# *A NEW MECHANISM OF INDUCTIVE ABSORPTION OF SOUND IN METALS*

R. G. MINTS

L. D. Landau Institute of Theoretical Physics, USSR Academy of Sciences

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The induction oscillations that develop in a conductor upon propagation of a low-frequency sound wave through it are considered. It is shown that in a quantizing magnetic field (the distance between the Landau levels is greater than the temperature), this induction change leads to sound absorption which is of the same order of magnitude as the value of induction damping of the sound.[2] The oscillatory dependence of the sound absorption mechanism on the magnetic field leads to oscillations in the absorption coefficient with a relative amplitude of the order of or greater than unity, provided that the induction absorption is dominant. It is shown that the latter condition exists in compensated metals, or in metals whose Fermi surface possesses a layer of open trajectories.

THE absorption of ultrasound in a metal in a quantizing magnetic field ($\pi^2 T < \hbar\Omega$, $\Omega = eB/mc$ is the cyclotron frequency, B the magnetic induction) is considered in the present work for the condition that the length of the free path of the conduction electrons $l$ satisfies the inequality $l < \lambda\!\!\!^-$, and $\mathbf{k}\cdot\mathbf{B} = 0$, where $2\pi\lambda\!\!\!^- = 2\pi/k$ is the sound wavelength. The damping of the sound in a strong magnetic field has been considered previously a number of times; here we have studied two absorption mechanisms: deformation and induction.[1,2] The deformation mechanism determines the sound absorption in the absence of a magnetic field. Physically, it is connected with the retardation in establishment of the equilibrium distribution of the conduction electrons, the dispersion law for which has the form

$$\varepsilon = \varepsilon_0(\mathbf{p}) + \lambda_{ik}(\mathbf{p})u_{ik} + p_i v_k \frac{\partial u_i}{\partial x_k} + \mathbf{pu} = \varepsilon_0(\mathbf{p}) + \delta\varepsilon(\mathbf{p})$$

in the field of the sound wave $\mathbf{u}$. Here $\lambda_{ik}(p)$ is the deformation potential, $u_{ik} = \frac{1}{2}(\partial u_i/\partial x_k + \partial u_k/\partial x_i)$ is the deformation tensor. This retardation, which is due to the finite velocity of sound, leads to dissipation of energy.

In addition to the direct deformation mechanism, the electric field, which develops in the conductor upon passage of the sound field through it, also contributes to the sound absorption. This field is found from the Maxwell equations. Estimates show that the contribution to the absorption from the electric field is, in any case, not larger than the direct contribution from the deformation mechanism. The induction mechanism of sound absorption arises in a magnetic field. It is associated with the electric field $\mathbf{G} = \mathbf{u}\times\mathbf{B}/c$, which is induced when sound passes through a deformed conductor moving in a magnetic field. Here the absorption and the electric field are determined by the total electric field $\vec{\mathscr{E}} = \mathbf{E} + \mathbf{G}$ (see[2]), and the field $\mathbf{E}$ satisfies the Maxwell equations:

$$\operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t},$$

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\{\hat{\sigma}\vec{\mathscr{E}} + \mathbf{j}_{\text{def}}\}.$$

For what follows, it will be convenient for us to write down Maxwell's equations for the field $\vec{\mathscr{E}}$. This is not difficult to do if we take it into account that*

$$\operatorname{rot}\mathbf{G} = \frac{1}{c}\operatorname{rot}[\mathbf{uB}] = \frac{1}{c}\frac{\partial}{\partial t}\{(\mathbf{B}\nabla)\mathbf{u} - \mathbf{B}\operatorname{div}\mathbf{u}\}.$$

By using the latter relation, we get

$$\operatorname{rot}\vec{\mathscr{E}} = -\frac{1}{c}\frac{\partial}{\partial t}\{\mathbf{B}(1 + \delta_{ik}u_{ik})\},$$

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\hat{\sigma}\vec{\mathscr{E}} + \frac{4\pi}{c}\mathbf{j}_{\text{def}}.$$

It is then seen that the inductive absorption of sound in the case $\mathbf{k}\cdot\mathbf{B} = 0$ corresponds to a "change" in the induction by the amount $\delta B_{\text{ind}} \sim B\delta_{ik}u_{ik}$.

In the preceding discussion it was assumed that the induction $\mathbf{B}$ remains constant. However, being a self-consistent magnetic field, the induction "feels" the change in the dispersion law of conduction electrons, i.e., a dependence of the induction on the time appears. This leads to the appearance of an induced electric field and, at the same time, to additional sound absorption. This is the physical reason of the new induction mechanism of sound absorption.

We now estimate the corresponding value of the change in induction. Both the Fermi energy $\epsilon_0$ and the form of the Fermi surface change in the field of the sound wave. As will be shown below, both these factors lead to induction changes of the same order of magnitude. Therefore, we shall estimate here, for simplicity, only the effect of the change of the Fermi energy. From the condition of electrical neutrality, it is easy to obtain the result that (with accuracy up to $(\hbar\Omega/\epsilon_0)^2$) $\delta\epsilon_0 = \bar{\lambda}_{ik}u_{ik}$, where

$$\bar{\lambda}_{ik} = \oint\frac{\lambda_{ik}\,d\Sigma}{v} \Big/ \oint\frac{d\Sigma}{v}$$

is the value of $\lambda_{ik}$ averaged over the Fermi surface. We then get for $\delta B$

$$\delta B \sim \frac{\partial B}{\partial\varepsilon_0}\delta\varepsilon_0 \sim \frac{\partial B}{\partial\varepsilon_0}\bar{\lambda}_{ik}u_{ik} \sim 4\pi\frac{\partial M}{\partial\varepsilon_0}\bar{\lambda}_{ik}u_{ik}.$$

The largest values of the magnetic moment and its derivative occur in those regions of the magnetic field and temperature where they are determined by the

*$[\mathbf{uB}] \equiv \mathbf{u}\times\mathbf{B}$.

de Haas—van Alphen effect ($\pi^2 T < \hbar\Omega$).[3] As a consequence of the oscillatory dependence of the magnetic field on the Fermi energy, we get

$$4\pi\frac{\partial M}{\partial\varepsilon_0} \sim \frac{4\pi M}{\hbar\Omega} \sim 4\pi\chi\frac{B}{\varepsilon_0},$$

where

$$\chi = \frac{\partial M}{\partial B} \sim \left(\frac{v}{c}\right)^2\left(\frac{\varepsilon_0}{\hbar\Omega}\right)^{3/2}.$$

Thus,

$$\delta B \sim B4\pi\chi\frac{\lambda_{ik}u_{ik}}{\varepsilon_0}, \quad \frac{\delta B}{\delta B_{\text{ind}}} \sim 4\pi\chi\frac{\bar{\lambda}}{\varepsilon_0},$$

and if $4\pi\chi \sim 1$, then the contribution to the absorption from the mechanism considered is of the order of the value of the induction absorption. In this connection, we shall assume in what follows that the induction mechanism of sound absorption is dominant.

It is clear from what has been pointed out that for an exact solution of the problem it is necessary to compute the induction B in the field of the sound wave and to substitute the resultant expression in the Maxwell equation for the determination of the electric field $\vec{\mathscr{E}}$. In the case in which the induction absorption of sound is dominant, $j_{\text{def}}$ in the Maxwell equations can be set equal to zero, and the value of the energy dissipated per unit time is

$$Q = {}^1/_2 \operatorname{Re}\sum_{i,k}\sigma_{ik}\mathscr{E}_i\mathscr{E}_k^*.$$

The coefficient of sound absorption is thus equal to

$$\Gamma = \frac{1}{2\rho\omega^2u^2s}\operatorname{Re}\sum_{i,k}\sigma_{ik}\mathscr{E}_i\mathscr{E}_k^*,$$

where s is the group velocity of sound, $\omega$ the sound frequency, and $\rho$ the density.

We now proceed to the quantitative solution of the problem.

## 2. MAGNETIC MOMENT IN THE FIELD OF THE SOUND WAVE

The calculation of the magnetic moment requires a knowledge of the energy levels of the electron, which we shall determine in the quasiclassical approximation. For this purpose, we first consider the classical motion of the conduction electron. Let $z \parallel B$ and $y \parallel k$, and let the vector potential **A** be written in the form $A_y = A_z = 0$, $A_x = -\int B(y')dy'$. Then the classical motion is reduced to the one-dimensional motion of a particle with a Hamiltonian

$$\varepsilon(\mathbf{P}, y) = \varepsilon\left(\mathbf{P} - \frac{e}{c}\mathbf{A}(y), y\right),$$

and the energy levels are determined in the quasiclassical approximation from the condition

$$\oint P_y\,dy = 2\pi n\hbar.$$

By using the fact that $P_y = p_y$ and $dy = dp_x/ec^{-1}B(y)$, we have

$$\oint\frac{p_y\,dp_x}{B(y)} = \frac{2\pi n\hbar e}{c}$$

The condition $kR \ll 1$ allows us to take $B(y)$ outside the integral sign. Finally, we get the relation

$$\oint p_y\,dp_x = S(\varepsilon) = \frac{2\pi n\hbar eB}{c}$$

for the determination of the energy levels.

Further calculations of the magnetic moment are made in standard fashion (see the work of I. M. Lifshitz and A. M. Kosevich[3]). As a result, we obtain the following expression:

$$\mathbf{M} = \sum_\alpha \mathbf{M}_{\mathrm{LK}^\alpha}\left\{\frac{cS^\alpha(\mu)}{e\hbar B}\right\},$$

where $S^\alpha(\mu)$ is the area of the extremal cross section of the Fermi surface in $p_z$, and $M_{LK}$ is the magnetic moment computed in[3]. We now describe the quantity $S^\alpha(\mu)$ in more detail. For this purpose, we represent $p_y$ in the form $p_y = p_y^{(0)}(\epsilon_0, p_z) + p_y^{(1)}$, where $p_y^{(0)}$ satisfies the equation $\epsilon_0 = \epsilon_0(p_y^{(0)}, p_z, p_x)$. Then we get for $p_y^{-1}$ the expression $v_y p_y^{(1)} = \lambda_{ik}u_{ik} - \delta\epsilon(p)$, whence $S^\alpha = S_0^\alpha(\epsilon_0) - \delta S^\alpha$ where $\delta S^\alpha = -S^\alpha\delta_{ik}u_{ik} - \bar{\Lambda}_{ik}^\alpha u_{ik}(\partial S^\alpha/\partial\epsilon)$, $\Lambda_{ik} = \lambda_{ik} - \bar{\lambda}_{ik}$, $\tilde{\Lambda}^\alpha$ is the mean value of $\Lambda$ over the perimeter of the extremal cross section. Thus

$$\mathbf{M} = \sum_\alpha \mathbf{M}_{\mathrm{LK}\alpha}\left\{\frac{(S_0{}^\alpha + \delta S^\alpha)c}{e\hbar B}\right\}.$$

## 3. SOUND ABSORPTION COEFFICIENT

We shall first transform the sound absorption coefficient. From $j_y = 0$, which is equivalent to the condition of electrical neutrality, we have $\mathscr{E}_y = -\sigma_{y\beta}\mathscr{E}_\beta/\sigma_{yy}$. We then have for $j_\alpha$ and $\Gamma$

$$j_\alpha = \left(\sigma_{\alpha\beta} - \frac{\sigma_{\alpha y}\sigma_{y\beta}}{\sigma_{yy}}\right)\mathscr{E}_\beta = \tilde{\sigma}_{\alpha\beta}\mathscr{E}_\beta,$$

$$\Gamma = \frac{1}{2\rho u^2\omega^2 s}\operatorname{Re}\sum_{\alpha,\beta}\tilde{\sigma}_{\alpha\beta}\mathscr{E}_\alpha\mathscr{E}_\beta^* \quad (\alpha, \beta = y, z). \tag{1}$$

For simplicity, we shall assume in what follows that the magnetic field is applied along one of the axes of the crystal. Then $h_x = h_y = \mathscr{E}_z = 0$ and only the component with $\mathscr{E}_x$ in the sum in Eq. (1) remains. We write the expression for curl $\vec{\mathscr{E}} = -1/c \times \partial/\partial t\,[\mathbf{B}(1 + \operatorname{div}\mathbf{u})]$ in the following fashion:

$$\operatorname{rot}\vec{\mathscr{E}} = -\frac{1}{c}\frac{\partial}{\partial t}\left\{\mathbf{B}\delta_{ik}u_{ik} + \left(\frac{\partial B}{\partial H}\right)_{u_{ik}=0}\mathbf{h} + \left(\frac{\partial\mathbf{B}}{\partial S^\alpha}\right)_{h=0}\delta S^\alpha\right\},$$

$$\left(\frac{\partial B}{\partial H}\right)_{u_{ik}=0} = \frac{1}{1-4\pi\chi} = \mu, \quad \left(\frac{\partial\mathbf{B}}{\partial S^\alpha}\right)_{\mathrm{h}=0} = -\frac{4\pi\chi^\alpha}{S^\alpha}\mathbf{B}.$$

After substitution of the explicit expression for $\delta S^\alpha$, the final set of equations, from which the field $\vec{\mathscr{E}}$ is determined, has the form

$$\operatorname{rot}\vec{\mathscr{E}} = -\frac{\mu}{c}\left\{\frac{\partial\mathbf{h}}{\partial t} + \mathbf{B}\alpha_{ik}u_{ik}\right\},$$

where

$$\operatorname{rot}\mathbf{H} = \frac{4\pi}{c}\hat{\tilde{\sigma}}\vec{\mathscr{E}},$$

$$\alpha_{ik} = \delta_{ik} + 4\pi\chi^\alpha\frac{dS^\alpha}{d\varepsilon}\frac{1}{S^\alpha}\tilde{\Lambda}_{ik}{}^\alpha. \tag{2}$$

Solving this system, we get the following result for $\mathscr{E}_x$:

$$\mathscr{E}_x = -\frac{\mu Bs}{c}\alpha_{ik}u_{ik}\left(1 - \frac{2i\mu}{(k\delta_0)^2}\right)^{-1},$$

where $(k\delta_0)^2 = \omega_c^2/2\pi\tilde{\sigma}s^2$.

Substituting this expression in Eq. (1), we get

$$\Gamma = \frac{1}{2}\frac{\tilde{\sigma}B^2\mu^2}{\rho c^2 s}\frac{|v_k\alpha_{yk}|^2}{1 + 4\mu^2/(k\delta_0)^4}, \quad \tilde{\sigma} = \sigma_{xx} - \frac{\sigma_{xy}\sigma_{yx}}{\sigma_{yy}}, \quad v_k = \frac{u_k}{u}. \tag{3}$$

We now proceed to the analysis of the sound absorption coefficient for different cases.

First, we note that for $4\pi\chi \sim 1$, both components in $\alpha_{yk}$ (see (2)) are generally of the same order. From the oscillatory dependence on the magnetic field of the second component, the absorption coefficient at its minimum can be much smaller than its value at maximum for a longitudinal wave, and the minimum value of the absorption coefficient is exactly equal to zero in the case of a transverse sound wave. Thus, gigantic oscillations of the sound absorption coefficient can develop. In this case, the minimum value of the absorption is determined by the deformation mechanism, and the relative amplitude of the oscillations is $\Gamma_{ind}^{max}/\Gamma_{def}$.

Let us consider the value of the ratio $\Gamma_{ind}^{max}/\Gamma_{def}$ in more detail. In the case of a normal metal (case a)), $\Gamma_{def} \sim n\epsilon_0\omega^2\tau/\rho s^2$ and $\overline{\sigma} \sim \sigma_0 \sim ne^2\tau/m$. For a compensated metal (case b)), $\Gamma_{def} \sim n\epsilon_0\omega^2\tau/\rho s^3$ and $\overline{\sigma} \sim \sigma_0(R/l)^2$ (since $n_\epsilon = n_h)l$, where R is the radius of the electronic orbit. If, for a given direction of the magnetic field on the Fermi surface there is a layer of open trajectories, and the number of electrons on open trajectories is of the order of the number of electrons on closed trajectories (case c)), then $\Gamma_{def} \sim n\epsilon_0\omega/\rho s^2 v$ and $\overline{\sigma} \sim \sigma_0(R/l)^2$. Using these formulas, it is easy to obtain

a) $$\frac{\Gamma_{ind}^{max}}{\Gamma_{def}} \sim \left(\frac{1}{kR}\right)^2 \frac{\mu^2}{1+4\mu^2/(k\delta_0)^4},$$

b) $$\frac{\Gamma_{ind}^{max}}{\Gamma_{def}} \sim \left(\frac{1}{kl}\right)^2 \frac{\mu^2}{1+4\mu^2/(k\delta_0)^4},$$

c) $$\frac{\Gamma_{ind}^{max}}{\Gamma_{def}} \sim \frac{1}{kl} \frac{\mu^2}{1+4\mu^2/(k\delta_0)^4}.$$

It is not difficult to establish the fact for a normal metal we always have $(k\delta_0)^2 \gg 1$ and $\Gamma_{ind}^{max}/\Gamma_{def} \sim (k\delta_0)^4/(kR)^2 \lesssim 1$ in the region of interest to us.

In cases b) and c), we can have both $(k\delta_0)^2 < 1$ and also $(k\delta_0)^2 > 1$ because of the small value of $\overline{\sigma} \sim \sigma_0(R/l)^2$.

If $(k\delta_0)^2 < 1$, then $\Gamma_{ind}^{max}/\Gamma_{def} \sim 1/(kl)^2 \gg 1$ and $\Gamma_{ind}^{max}/\Gamma_{def} \sim 1/kl > 1$ for cases b) and c), respectively; in the opposite limiting case, $\Gamma_{ind}^{max}/\Gamma_{def} \lesssim 1$. Thus, in compensated metals and in metals whose Fermi surface has a layer of open trajectories, strong oscillations of the ultrasonic absorption coefficient arise:

$$1 \lesssim \Gamma_{ind}^{max}/\Gamma_{ind}^{min} \lesssim \Gamma_{ind}^{max}/\Gamma_{def}$$

We now consider the effect of the value of $\mu$ on the oscillations of the sound absorption coefficient. From the condition of thermodynamic stability of a uniformly magnetized state, it follows that $\mu > 0$, i.e., $4\pi\chi \leq 1$. Here, as in the liquid-vapor case, one can show that a system of critical points arises (because of the periodicity of the magnetic field). These points ($T_0^{(i)}$, $B_0^{(i)}$) are defined by the condition[4]

$$4\pi\chi(B_0, T_0) = 1, \quad 4\pi\frac{\partial\chi}{\partial B}(T_0, B_0) = 0.$$

Under conditions sufficiently close to the critical point, the value of $\mu$ begins to increase, and its maximum value for a given temperature is $\mu_{max} \sim (\Delta T/T_0)^{-1}$, where $\Delta T = T - T_0 > 0$, while the width of the induction range, where $\mu$ is significantly different from unity,[5] is

$$\Delta B \sim B_0 \frac{h\Omega}{\varepsilon_0}\sqrt{\frac{\Delta T}{T}}.$$

We now assume that $(k\delta_0)^2 \gg 1$ and the temperature satisfies the condition $\Delta T/T_0 < 2/(k\delta_0)^2$. Then, as is easy to see from Eq. (3), the absorption coefficient in the region close to the critical point increases by a factor $(k\delta_0)^4/4$. Thus, in the indicated temperature interval, a narrowing ($\Delta B \ll B_0\hbar\Omega/\epsilon_0$ is the period of the oscillations) of the absorption peak is superimposed on the considered oscillations.

## 4. DISCUSSION OF THE RESULTS

The experimental observation of strong oscillations of the sound absorption coefficient $\Gamma_{max}/\Gamma_{min} \gtrsim 1$ in a quantized magnetic field ($\mathbf{k} \cdot \mathbf{B} = 0$) allows us to conclude that the induction absorption is of the order of the deformation absorption in any case. The presence of gigantic oscillations of the sound absorption should mean that the dominant mechanism of absorption is the induction mechanism. We recall that if the induction mechanism of absorption is actually dominant, then the gigantic sound absorption oscillations arise for transverse polarization of the sound wave. In this case, the amplitude of the gigantic oscillations gives the value of the inductive sound absorption, while the minimum value of the absorption coefficient is identical with the value of the deformation sound absorption.

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