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EFFECT OF A HELICON ON PLASMA INSTABILITIES

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The effect of a helicon moving along a stationary magnetic field H_0 on the loss-cone and drift-loss-cone instabilities of a plasma is investigated. It is shown that at a relatively low helicon field strength the instabilities may be suppressed.

THE cone and cone-drift instabilities that arise in a plasma contained by a magnetic field lead to the perturbations of the equilibrium values of the density, potential, etc., in the form of grooves (flutes) elongated along the magnetic force lines. In other words, waves are built up, in which the projection of the wave vector on the direction of the magnetic field is small. If simultaneously with the indicated instabilities, the frequency of which is ω, there propagates in the plasma along the constant magnetic field H_0 a magnetosonic wave (helicon) with frequency $\Omega \gg \omega$, min $(\omega_{pi}, \omega_{i.e}) \ll \Omega < \omega_{He}$, which influences only the motion of the electrons, then the latter, executing electric drift across the grooves of the perturbation, will decrease the electric fields, and consequently will stabilize the instability.

The condition under which the high-frequency (HF) field exerts a noticeable influence on the instability is clear from the foregoing: within a time on the order of the period of the HF field, the magnetized electrons should drift across the grooves of the instabilities (in practice, across the constant magnetic field) and traverse a distance of the order of the transverse wavelength of the instability $\lambda_\perp \sim k_\perp^{-1}$, i.e.,

$$\frac{cE_1}{H_0}\frac{k_\perp}{\Omega}=\frac{H_1}{H_0}\frac{k_\perp}{k_0}>1$$

(E_1 and H_1 are the amplitudes of the fields of the magnetosonic wave, k_0 is its wave vector along H_0). If it is recognized that in the frequency limit indicated above the longitudinal wave vector of the fast magnetosonic wave is

$$k_0=\frac{\omega_{pe}}{c}\sqrt{\frac{\Omega}{\omega_{He}}},$$

then the inequality can be written in the form

$$\frac{H_1}{H_0}\frac{k_\perp}{\Omega}v_{Te}\frac{c}{v_{Te}}\frac{\sqrt{\Omega\omega_{He}}}{\omega_{pe}}>1.$$

Ivanov, Rudakov, and Teichmann[1], who considered the stabilization by a HF magnetic field, have shown that the condition for a noticeable influence of this field on the instability is

$$\frac{H_1}{H_0}\frac{k_\perp}{\Omega}v_{Te}\gtrsim 1.$$

We see that under the condition

$$c\sqrt{\Omega\omega_{He}}/v_{Te}\omega_{pe}>1$$

which is easily realized experimentally, stabilization by means of a helicon can be realized at lower HF field amplitudes. In this connection, a clarification of the possible stabilization of the instability by means of weakly-damped helicons is of definite interest.

CALCULATION OF THE CORRECTION TO THE ELECTRON DISTRIBUTION FUNCTION

We consider a plasma which is not bounded in space and has a density that varies along the x axis, contained by a constant magnetic field H_0 directed along the z axis. A fast magnetosonic wave with frequency Ω $(\omega_{Hi} \ll \Omega \ll \omega_{He})$, propagating along H_0, has in this frequency interval a wave vector $k_0 = \omega_{pe}c^{-1}\sqrt{\Omega/\omega_{He}}$[2] and a circular polarization

$$\mathbf{H}_1(z,t)=H_1[\mathbf{e}_x\sin(k_0z-\Omega t)+\mathbf{e}_y\cos(k_0z-\Omega t)],$$
$$\mathbf{E}_1(z,t)=\frac{\Omega}{k_0c}H_1[\mathbf{e}_x\cos(k_0z-\Omega t)-\mathbf{e}_y\sin(k_0z-\Omega t)].$$

We investigate furthermore the stability of such a stationary state of the plasma against perturbations of the loss-cone and drift-loss-cone type, with frequencies ω and wave vectors k_z satisfying the inequalities $\omega \ll \Omega$ and $k_z \ll k_0$.

The equilibrium electron distribution function, satisfying the equation

$$\frac{\partial f_0}{\partial t}+(\mathbf{v}\nabla)f_0-\frac{e}{m}\left(\mathbf{E}_1+\frac{1}{c}[\mathbf{v},\mathbf{H}_0+\mathbf{H}_1]\right)\frac{\partial f_0}{\partial \mathbf{v}}=0, \qquad (1)^*$$

should be a function of the integrals of the characteristic equations

$$\frac{d\mathbf{r}}{dt}=\mathbf{v},\quad \frac{d\mathbf{v}}{dt}=-\frac{e}{m}\left(\mathbf{E}_1+\frac{1}{c}[\mathbf{v},\mathbf{H}_0+\mathbf{H}_1]\right). \qquad (2)$$

The integrals are the initial coordinates x_0, y_0, z_0, and also, accurate to terms of order $(H_1/H_0)^2$, the quantity

$$2\varepsilon\equiv\left(v_x-\frac{H_1}{H_0}\frac{\Omega}{k_0}\sin[k_0z_0-t(\Omega-k_0v_z^0)]\right)^2+\left(v_y+\frac{H_1}{H_0}\frac{\Omega}{k_0}\cos[k_0z_0-t(\Omega-k_0v_z^0)]\right)^2+v_z^2.$$

Here v_z^0 is the constant velocity component along H_0. In the absence of the helicon, the equilibrium distribution function depends on ϵ and on the coordinate x_0. We can therefore put $f_0 = f_0(\epsilon, x_0)$.

The small correction to the stationary distribution function, corresponding to the instabilities in question, satisfies the equation

$$\frac{df_1}{dt}=-\frac{e}{m}\nabla\varphi\frac{\partial f_0}{\partial \mathbf{v}}, \qquad (3)$$

*$[\mathbf{v}, \mathbf{H}_0 + \mathbf{H}_1] \equiv \mathbf{v}_0 \times (\mathbf{H}_0 + \mathbf{H}_1)$.

where the total derivative with respect to t denotes differentiation along the trajectory (2), and φ is the perturbation potential, Integrating along the trajectory, we obtain

$$f_1(\mathbf{v}, \mathbf{r}, t) = -\frac{e}{m}\int_{-\infty}^{t} \nabla\varphi \frac{\partial f_0}{\partial \mathbf{v}} dt'. \tag{4}$$

The derivative with respect to f_0 must be expressed in terms of the integrals of motion, which are constants along the trajectory:

$$\begin{aligned}\frac{\partial f_0}{\partial \mathbf{v}} = \frac{\partial f_0}{\partial \varepsilon}\Big\{\mathbf{e}_x\Big[v_\perp^0 \cos(\omega_{He}t + \vartheta) + v_z^0 \frac{H_1}{H_0}\sin(\Omega t - k_0 v_z^0 t - k_0 z_0)\Big] \\ + \mathbf{e}_y\Big[-v_\perp^0 \sin(\omega_{He}t + \vartheta) + v_z^0\frac{H_1}{H_0}\cos(\Omega t - k_0 v_z^0 t - k_0 z_0)\Big] \\ + \mathbf{e}_z\Big[v_z^0 - v_\perp^0 \frac{H_1}{H_0}\sin(\Omega t - k_0 v_z^0 t - \omega_{He} t - \vartheta - k_0 z_0)\Big]\Big\} - \mathbf{e}_y \frac{\partial f_0}{\partial x_0}\frac{1}{\omega_{He}}.\end{aligned} \tag{5}$$

All the quantities connected with the perturbation are assumed to be proportional to $\exp(i\mathbf{k}\cdot\mathbf{r} - i\omega t)$. In the presence of a helicon, however, the Fourier coefficients are themselves periodic functions of z and t. We therefore put

$$\varphi_{k\omega} = \sum_{n=-\infty}^{+\infty} \varphi_n \exp\{i[\mathbf{kr} - \omega t + n(k_0 z - \Omega t)]\},$$

$$f_{1k\omega} = \sum_{s=-\infty}^{+\infty} f_{1s}\exp\{i[\mathbf{kr} - \omega t + s(k_0 z - \Omega t)]\}. \tag{6}$$

Using (4)–(6), we can obtain expression for $f_{1s}(v_\perp^0, v_z^0)$, which reduces, after integration with respect to the transverse velocities, to the following:

$$\begin{aligned}f_{1s}(v_z^0) = ie\frac{\Omega}{2\pi}\int_0^{2\pi/\Omega} dt \sum_{n, m=-\infty}^{+\infty} \varphi_n I_m(\nu) e^{-\nu}\int_{-\infty}^{t} dt' \frac{f_0(v_z^0)}{T} \\ \times\Big[m\omega_{He} + (k_z + nk_0) + k_\perp v_z^0 \frac{H_1}{H_0}\cos(k_0 z_0 - \beta - \Omega t' + k_0 v_z^0 t') \\ + \frac{ck_y}{eH_0}\frac{\partial f_0(v_z^0)}{\partial x_0}\Big]\exp\Big\{i\Big[k_0 z_0(n - s) + (t' - t)(k_z v_z^0 - \omega + m\omega_{He}) \\ + (k_0 v_z^0 - \Omega)(nt' - st) + \frac{H_1}{H_0}\frac{k_\perp}{k_0}\sin(k_0 z_0 - \beta - \Omega t' + k_0 v_z^0 t') \\ - \frac{H_1}{H_0}\frac{k_\perp}{k_0}\sin(k_0 z_0 - \beta - \Omega t + k_0 v_z^0 t)\Big]\Big\},\end{aligned} \tag{7}$$

where $I_m(\nu)$ is the modified Bessel function,

$$\nu = \frac{k_\perp^2 v_\perp^2}{\omega_{He}^2} \ll 1, \quad \beta = \operatorname{arctg}\left(-\frac{k_x}{k_y}\right), \quad k_\perp = \sqrt{k_x^2 + k_y^2} \gg k_z.$$

After integration, expression (7) can be reduced, assuming $|k_z v_z^0 - \omega| \ll |k_0 v_z^0 - \Omega|$, to the form

$$\begin{aligned}f_{1s}(v_z^0) = \frac{e\varphi_s}{m}\frac{k_\perp^2}{\omega_{He}^2} f_0(v_z^0) + \frac{e\varphi_s}{T}\frac{v_z^0}{v_z^0 - \Omega/k_0} f_0(v_z^0) \\ + \frac{e}{T}\Big[\frac{v_z^0(\omega k_0 - k_z\Omega) f_0(v_z^0)}{(k_z v_z^0 - \omega)(k_0 v_z^0 - \Omega)} + \frac{cTk_y}{eH_0}\frac{1}{k_z v_z^0 - \omega}\frac{\partial f_0(v_z^0)}{\partial x_0}\Big] \\ \times\sum_{n=-\infty}^{+\infty}(-1)^{n+s}\varphi_n J_n(\mu) J_s(\mu) e^{i(n-s)\beta},\end{aligned} \tag{8}$$

where $J_n(\mu)$ is a Bessel function of $\mu = k_\perp H_1/k_0 H_0$.

The obtained expression can be used to investigate the result of the action of a helicon on instabilities whose parameters satisfy the conditions for the applicability of (8). We shall use expression (8) to investigate the stabilization of the cone[3] and drift-cone[4] instabilities.

CONE AND DRIFT-CONE INSTABILITIES

A. A homogeneous plasma, $\partial f_0/\partial x = 0$, contained in a trap with magnetic mirrors, has an instability due to the anisotropy of the ion distribution function, resulting from the lost cone (cone instability). The dispersion equation of this instability, for a plasma of a density such that $\omega_{pi} \gg \omega_{Hi}$ and $k_z \ll k_\perp$, has the following form in the limit of low electron temperature[3,5]

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2}{\omega^2}\frac{k_z^2}{k^2} = \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} F\left(\frac{\omega}{kv_{Ti}}\right). \tag{9}$$

Equation (9) has been derived under the assumption $\gamma \equiv \operatorname{Im}\omega > \omega_{Hi}$. The maximum increment is possessed by oscillations with

$$\omega \approx kv_{Ti} \approx \omega_{pi} \approx k_z v_{Ti}\left(\frac{M}{m}\right)^{1/2}.$$

The dispersion equation of the cone instability, with allowance for the helicon, is obtained in the following manner. Substituting in the Poisson equation for each of the components of the expansion (6) the perturbation of the electron density, obtained by integrating (8) with respect to the longitudinal velocities, and the perturbation of the ion density, used in (9) (we take into account the fact that a helicon with $\Omega \gg \omega_{Hi}$ acts only on the electrons), we obtain an infinite system of equations with respect to φ_s:

$$\begin{aligned}k_\perp^2\varphi_s = \frac{\omega_{pi}^2}{v_{Ti}^2}F\varphi_s\delta_{s0} - \frac{\omega_{pe}^2}{\omega_{He}^2}k_\perp^2\varphi_s - \frac{4\pi e^2}{T}\varphi_s\int_{-\infty}^{+\infty}\frac{v_z^0 f_0(v_z^0)}{v_z^0 - \Omega/k_0}dv_z^0 \\ - \frac{4\pi e^2}{T}\int_{-\infty}^{+\infty}\Big[\frac{v_z^0(\omega k_0 - k_z\Omega) f_0(v_z^0)}{(k_z v_z^0 - \omega)(k_0 v_z^0 - \Omega)} + \frac{cTk_y}{eH_0}\frac{1}{k_z v_z^0 - \omega}\frac{\partial f_0(v_z^0)}{\partial x_0}\Big]dv_z^0 \\ \times\sum_{n=-\infty}^{+\infty}(-1)^{n+s}\varphi_n J_n(\mu) J_s(\mu) e^{i(n-s)\beta}.\end{aligned} \tag{10}$$

Here δ_{s0} is the Kronecker symbol. From the system (10), using the property of the Bessel functions

$$\sum_{s=-\infty}^{+\infty} J_s^2 = 1,$$

we obtain a dispersion equation, which after calculation of the integrals under the assumption that Ω/k_0 and $\omega/k_z \gg v_{Te}$, takes the form

$$\begin{aligned}\Phi(\omega) \equiv \Big[1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2}{\Omega^2}\frac{k_0^2}{k^2} - \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}F\left(\frac{\omega}{kv_{Ti}}\right)\Big]\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\right. \\ \left. - \frac{\omega_{pe}^2}{\omega^2}\frac{k_z^2}{k^2}\right) + J_0^2(\mu)\frac{\omega_{pi}^2}{k^2 v_{Ti}^2}F\left(\frac{\omega}{kv_{Ti}}\right)\frac{\omega_{pe}^2}{k^2}\left(\frac{k_0^2}{\Omega^2} - \frac{k_z^2}{\omega^2}\right) = 0.\end{aligned} \tag{11}$$

The number of zeroes of this equation in the upper half of the complex ω plane, i.e., the number of unstable solutions, is determined by the formula[6]

$$N = \frac{1}{2\pi i}\int_C \frac{d\ln\Phi}{d\omega}d\omega$$

(the argument principle), where C is the contour in the upper half-plane of ω, inside which there are no singularities of the function $\Phi(\omega)$.

Choosing the contour C along the real ω axis from $-\infty$ to $+\infty$, with an infinitesimally small half-circle above the pole $\omega = 0$ and closed by a circle of infinitely large radius, we can obtain for N the expression

$$N = 1 + \frac{1}{2\pi}[\operatorname{Arg}\Phi(+\infty) - \operatorname{Arg}\Phi(-\infty)].$$

Analyzing the increment of the argument of the vector Φ when ω changes from $-\infty$ to $+\infty$ at different values of $J_0(\mu)$, bearing in mind a form of the function $F(\omega/kv_{Ti})$ which is typical for the ion distribution in a trap[3,5], we can show that the unstable solutions vanish if

$$\operatorname{Re}\Phi(\omega_1) < 0, \tag{12}$$

where $\omega_1 = kv_{Ti}y_1$, and the point y_1 is characterized by the fact that in it $\operatorname{Im} F(y_1) = 0$. However, the waves are stabilized only in a limited range of k_z:

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_z^2}{\omega_1^2 k^2} < 0 \tag{13}$$

and at definite helicon parameters:

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_0^2}{\Omega^2 k^2} > 0.$$

On the other hand, if the inequality sign in (13) is reversed, then the increment remains positive and proportional to $J_0^2(\mu)$ ($J_0^0 \ll 1$).

It is possible to write down for k_z formal conditions that determine the stable solutions. At small J_0^2 we obtain from (11) the following approximate expression for the increment:

$$\gamma \approx \frac{1}{2}\omega_0\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_0^2}{k^2\Omega^2}\right)^2 \frac{\omega_{pi}^2}{k^2 v_{Ti}^2} \operatorname{Im} F\left(\frac{\omega_0}{kv_{Ti}}\right) J_0^2(\mu)$$
$$\times\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{-1}\left\{\left[1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_0^2}{\Omega^2 k^2} - \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\operatorname{Re} F\left(\frac{\omega_0}{kv_{Ti}}\right)\right]^2 + \left(\frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\right)^2\left[\operatorname{Im} F\left(\frac{\omega_0}{kv_{Ti}}\right)\right]^2\right\}^{-1}. \tag{14}$$

Here

$$\omega_0^2 = \frac{k_z^2}{k^2}\omega_{pe}^2\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{-1} \tag{15}$$

is the spectrum of the stable solutions of (11) when $J_0(\mu) = 0$.

The increment (14) is positive if $\operatorname{Im} F(\omega_0/kv_{Ti}) > 0$. It is known[3] that this is the region of frequencies ω_0 from 0 to $\omega_1 = kv_{Ti}y_1$. Alternately, if we take (15) into account, this is the k_z region from 0 to k_{z1}, satisfying the relation

$$\omega_1^2 = \frac{k_{z1}^2}{k^2}\omega_{pe}^2\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{-1}$$

or

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_{z1}^2}{\omega_1^2 k^2} = 0.$$

In other words, the range of k_z in which $\gamma > 0$, as already determined, is

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_z^2}{\omega_1^2 k^2} \geqslant 0.$$

If we calculate γ from (11) with greater accuracy, then we can readily find that with increasing $J_0^2(\mu)$ the region of k_z in which $\gamma > 0$ broadens in the direction of larger k_z. In other words, the range of k_z

$$1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_z^2}{\omega_1^2 k^2} < 0$$

yields stable solutions only in the limit $J_0^2(\mu) \approx 0$ (sufficiently large amplitudes of the HF fields). On the other hand, with increasing $J_0^2(\mu)$, a region of small positive increments appears also in this range. But small increments can be disregarded, since the instability in question exists only when $\gamma > \omega_{Hi}$[5,7].

Thus, with increasing amplitude of the helicon, i.e., with decreasing $J_0(k_\perp H_1/k_0 H_0)$, the increment decreases for all k_z, albeit differently for different k_z. And since the increment γ vanishes everywhere when $J_0 = 0$, there exists a minimum $J_0(\mu)$ (maximum H/H_0) such that the condition for the existence of the instability ceases to be satisfied even for the maximum increment ($\gamma_{max} > \omega_{Hi}$).

Let us estimate this necessary value of H_1/H_0 (it is important that H_1/H_0 turn out to be much smaller than unity). Using the expression for γ given by (14), we estimate the upper bound of γ_{max}:

$$\gamma_{max} < \omega_{pi} J_0^2(\mu). \tag{16}$$

It follows therefore that the condition for the existence of the instability is certainly violated when

$$J_0^2\left(\frac{k_\perp}{k_0}\frac{H_1}{H_0}\right) < \frac{\omega_{Hi}}{\omega_{pi}}. \tag{17}$$

Recognizing that for the cone instability $k_{\perp\min} \lesssim 1/r_{Di}$ (r_{Di} is the Debye radius), the wave vector of the helicon is equal to $k_0 = \omega_{pe}c^{-1}\sqrt{\Omega/\omega_{He}}$, and the minimum frequency is $\Omega \gtrsim \omega_{pi}$, we obtain

$$\mu_{\min} = \left(\frac{k_\perp}{k_0}\frac{H_1}{H_0}\right)_{\min} \sim \frac{c}{v_{Ti}}\sqrt{\frac{\omega_{Hi}}{\omega_{pi}}}\frac{H_1}{H_0}.$$

Since $\omega_{Hi} \ll \omega_{pi}$ for the instability in question, the inequality (17) is satisfied when $\mu_{\min} \gtrsim 2$, i.e., when

$$\frac{H_1}{H_0} \gtrsim 2\sqrt{\frac{\omega_{pi}}{\omega_{Hi}}}\frac{v_{Ti}}{c}.$$

Thus, if $\omega_{Hi}/\omega_{pi} = 0.2$, then to satisfy the inequality (17) in the entire range $k_\perp \gtrsim r_{Di}^{-1}$ it is necessary to have

$$\frac{H_1}{H_0} \gtrsim \frac{1.6}{\sqrt{0.2}}\frac{v_{Ti}}{c} \simeq 4\frac{v_{Ti}}{c}.$$

We note for comparison, that to stabilize loss-cone instability in a plasma with cold electrons by means of an HF magnetic field[1] the amplitudes required were $H_1/H_0 > v_{Ti}/v_{Te}$, i.e., much larger.

B. In a spatially inhomogeneous plasma there can develop an instability with $k_z = 0$ and $\omega_{Hi} \ll \omega \ll \omega_{He}$ (drift-loss-cone instability)[4]. By a method analogous to that proposed in subsection A above, using (8), we can obtain a dispersion equation for this instability in the presence of a helicon:

$$\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2} + \frac{1}{k^2 r_{Di}^2}\frac{\omega^*}{\omega}\right)\left(1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_0^2}{\Omega^2 k^2}\right) \tag{18}$$
$$= i\frac{\omega}{|k| v_c}\frac{b}{k^2 r_{Di}^2}\left\{1 + \frac{\omega_{pe}^2}{\omega_{He}^2} - \frac{\omega_{pe}^2 k_0^2}{\Omega^2 k^2} + [1 - J_0^2(\mu)] \times\left(\frac{\omega_{pe}^2 k_0^2}{\Omega^2 k^2} + \frac{\omega^*}{\omega}\frac{1}{k^2 r_{Di}^2}\right)\right\}$$

(the notation is the same as in[4]). It turns out that the maximum increment corresponds to the same k_m as in a plasma without helicons, and the maximum increment itself has an order of magnitude

$$\gamma \approx J_0^2\left(\frac{k_m H_1}{k_0 H_0}\right)\left(\frac{\rho_i}{a}\right)^{3/4}\left(\frac{v_c}{v_T}\right)^{1/4}\frac{\omega_{pi}}{(1 + \omega_{pe}^2/\omega_{He}^2)^{1/2}} = J_0^2\gamma_0 \tag{19}$$

i.e., it decreases by a factor $J_0^2(\mu)$ in the presence of the helicon. On the other hand, the condition for eliminating the instability ($\gamma < \omega_{Hi}$[4]) takes the form

$$J_0^2\left(\frac{k_m H_1}{k_0 H_1}\right) < \frac{\omega_{Hi}}{\omega_{pi}}\left(1+\frac{\omega_{pe}^2}{\omega_{He}^2}\right)^{1/2}\left(\frac{v_T}{v_c}\right)^{1/4}\left(\frac{a}{\rho_i}\right)^{3/4}. \quad (20)$$

Let us estimate the necessary helicon amplitude under concrete assumptions:

$$\omega_{pe}^2 \gtrsim \omega_{He}^2, \quad v_T \sim v_c, \quad \omega_{Hi} \sim 0.1\Omega \sim 0.1\gamma_0.$$

We get

$$J_0^2\left(\frac{k_m H_1}{k_0 H_0}\right) \lesssim 0{,}1,$$

$$\frac{k_m H_1}{k_0 H_0} \gtrsim 2, \quad \frac{H_1}{H_0} \gtrsim 2\sqrt{\frac{\omega_{pi}}{\omega_{Hi}}}\frac{v_{Ti}}{c}\frac{(1+\omega_{pe}^2/\omega_{He}^2)^{1/4}}{(av_T/\rho_i v_c)^{1/8}} \approx 2\sqrt{\frac{\omega_{pi}}{\omega_{Hi}}}\frac{v_{Ti}}{c}$$

(v_T is the thermal velocity of the ions).

DISCUSSION OF RESULTS

Thus, the effect of stabilization of instabilities by means of a helicon occurs and is observed at lower HF field amplitudes than in the case of stabilization with an HF magnetic field (see[1]). If we speak, for concreteness, of stabilization of loss-cone instability of a plasma with cold electrons, for which it is necessary in both methods to have

$$\mu_r = \frac{k_\perp H_1}{k_0 H_0} \gtrsim 1, \quad \mu_H = \frac{k_\perp}{\Omega} v_{Te}\frac{H_1}{H_0} \gtrsim 1,$$

then, even by choosing the worst conditions (from the point of view of stabilization) for the helicon ($k_{0max} \sim \omega_{pe}/c$) and the best ones ($\Omega_{min} \sim \omega_{pi}$) for the HF magnetic field[1], we find that $\mu_{hel} > \mu_H$ at the same value of H_1. Thus, indeed, in the case of a helicon smaller HF field amplitudes are necessary. This is connected with the fact that the motion of the electrons in the HF field across H_0, and consequently (for the instabilities in question) across the grooves of the instabilities, which lead to the smoothing of the potential, at identical HF field amplitudes, occurs with a larger velocity cE_1/H_0) in the case of stabilization by means of a helicon than in the case of stabilization by means of an HF magnetic field ($v_{Te}H_1/H_0$). (This follows from the fact that the phase velocity of the stabilizing helicon is larger than the thermal velocity of the electrons.)

From the computational point of view, the use of the helicon is also successful. Owing to the fact that the arguments of the Bessel functions do not depend on the velocities in the case under consideration, it is possible to obtain in a simple manner a dispersion equation that takes into account the influence of all the harmonics of the helicon.

The harmonics of the HF field can be easily taken into account also in the case of stabilization by means of an HF magnetic field[1]. By a method analogous to that proposed above, it is possible to obtain an infinite system of equations with respect to HF harmonics of the perturbations of the potential. However, owing to the resultant dependence of the arguments of the Bessel functions on the velocities, the dispersion equation corresponds to the vanishing of an infinite determinant. The convergence of the determinant is ensured by the rapid decrease of the Bessel functions with increasing order. Taking this into account, it is possible to confine oneself to approximate dispersion equations, as was done indeed in[1]. We note that for universal instability it is possible to obtain also in this case a dispersion equation in closed form (the arguments of the Bessel functions do not depend on the velocities), from which it follows that the role of the higher harmonics is slight.

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