EVOLUTION OF A CHARGED SPHERE AFTER COLLAPSE UNDER A SCHWARZSCHILD SPHERE

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Replacement of the relativistic collapse of a charged sphere by expansion inside a Schwarzschild sphere is considered. It has been shown in[2] that if the field outside the sphere is always a Reisner-Nordstrom field, then sphere contraction will always be replaced by expansion into another outer space. It is shown in the present paper that a solution can be set up for which passage into another outer space does not occur. For this purpose one has to change the conditions in the space-time region within the Schwarzschild sphere (about which an outer observer will never learn). Another charged sphere can be put into this region, its "explosion" changing the solution for the first sphere in such a way that passage into the other outer space will be hindered. For an observer in the first outer space the total picture of the collapse will not change and he sees an asymptotic approach of the sphere's surface to the Schwarzschild sphere.

IN earlier papers[1,2], the author constructed a solution of the problem of the relativistic collapse of a charged sphere. After compression to below the Schwarzschild sphere, the sphere again expands, but now in a different external space, lying in the absolute future relative to the space from which the compression took place. It is clear that the evolution of the sphere in this second space depends on the processes occurring in that space, and is not determined completely by the initial conditions in the first space (the absence of a Cauchy hypersurface). In the present article we wish to emphasize that even the very fact of the expansion of the sphere in the second outer space cannot be determined completely, by the conditions in the first space. The evolution of the sphere, its emergence to the second space (and thus, the very existence of the second space for the sphere) depend on the conditions in the space-time region "between" the first and second spaces; these conditions must be specified in addition to the conditions in the first space.

We shall consider below an example in which, under identical conditions for the first space, the sphere expands in the second space in one case, and does not in another.

We consider the collapse of a charged sphere. At the initial instant the matter in the sphere has a low density. Let the charge of the sphere be $\epsilon < mG^{1/2}$, where G is Newton's gravitational constant and m is the mass of the sphere. The sphere will collapse.

Assume that there is still nothing outside the sphere, with the exception of its electric field. The solution of the problem is given in[1,2]. The space-time metric *outside the sphere* is the Reisner-Nordstrom metric. The world line of the surface of the sphere is shown in Fig. 1. The vertical axis represents the proper time τ, and the horizontal the radial coordinate R of the system, co-moving with the matter inside the sphere and continuously continued by trial particles outside the sphere (for details see[2]). The region occupied by the matter of the sphere is shown cross-hatched in the figure.

The sphere compresses from an external space A, its surface crosses the Schwarzschild sphere

$$r = r_g = \frac{Gm}{c^2}\left(1 + \sqrt{1 - \frac{\varepsilon^2}{Gm^2}}\right),$$

(r—length of the circle at a fixed radial coordinate), continues to be compressed in the nonstatic region T_-, and crosses its boundary

$$r = r_1 = \frac{Gm}{c^2}\left(1 - \sqrt{1 - \frac{\varepsilon^2}{Gm^2}}\right).$$

In the region B, the compression of the sphere gives way to expansion, and after crossing r_1' and r_g' the sphere goes out into a second external space C.

In the internal region B there is a true time-like singularity r = 0 located outside the sphere. We note that the spatial section (a, b, c) is closed (in analogy with the closed Friedmann cosmological model), and the true singularity is located at the pole opposite to the center of the sphere. The dashed lines in region B are the world lines r = const, which have an infinite (proper) length, i.e., particles with r = const can exist

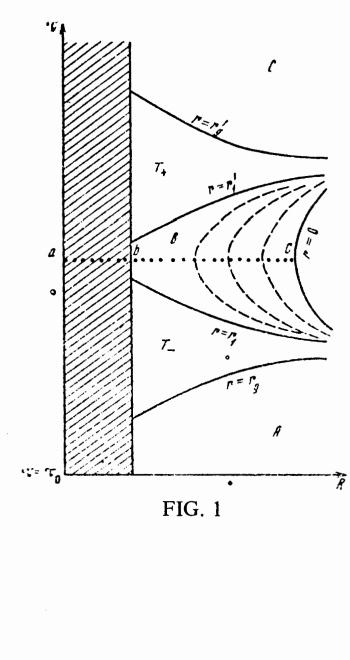

FIG. 1

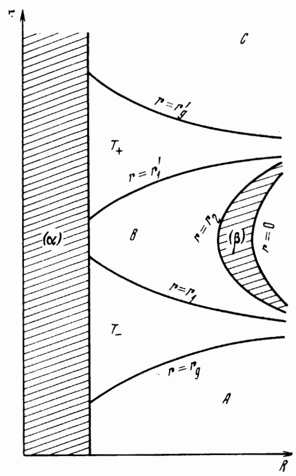

FIG. 2

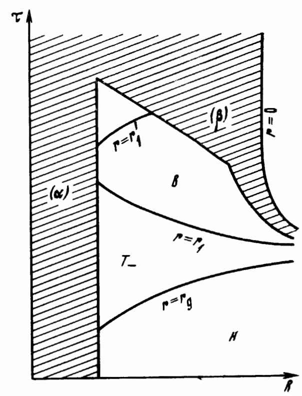

FIG. 3

forever in this region "between" the two ordinary spaces A and C, which are Euclidean at infinity.

We note now that by specifying the initial data in space A, for example at $\tau = \tau_0$ (Fig. 1), we determine the evolution only to the left of the line $r = r_1$, since this line is the last characteristic (the zeroth geodesic) arriving from the space A from $r = +\infty$ at $\tau = \tau_0$. Owing to the relativistic slowing down of the time, this occurs after a finite τ. Whatever occurs to the right and above this characteristic $r = r_1$ is no longer determined completely by the initial conditions on the section $\tau = \tau_0$ (there is no Cauchy hypersurface). It is possible, without changing anything in the region to the left of $r = r_1$, to specify different conditions in the region to the right of $r = r_1$ (of course, satisfying the Einstein equations and continuing smoothly at $r = r_1$ into the remaining solution). We assume now that matter from a different charged static sphere β[1] is located to the right of $r = r_1$, in region B, in place of the singularity $r = 0$ around the pole opposite to our sphere α. The possibility of this fact is discussed in the Appendix below. It is important to note that in this case the gravitational and electric field outside the sphere β do not change in any way, i.e., the entire evolution of the sphere α remains the same as before. The world line of the surface of the sphere β will be $r_2 = \text{const}$, the sphere β can exist infinitely long in its proper time. The corresponding space-time is shown in Fig. 2.

We note here that if there are no true singularities in the solution, then the matter of sphere β (or at least part of the matter) should be not under pressure but under strong tension (generally speaking, anisotropic), but this does not contradict the physical laws (for details see the Appendix).

We now stipulate that the sphere β be static not everywhere, but only up to an arbitrary instant of its proper time, after which it explodes. Its matter expands and collides with the expanding sphere α, see Fig. 3. The evolution of the sphere α now changes. Clearly, after collision, emergence to the outer space C is impossible. The evolution of the entire matter of both spheres after the collision is the evolution of a closed world, in which there is no boundary surface and which can appear in the outer space C[2]. By the same token, we have constructed examples in which, under absolutely identical conditions in space A, where the sphere α begins to collapse, in one case the sphere emerges to the second space C, and in the second case it does not, of course, such a possibility is due to the absence of a Cauchy hypersurface in space A.

We note that the very possibility of the absence of an initial Cauchy hypersurface in relativistic problems has been noted many times in the literature (see, for example,[3,4]). Usually such a situation was regarded as exotic, incapable of occurring in real processes of a real universe.

The foregoing examples show that, in principle, such a situation, which leads to unique consequences, can occur in reality (of course, assuming that relativity theory is valid).

In conclusion we note that the absence of a Cauchy hypersurface probably has no significant influence on the occurrence of a singularity in relativistic collapse (Thorne[3]). Indeed, according to Hawking's theorem 2[5], a singularity arises during the course of the collapse under natural physical requirements, regardless of the presence or absence of a Cauchy hypersurface.

APPENDIX

Different aspects of the collapse of short spheres were discussed many times (see, for example[6-12]). We are interested here in a static solution for a charged sphere with radius $r_2 < r_1$ and the subsequent expansion of this sphere.

We write the interval in the form

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2). \quad (1)$$

Outside the sphere we have the Reisner-Nordstrom metric

$$e^{\nu} = e^{-\lambda} = 1 - \frac{2Gm}{c^2 r} + \frac{G\varepsilon^2}{cr^2}. \quad (2)$$

For the material inside the sphere we stipulate

$$T_0^0 = \varepsilon > 0, \ |T_1^1| \leqslant \varepsilon, \ |T_2^2| = |T_3^3| \leqslant \varepsilon. \quad (3)$$

We shall show that at least in part of the matter of such a static sphere there takes place the inequality[3]

$$T_0^0 - T_1^1 - T_2^2 - T_3^3 < 0, \quad (4)$$

i.e., strong tension is present.

At the surface of the sphere we have[4] $\nu' < 0$. The function $\nu(r)$ is bounded and has a maximum at $r = r_0$, $0 \le r \le r_1$. At $r = r_0$ we have $\nu'' < 0$ and $\nu' = 0$, and we obtain from gravitational equations

$$\varkappa(T_0^0 - T_1^1 - T_2^2 - T_3^3) = e^{-\lambda}[\nu'' + 2\lim_{r\to r_0}(\nu'/r)] < 0, \quad (5)$$

where κ is the Einstein constant. The quantity

1) The charges of the spheres β and α are opposite, but equal in absolute magnitude.

2) According to a remark by Ya. B. Zel'dovich, an interesting situation arises if the time is reversed ($\tau \to -\tau$). We then obtain an example of "opening" of a closed world.

3) This inequality follows indirectly in our problem from Hawking's second theorem [5]. It is obvious that ordinary matter and an electromagnetic field do not satisfy this inequality.

4) The prime denotes the derivative with respect to r.

$\lim_{\nu \to \nu_0}(\nu'/r)$ vanishes at $r_0 \neq 0$ and is negative when $r_0 = 0$[5)].

A static sphere with an equation of state satisfying (5) everywhere inside the sphere (except the regions at the surface itself), corresponding to gravitational repulsion, can be described, for example, by Bardin's solution[4], which joins the external solution (2) on the surface of the sphere. A solution satisfying (3) and describing gravitational attraction near the center may be, for example, the following:

$$e^{\nu} = e^{-\lambda} = B^2 e^{-A^2 \cos \alpha r}, \tag{6}$$

where A^2, B^2, and α are suitably chosen constants. At the surface of the sphere, the solution should join smoothly with the external solution (2).

5)If $\nu'' = 0$ when $r = r_0$, then inequality (5) is satisfied at neighboring points.

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