$A_2=-A_1+A_3,\quad \rho^{(1,2)}=\frac{2\pi i}{\omega}\sigma_N\,dQ^{(1,2)}. \qquad (45)$$

It follows from (45) that the planes of polarization of the reflected and transmitted waves are rotated if current flows in the film. For thick films ($d\gtrsim\delta$) an analogous phenomenon should be observed also in a magnetic field.

In the general form, the connection between the current and the field (formulas (40) and (41) turns out to be quite complicated and it is therefore useful to consider particular cases, in which this connection turns out to be much simpler and clearer. Let us consider several such cases:

a) Low temperature:

$$T\ll\Delta(\Gamma/\Delta)^{2/3},\quad \omega\ll\Delta(\Gamma/\Delta)^{2/3},\quad \omega\ll\Delta-\Gamma,\quad \Gamma<\Delta.$$

Under the foregoing conditions, the imaginary parts of the kernels $Q^{(1)}$ and $Q^{(2)}$ turn out to be exponentially small and is determined by the behavior of the coefficients α and β near the threshold point $t=\epsilon$, where ϵ is the gap in the excitation spectrum. A similar property is possessed also by the function $\mathscr{F}(\omega)$. Omitting the intermediate derivations, we present immediately an expression for the kernels $Q^{(1)}(\omega)$ and $Q^{(2)}(\omega)$ and the function

$$\mathscr{F}(\omega)=-\frac{\pi}{2(1-\pi\Gamma/4\Delta)}+ia(\omega)$$
$$\times\frac{(1-2(\Gamma/\Delta)^{2/3})-(3\pi\Gamma/4\Delta)(1-(\Gamma/\Delta)^{2/3})}{2\Delta(1-\pi\Gamma/4\Delta)^2(1-(\Gamma/\Delta)^{2/3})}$$
$$Q^{(1)}(\omega)=(\pi\Delta-4\Gamma/3)-i9a(\omega)(1-{}^1/_2(\Gamma/\Delta)^{2/3}),$$
$$Q^{(2)}(\omega)=-\Gamma\left(\frac{8}{3}+\frac{\pi^2}{2(1-\pi\Gamma/4\Delta)}\right)$$
$$+i8a(\omega)\frac{(1-{}^5/_4(\Gamma/\Delta)^{2/3})-(3\pi\Gamma/8\Delta)(1-(\Gamma/\Delta)^{2/3})}{(1-\pi\Gamma/4\Delta)^2(1-(\Gamma/\Delta)^{2/3})}, \qquad (46)$$
$$\varepsilon=\Delta(1-(\Gamma/\Delta)^{2/3})^{3/2},$$
$$a(\omega)=\frac{8\omega T}{27\Delta}\left(\frac{\Delta}{\Gamma}\right)^{4/3}\mathrm{sh}\left(\frac{\omega}{2T}\right)K_1\left(\frac{\omega}{2T}\right)\exp\left(-\frac{\varepsilon}{T}\right)(1-(\Gamma/\Delta)^{2/3})^{-1/2}.$$

The imaginary part of the kernel $Q^{(2)}(\omega)$ reverses sign when $(\Gamma/\Delta)^{2/3}\sim{}^5/_8$. It can be shown that the absolute value of the imaginary part of the kernel

$Q^{(2)}(\omega)$ is always smaller than that of the kernel $Q^{(1)}(\omega)$. Therefore the instabilities with respect to a weak rapidly alternating field do not arise, but the presence of $Q^{(2)}(\omega)$ can lead to a noticeable attenuation of the absorption in the region of $(\Gamma/\Delta)^{2/3} < 5/8$. This effect can be observed experimentally in the study of the dependence of the absorption coefficient in a thin film as a function of the angle between the direction of the current and the plane of polarization of the incident electromagnetic wave. An analogous effect arises also with respect to the magnetic field, if the thickness of the film satisfies the condition $d \gtrsim \delta$, where δ is the depth of penetration.

b) High temperature

$$T \to T_c, \quad \omega \ll \Delta(\Gamma/\Delta)^{2/3}, \quad \Gamma \ll \Delta.$$

Unlike the static case, where the expansion of all the quantities is carried out with respect to temperature, in the presence of a rapidly alternating field there appear "anomalous" terms, which do not have this property. The "anomalous" terms depend essentially on the ratio of ω to $\Delta^{1/3}\Gamma^{2/3}$. Omitting the intermedisteps, we present immediately the final result:

$$\mathcal{F}(\omega) \approx -1,$$

$$Q^1(\omega) = -i\omega\left(1 + \frac{\Delta}{3T}\ln\frac{8\Delta}{\Gamma}\right) + \frac{\pi\Delta^2}{2T},$$

$$Q^2(\omega) = \frac{\Gamma\Delta}{T}\left(\frac{\Delta}{2\Gamma}\right)^{1/3}\frac{3\sqrt[3]{3}\pi^2}{10\Gamma^3(2/3)} + \frac{2i\omega\Delta}{3T}, \tag{47}$$

where $\Gamma(2/3)$ is the Euler Gamma function. For comparison, we present the value of the kernel Q in the frequency region $\Delta(\Gamma/\Delta)^{2/3} \ll \omega \ll \Delta$:

$$Q_1(\omega) = -i\omega\left(1 + \frac{\Delta}{2T}\left(\ln\frac{8\Delta}{\omega} - 1\right)\right) + \frac{\pi\Delta^2}{2T}.$$

We note that the static limit does not coincide at all with the dynamic limit as $\omega \to 0$. For example, for the function F_{st} in the region of weak fields as $T \to T_c$ we can easily obtain the following expression

$$\mathcal{F}_{st} = -2\pi^3 T / 7\zeta(3)\Delta, \tag{48}$$

where $\zeta(x)$ is the Reimann Zeta function. Such a strong difference between expressions (46) and (47) for the function F means that all of the obtained formulas are applicable only for sufficiently high frequencies[7] $\omega \gg \omega_{rel}$, where ω_{rel} is the frequency connected with the relaxation time in the superconductor. It can be shown that as $T \to 0$ the static limit and the dynamic limit $\omega \to 0$ coincide.

c) Region of strong fields $\Gamma \gg \Delta$.

This case corresponds to gapless superconductivity and can be actually realized only in the presence of a magnetic field. The region $\Gamma > \Delta$ in the presence of only current corresponds to the metastable state. On the (j, A_0) diagram this region lies to the right of the point at which the critical current is reached. The region of strong fields turns out to be simplest and in it is relatively easy to obtain an expression for the kernels $Q^{(1)}$ and $Q^{(2)}$ and the function $F(\omega)$ at arbitrary values of both the frequency ω and the temperature T. Omitting the rather simple calculations, we obtain

$$\mathcal{F}(\omega) = \frac{4i\Delta}{\omega}[\psi(x) - \psi(x_0)]\Big\{-2\Big[\psi\Big(x_0 - \frac{i\omega}{4\pi T}\Big) - \psi(x_0)\Big] - \frac{\Delta^2}{\Gamma(2\pi T)}\Big[\psi'(x_0) - \frac{\Gamma^2}{3(2\pi T)^2}\psi'''(x_0)\Big]\Big\}^{-1},$$

$$Q^{(1)}(\omega) = -i\omega + \Delta^2\left\{\left(\frac{i}{\omega} + \frac{1}{2\Gamma - i\omega}\right)(\psi(x) - \psi(x_0)) + \frac{1}{2\pi T}\psi'(x)\right\},$$

$$Q^{(2)}(\omega) = \frac{2i\Gamma\Delta^2}{\pi\omega T}[\psi'(x) - \psi'(x_0)] + \frac{4i\Delta\Gamma\mathcal{F}(\omega)}{\omega}[\psi(x) - \psi(x_0)],$$

$$x = 1/2 + (\Gamma - i\omega)/2\pi T, \quad x_0 = 1/2 + \Gamma/2\pi T, \tag{49}$$

where $\psi(x)$ is the Psi function.

Let us consider in detail the case of high and low temperatures at $\omega \ll \Gamma$. Near T_c we get

$$\mathcal{F}(\omega) = \frac{4\Delta\Gamma}{i\omega\Gamma - \Delta^2}, \quad Q^{(1)}(\omega) = \frac{\pi\Delta^2}{2T} - i\omega\left(1 + \frac{\pi\Delta^2}{8\Gamma T}\right),$$

$$Q^{(2)}(\omega) = \frac{4\pi\Delta^2\Gamma^2}{T(i\omega\Gamma - \Delta^2)}. \tag{50}$$

The absorption connected with the presence of the kernel $Q^2(\omega)$ decreases rapidly with frequency. For thin films $(d \ll \delta)$, the ratio of the absorption is connected with the kernels Q^2 and Q^1 assumes at the critical-current point the value

$$I = \frac{112\zeta(3)}{\pi^4}\cos^2\varphi\left(1 - \frac{T}{T_c}\right)^2, \quad \Gamma \gg \Delta, \quad \omega\Gamma \ll \Delta^2. \tag{51}$$

At low temperatures and at $\omega \ll \Gamma$, we get from formulas (49)

$$\mathcal{F}(\omega) = \frac{12\Delta\Gamma}{3i\omega\Gamma - \Delta^2},$$

$$Q^1(\omega) = -i\omega\left(1 - \frac{\Delta^2}{\Gamma^2}\right) + \frac{2\Delta^2}{\Gamma}, \quad Q^2(\omega) = \frac{48\Delta^2\Gamma}{3i\omega\Gamma - \Delta^2} - \frac{4\Delta^2}{\Gamma}. \tag{52}$$

In analogy with formula (51), for the ratio of the absorptions connected with the kernels Q^2 and Q^1 at the critical-current point we get

$$I = 6\cos^2\varphi, \quad \omega\Gamma \ll \Delta^2, \quad \Gamma \gg \Delta, \quad T \ll T_c.$$

4. CONCLUSION

For contaminated films $(l \ll \xi_0)$ we have obtained an expression for the ordering parameter Δ and for the current density under the condition $eAl \ll 1$ and $eAd \ll 1$. In a wide range of free paths $d^2/\xi \ll 9l \ll (d\xi)^{1/2}$, the conditions $eaL \ll 1$ and $eAd \ll 1$ are satisfied for all fields and currents up to their critical values. The film thickness is assumed small compared with ξ and arbitrary relative to the penetration depth δ. For films whose thickness d satisfies the condition $d \ll \delta$, the dependence of the quantities $\Delta = \Delta(H, T, \gamma)$, $\gamma_c(H, T)$, and $H_c(T)$ on the free path and on the character of the reflection of the electrons from the boundaries of the sample is described with the aid of two constants b_1 and b_2, which have the meaning of the effective free paths. The constant b_1 is connected in simple fashion with the residual resistance of the film, while b_2 can be obtained, for example, from the value of the critical field. When a rapidly alternating field is applied, a current flows in the film, and its magnitude depends to a considerable degree on *the relative orientation* of the static and alternating fields. The connection between the rapidly alternating current and the vector potential is described with the aid of a kernel, the properties of which depend on the magnitude of the static current and the magnetic field. A general expression is obtained for this kernel, and its properties are investigated for high and low temperatures in the region of both strong and weak fields. For thin films in the intermediate region of temperatures, a minimum in the absorption coefficient may be observed with increasing current. In thick films $d \gtrsim \delta$, and similar phenomena should be observed also with respect to the magnetic field. This phenomenon is connected with the fact that in the presence of static fields there appears in the kernel an additional term, whose imaginary part has a positive maximum.

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