SECOND SOUND IN ANTIFERROMAGNETIC SUBSTANCES

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The question of the existence of second sound in uniaxial antiferromagnetic substances of the "easy plane" and "easy axis" type is considered, taking the interaction between the magnon and phonon subsystems into account. In the majority of cases the contribution of the magnon subsystem to the second sound is found to be important in antiferromagnetics with small values of the ratio of the Neel and Debye temperatures: $\Theta_N/\Theta_D < 1$. But in certain situations, e.g., in magnetic fields close to the effective fields of the first- or second-order phase transition, such a contribution can be found in antiferromagnetics with any value of Θ_N/Θ_D up to $\Theta_N/\Theta_D \gg 1$. The temperature dependence of the second sound velocity is studied in a broad range of magnetic fields. It is shown, in particular, that in antiferromagnetics of the "easy axis" type in the critical field for "inverting" the magnetic moments of the sublattices (a first-order phase transition), a discontinuity in the second sound velocity should be observed at sufficiently low temperatures.

1. INTRODUCTION

It is well known that second sound consists of slowly decaying oscillations of the density of thermal excitations. For its existence it is necessary that normal collisions between quasi-particles be much more probable than collisions with loss of quasi-momentum; this condition is usually fulfilled in dielectrics at sufficiently low temperatures. Only under this condition is it possible for such temperature oscillations, analogous to elastic oscillations of the density of matter, to arise in a system of quasi-particles.

The possibility of the existence of two types of oscillation of different nature (ordinary sound and temperature waves) in superfluid helium was first pointed out by Landau.[1] After this, a large number of papers appeared in which the possibility of similar oscillations in solids was considered. In the larger part of these papers second sound in a phonon gas was investigated, and only a few papers[2-4] are concerned with the study of second sound in a magnon gas. In the latter papers, which are devoted to second sound in ferromagnetics, an isolated system of magnons was considered without taking account of the coupling with the phonon system. Results of a calculation performed with allowance for the interaction of the magnon and phonon subsystems are given in the monograph,[5] whence it is clear that it is extremely important to allow for this interaction when the subsystems are in a state of local equilibrium.

In the present paper the question of the existence of second sound in antiferromagnetics is investigated. The temperature and magnetic field dependences of the second sound velocity in a coupled system of magnons and phonons are obtained when these subsystems are in a local equilibrium state. Similar results are also obtained for the isolated magnon subsystem. In this case the damping constants of the temperature waves are also calculated, in the τ-approximation. The calculations are performed for uniaxial antiferromagnetics of the "easy plane" (EP AFM) and "easy axis" (EA AFM) types.

We shall consider an antiferromagnetic in a variable and non-uniform temperature field $\tilde{T}(\mathbf{r}, t) = T[1 + \vartheta(\mathbf{r}, t)]$, where $|\vartheta| \ll 1$. The magnon distribution function n_i ($i = 1, 2$ corresponds to the two magnetic subsystems) and that for the phonons N_j (j is the polarization index) will obey the following kinetic equations:

$$\frac{\partial n_i}{\partial t} + \mathbf{v}_i \frac{\partial n_i}{\partial \mathbf{r}} = L_i\{n, N\} + \mathscr{L}_i\{n, N\},$$
$$\frac{\partial N_j}{\partial t} + \mathbf{s}_j \frac{\partial N_j}{\partial \mathbf{r}} = L_j\{N, n\} + \mathscr{L}_j\{N, n\}, \qquad (1)$$

where L and $\mathscr{L}$ are respectively the collision integrals for the quasi-particles with and without conservation of the quasi-momentum. It is assumed that $L \gg \mathscr{L}$. It is known (see, e.g.,[5]) that in this case the initial distribution functions are the local-equilibrium magnon and phonon distribution functions

$$n_i^{(0)} = \left\{\exp\left[\frac{\varepsilon_i(\mathbf{k}) - \mathbf{uk}}{T(1+\vartheta)}\right] - 1\right\}^{-1}, \quad N_j^{(0)} = \left\{\exp\left[\frac{\omega_j(\mathbf{k}) - \mathbf{uk}}{T(1+\vartheta)}\right] - 1\right\}^{-1}, \qquad (2)$$

which depend on the local temperature $\vartheta(\mathbf{r}, t)$ and on the local velocity $\mathbf{u}(\mathbf{r}, t)$ of the ordered motion of the quasi-particles. The functions ϑ and $\mathbf{u}$ are determined by the local conservation laws for energy and momentum, which, starting from (1), can be represented in the form

$$\sum_{\mathbf{k}} \Big\{ \frac{\partial}{\partial t} \Big[\sum_i \varepsilon_i(\mathbf{k}) n_i^{(0)}(\mathbf{k}) + \sum_j \omega_j(\mathbf{k}) N_j^{(0)}(\mathbf{k}) \Big] + \frac{\partial}{\partial \mathbf{r}} \Big[\sum_i \varepsilon_i(\mathbf{k}) \mathbf{v}_i n_i^{(0)}(\mathbf{k}) + \sum_j \omega_j(\mathbf{k}) \mathbf{s}_j N_j^{(0)}(\mathbf{k}) \Big] \Big\} = 0,$$

$$\sum_{\mathbf{k}} \mathbf{k} \Big\{ \frac{\partial}{\partial t} \Big[\sum_i n_i^{(0)}(\mathbf{k}) + \sum_j N_j^{(0)}(\mathbf{k}) \Big] + \frac{\partial}{\partial \mathbf{r}} \Big[\sum_i \mathbf{v}_i n_i^{(0)}(\mathbf{k}) + \sum_j \mathbf{s}_j N_j^{(0)}(\mathbf{k}) \Big] - \sum_i \mathscr{L}_i\{n^{(0)}, N^{(0)}\} - \sum_j \mathscr{L}_j\{N^{(0)}, n^{(0)}\} \Big\} = 0,$$

where $\epsilon(\mathbf{k})$ and $\omega(\mathbf{k})$ are the energies of a spin wave and a phonon with wave-vector $\mathbf{k}$; $\mathbf{v}_i = \partial\epsilon_i/\partial\mathbf{k}$, $\mathbf{s}_j$

$= \partial\omega_i/\partial \mathbf{k}$, $\hbar = 1$ and the temperature is given on the energy scale.

Since $|\vartheta| \ll 1$ and $|\mathbf{k}\cdot\mathbf{u}/T| \ll 1$, these equations can be given in a simpler form:

$$(c_s + c_l)\dot{\vartheta} + {}^1/_3(a_s + a_l)\,\mathrm{div}\,\mathbf{u} = 0,$$

$$(B_s + B_l)\dot{\mathbf{u}} + (a_s + a_l)\nabla\vartheta + (A_s + A_l)\mathbf{u} = 0. \quad (3)$$

Here the spin parameters are:

$$c_s = -\frac{1}{TV}\sum_{i,\mathbf{k}} \varepsilon_i^2(\mathbf{k})\frac{\partial \bar{n}_i}{\partial \varepsilon_i}, \quad a_s = -\frac{1}{TV}\sum_{i,\mathbf{k}} \varepsilon_i(\mathbf{k}) v_i k \frac{\partial \bar{n}_i}{\partial \varepsilon_i},$$

$$B_s = -\frac{1}{TV}\sum_{i,\mathbf{k}} k^2 \frac{\partial \bar{n}_i}{\partial \varepsilon_i}, \quad A_s = -\frac{1}{TV}\sum_{i,\mathbf{k}} k^2 \tau_{iU}^{-1}\frac{\partial \bar{n}_i}{\partial \varepsilon_i}; \quad (4)$$

the phonon parameters are:

$$c_l = -\frac{1}{TV}\sum_{j,\mathbf{k}} \omega_j^2(\mathbf{k})\frac{\partial \bar{N}_j}{\partial \omega_j} = \frac{2\pi^2}{15}\sum_j \left(\frac{T}{s_j}\right)^3 \approx \frac{2\pi^2}{5a^3}\left(\frac{T}{\Theta_D}\right)^3,$$

$$a_l = -\frac{1}{TV}\sum_{j,\mathbf{k}} \omega_j(\mathbf{k}) s_j k \frac{\partial \bar{N}_j}{\partial \omega_j} = c_l,$$

$$B_l = -\frac{1}{TV}\sum_{j\mathbf{k}} k^2 \frac{\partial \bar{N}_j}{\partial \omega_j} = \frac{2\pi^2}{15}\sum_j \frac{1}{s_j^2}\left(\frac{T}{s_j}\right)^3 \approx \frac{2\pi^2}{5a^5\Theta_D^2}\left(\frac{T}{\Theta_D}\right)^3 \quad (5)$$

$$A_l = -\frac{1}{TV}\sum_{j,\mathbf{k}} k^2 \tau_{jU}^{-1}\frac{\partial \bar{N}_j}{\partial \omega_j} = \frac{2\pi^2}{15}\sum_j \frac{\tau_{jU}^{-1}}{s_j^2}\left(\frac{T}{s_j}\right)^3 \approx \frac{2\pi^2\tau_{lU}^{-1}}{5a^5\Theta_D^2}\left(\frac{T}{\Theta_D}\right)^3.$$

c_s and c_l are the spin and phonon specific heats, referred to one atom, $\bar{n}_i$ and $\bar{N}_j$ are the usual Bose magnon and phonon equilibrium distribution functions, τ_U^{-1} is the frequency of quasi-particle collisions with loss of quasi-momentum, $\Theta_D = s/a$ is the Debye temperature, and a is the lattice constant; the approximate equality signs in (5) correspond to the approximation $s_{t_1} = s_{t_2} = s_l = s$.

Assuming that ϑ and $\mathbf{u}$ are proportional to $\exp[i(\mathbf{q}\cdot\mathbf{r} - \omega t)]$, we can obtain from (3) the dispersion equation relating the frequency ω to the wave-vector q of the second sound in the system under consideration. The solution of this equation has the form

$$\omega = vq - i\Gamma, \quad (6)$$

where

$$v = (a_s + a_l)\,/\,\sqrt{3(B_s + B_l)(c_s + c_l)} \quad (7)$$

is the second sound velocity, and

$$\Gamma = (A_s + A_l)\,/\,2(B_s + B_l) \quad (8)$$

is the damping constant of the second sound.

The formulas given above are valid if there is local equilibrium between the spin and phonon subsystems (i.e., when the parameters ϑ and $\mathbf{u}$ are the same for both subsystems). In the case when we can regard the magnon subsystem as isolated from the phonon subsystem (assuming, however, that, although there is no *local equilibrium* between these subsystems there is such an equilibrium between the two magnon subsystems), for the velocities and damping constants of the second sound for each subsystem we have

$$v_s = \frac{a_s}{\sqrt{3B_s c_s}}, \quad \Gamma_s = \frac{A_s}{2B_s}; \quad (9)$$

$$v_l = \sqrt{\frac{c_l}{3B_l}}, \quad \Gamma_l = \frac{A_l}{2B_l}. \quad (10)$$

2. ANTIFERROMAGNETICS OF THE "EASY PLANE" TYPE

A. Weak Magnetic Fields ($H \ll H_E$)

In this case, when H is considerably smaller than the effective exchange field H_E, for an EP AFM with weak ferromagnetism the following relations are valid for the two spin-wave energy branches:

$$\varepsilon_1 = [\Theta_N^2(ak)^2 + \mu^2 H(H + H_D)]^{1/2},$$
$$\varepsilon_2 = [\Theta_N^2(ak)^2 + \mu^2 H_{AE}^2 + \mu^2 H_D(H + H_D)]^{1/2}, \quad (11)$$

where Θ_N is the Néel temperature, H_D is the Dzyaloshinskiĭ field, $H_{AE} = \sqrt{H_A H_E}$, and H_A is the uniaxial anisotropy field.

The spin parameters defined by the formulas (4) in this case take the form:

$$c_s = \frac{1}{2\pi^2 a^3}\left(\frac{T}{\Theta_N}\right)^3 \sum_i J_{1i}, \quad a_s = \frac{1}{2\pi^2 a^3}\left(\frac{T}{\Theta_N}\right)^3 \sum_i J_{2i},$$

$$B_s = \frac{1}{2\pi^2 a^5 \Theta_N^2}\left(\frac{T}{\Theta_N}\right)^3 \sum_i J_{2i}, \quad A_s = \frac{1}{2\pi^2 a^5\Theta_N^2}\left(\frac{T}{\Theta_N}\right)^3 \sum_i \tau_{iU}^{-1} J_{2i}, \quad (12)$$

where

$$J_{1i} = -\int_{X_i}^{\infty} dx \frac{\partial n}{\partial x}(x^2 - X_i^2)^{1/2} x^3, \quad J_{2i} = -\int_{X_i}^{\infty} dx \frac{\partial n}{\partial x}(x^2 - X_i^2)^{3/2} x; \quad (13)$$

$$X_1 = \frac{\mu}{T}\sqrt{H(H + H_D)}, \quad X_2 = \frac{\mu}{T}\sqrt{H_{AE}^2 + H_D(H + H_D)};$$

$$n = (\exp x - 1)^{-1}, \ i = 1, 2. \quad (14)$$

We shall consider the possible special cases.

1. $X_1 \ll 1$, $X_2 \ll 1$ ($T \gg \mu H$, $T \gg \mu H_E$; we assume that $H_D \sim H_{AE}$). In this case, $J_{1i} = J_{2i} = 4\pi^4/15$ and the second sound velocity is

$$v = \frac{1}{\sqrt{3}} a\Theta_N \left[\frac{1 + {}^3/_2(\Theta_N/\Theta_D)^3}{1 + {}^3/_2(\Theta_N/\Theta_D)^5}\right]^{1/2}. \quad (15)$$

If no account is taken of the magnon-phonon coupling, the second sound velocity v_s and damping constant Γ_s in the magnon gas have the form:

$$v_s = \frac{1}{\sqrt{3}} a\Theta_N, \quad \Gamma_s = {}^1/_4(\tau_{1U}^{-1} + \tau_{2U}^{-1}). \quad (16)$$

2. For a) $X_1 \ll 1$, $X_2 \gg 1$ ($T \gg \mu H$, $T \gg \mu\sqrt{HH_D}$, $T \ll \mu H_{AE}$), or for b) $X_1 \gg 1$, $X_2 \ll 1$ ($T \gg \mu H_{AE}$, $T \gg \mu\sqrt{HH_D}$, $T \ll \mu H$), the second sound velocity is

$$v = \frac{1}{\sqrt{3}} a\Theta_N \left[\frac{1 + 3(\Theta_N/\Theta_D)^3}{1 + 3(\Theta_N/\Theta_D)^5}\right]^{1/2}. \quad (17)$$

Without allowing for the magnon-phonon interaction,

$$v_s = a\Theta_N/\sqrt{3}; \ \text{a)}\ \Gamma_s = {}^1/_2\tau_{1U}^{-1}; \ \text{b)}\ \Gamma_s = {}^1/_2\tau_{2U}^{-1}. \quad (18)$$

3. $X_1 \gg 1$, $X_2 \gg 1$ (at least the condition $T \ll \mu\sqrt{HH_D}$ must be fulfilled). Putting $X_1 < X_2$ for simplicity, we obtain

$$v = \frac{1}{\sqrt{3}} a\Theta_N \frac{y_1(X_1) + (\Theta_N/\Theta_D)^3}{\{[y_1(X_1) + (\Theta_N/\Theta_D)^5][y_2(X_1) + (\Theta_N/\Theta_D)^3]\}^{1/2}}, \quad (19)$$

where

$$y_1(X) = \frac{30}{(2\pi)^{7/2}} X^{5/2} e^{-X}, \quad y_2(X) = \frac{10}{(2\pi)^{7/2}} X^{7/2} e^{-X}. \quad (20)$$

Here the following limiting cases are of interest (see Fig. 1).

a) $(\Theta_N/\Theta_D)^3 \ll y_1$ (region I in Fig. 1):

$$v = v_s = \frac{1}{\sqrt{3}} a\Theta_N \sqrt{y_1/y_2} = a\Theta_N \frac{T^{1/2}}{\mu^{1/2}(H^2 + HH_D)^{1/4}}, \qquad (21)$$

b) $(\Theta_N/\Theta_D)^5 \gg y_1$ (region II):

$$v = v_l = \frac{s_t}{\sqrt{3}} \sqrt{\frac{1 + {}^1/_2(s_t/s_l)^3}{1 + {}^1/_2(s_t/s_l)^5}} \approx \frac{1}{\sqrt{3}} a\Theta_D. \qquad (22)$$

It is clear from these formulas and from Fig. 1 that on change of H and T the character of the second sound also changes. On increase of the magnetic field or decrease of the temperature, the second sound passes over from purely magnon second sound in region I to purely phonon second sound in region II. In the transition region III, the change in the velocity v has an exponential character: $v \sim \exp\{\mu\sqrt{H(H + H_D)}/T\}$.

The approximate dependence of the velocity v on the magnetic field for an EP AFM is shown in Fig. 2 for the following parameter values: $\Theta_N/\Theta_D = 0.2$, $T = 0.13°K$, $H_D = 10^4$ Oe, $H_{AE} = 6 \times 10^4$ Oe, $H_E = 10^6$ Oe. The regions marked with Roman numerals are identical with the corresponding regions in Fig. 1.

It is clear from Fig. 1 that for an EP AFM in the case of sufficiently low temperatures and with $H \ll H_E$ a contribution of the magnons to the second sound can be found only for low values of the ratio Θ_N/Θ_D, not exceeding 0.5. For purely magnon second sound,

$$\Gamma_s = {}^1/_2 \tau_{1U}^{-1}. \qquad (23)$$

B. Strong Magnetic Fields ($H \lesssim H_E$)

In this case the main contribution to the phenomenon under consideration is made by the low-frequency branch of the spin oscillations

$$\varepsilon = \sqrt{\Theta^2(ak)^2 + \varepsilon_0^2}, \qquad (24)$$

where

$$\Theta = \Theta_N\sqrt{1 - (H/H_E^2)}, \quad \varepsilon_0 = \mu H_{AE}\sqrt{1 - (H/H_E)^2}. \qquad (25)$$

The second branch has an activation energy $\sim\mu H \gg T$

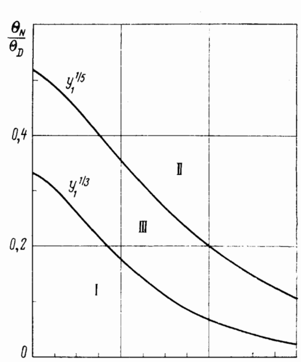

FIG. 1

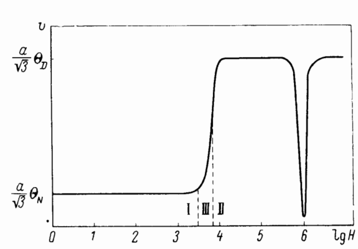

FIG. 2. Dependence of v on H in an EP AFM.

and therefore does not make an important contribution either to the spin parameters (4) or, consequently, to the second sound velocity.

The spin parameters for this case can be determined from the formulas (12), (13) if in these formulas we replace Θ_N by Θ and retain in the sums only one term, corresponding to the energy branch (i = 1) with small activation; the quantity

$$X_1 \equiv X = \varepsilon_0 / T. \qquad (26)$$

1. For $X \ll 1$ $(T \gg \epsilon_0)$:

$$v = \frac{1}{\sqrt{3}} a\Theta \left[\frac{1 + 3(\Theta/\Theta_D)^3}{1 + 3(\Theta/\Theta_D)^5}\right]^{1/2}. \qquad (27)$$

In an isolated magnon system,

$$v_s = \frac{1}{\sqrt{3}} a\Theta, \quad \Gamma_s = \frac{1}{2}\tau_{1U}^{-1}. \qquad (28)$$

2. For $X \gg 1$ $(T \ll \epsilon_0)$, the velocity v is determined by the expression (19) after making the replacements $\Theta_N \to \Theta$, $X_1 \to X$ in it. Corresponding to region I in Fig. 1 (where for this case the ratio Θ/Θ_D must be plotted along the ordinate axis) we shall have the following expression for v:

$$v = v_s = a\Theta_N(1 - H^2/H_E^2)^{1/4}\sqrt{T/\mu H_{AE}}, \quad \Theta/\Theta_D < y_1^{1/3}. \qquad (29)$$

We see that in strong magnetic fields the theoretical possibility arises of finding a magnon contribution to the second sound in an EP AFM with any value of the ratio Θ_N/Θ_D, even when $\Theta_N/\Theta_D \gtrsim 1$. The damping constant Γ_s in this case is expressed by formula (28).

3. ANTIFERROMAGNETICS OF THE "EASY AXIS" TYPE

It is known that EA AFM's can be found in three phase states, which we shall consider below, depending on the magnitude of the applied magnetic field (along the easy axis).

A. The Antiparallel Phase ($0 \le H < H_{AE}$)

In this case the spin wave spectrum has the form

$$\varepsilon_{1,2} = \sqrt{\Theta_N^2(ak)^2 + \mu^2 H_{AE}^2} \mp \mu H. \qquad (30)$$

At high temperatures $(T \gg \mu H_{AE})$, we have the same picture as in the EP AFM case; the quantities v, v_s and Γ_s are defined by the formulas (15) and (16).

Under the condition $\mu H \ll T \ll \mu H_{AE}$, the velocity v is given by the expression (19) in which the replacement $X_1 \to X_0 = \mu H_{AE}/T$ must be made. Under the condition $\mu H \sim T \ll \mu H_{AE}$, the analogous replacement in (19) appears thus: $y_i(X_1) \to y_i(X_0)\cosh(\mu H/T)$. The qualitative pattern in these latter two situations does not differ from case 3 in the EP AFM (for $H \ll H_E$). In particular, corresponding to the region I in Fig. 1 is the following expression for the second sound velocity:

$$v = v_s = a\Theta_N\sqrt{T/\mu H_{AE}}. \qquad (31)$$

We shall consider now the case of low temperature and the case of fields close to the field H_{AE} of the first order phase transition, i.e., when the condition

$$\mu(H_{AE} - H) \ll T \ll \mu H_{AE}. \qquad (32)$$

holds. In this case the velocity v takes the form

$$v=\frac{1}{\sqrt{3}}a\Theta_N\frac{2z_1(X_0)+(\Theta_N/\Theta_D)^3}{\{[z_2(X_0)+(\Theta_N/\Theta_D)^5][z_1(X_0)+(\Theta_N/\Theta_D)^3]\}^{1/2}} \qquad (33)$$

where

$$z_1(X)=\frac{75\zeta(5/2)}{2(2\pi)^{1/2}}X^{3/2},\quad z_2(X)=\frac{30\zeta(3/2)}{(2\pi)^{1/2}}X^{5/2}. \qquad (34)$$

The approximate boundary between the regions of magnon and phonon second sound is given here by the curve $z_1^{1/3}(X_0)=\alpha X_0^{1/2}$ ($\alpha\approx 0.5$). With the condition $T\ll\mu H_{AE}(\Theta_D/\Theta_N)^2$, which follows from the inequality $(\Theta_N/\Theta_D)^3\ll z_1(X_0)$, we have

$$v=v_s=\sqrt{\frac{5\zeta(5/2)}{3\zeta(3/2)}}a\Theta_N\sqrt{\frac{T}{\mu H_{AE}}}. \qquad (35)$$

If the temperature $T>\mu H_{AE}(\Theta_D/\Theta_N)^2$, which can be so, as is clear from a comparison of this inequality with inequality (32), only under the condition $\Theta_D^2\ll\Theta_N^2$, the second sound will be of purely phonon character (see formula (22)). In passing over from one limiting case to the other, the velocity v varies with temperature in accordance with a power law.

We see that in the case under consideration the region of existence of magnon second sound is greatly extended: it can be observed at any value of Θ_N/Θ_D right up to $\Theta_N/\Theta_D\gg 1$.

B. The Phase of "Collapse" of the Magnetic Moments ($H_{AE}<H<H_E$)

In this phase one of the spin wave energy branches is activationless:

$$\varepsilon_1=\sqrt{\Theta^2(ak)^2+\Theta_c^2(H)(ak)^4}, \qquad (36)$$

where the quantity Θ is determined by formula (25) and Θ_C, unlike Θ, remains finite at $H=H_E$ and is close to Θ_N in order of magnitude. The second energy branch has an activation, equal to $\mu(H-H_{AE})$, and consequently, in fields $H\sim H_E$ will give no contribution to the spin parameters (4).

In fields close to the field H_{AE} for the first-order phase transition, degeneracy ($\epsilon_1=\epsilon_2$) sets in and the two branches make the same contribution to the second sound velocity. In this case ($T\gg\mu(H-H_{AE})$) the quantities v, v_s and Γ_s are determined by the formulas (15) and (16).

From a comparison of the expressions for v to the left and to the right of the point $H=H_{AE}$ (formulas (35) and (15)), it is clear that, at sufficiently low temperatures, a discontinuity Δv should be observed in the second sound velocity at the first-order phase transition point (see Figs. 3 and 4).

With the conditions $\Theta_N<\Theta_D$ and $T\ll\mu H_{AE}$:

$$\Delta v=\frac{a\Theta_N}{\sqrt{3}}\left\{1-\left[\frac{5\zeta(5/2)}{\zeta(3/2)}\frac{T}{\mu H_{AE}}\right]^{1/2}\right\}. \qquad (37)$$

If $\Theta_N>\Theta_D$ and $T\ll\mu H_{AE}(\Theta_D/\Theta_N)^2$, then

$$\Delta v=\frac{a\Theta_N}{\sqrt{3}}\left\{1-\frac{\Theta_N}{\Theta_D}\left[\frac{5\zeta(5/2)}{\zeta(3/2)}\frac{T}{\mu H_{AE}}\right]^{1/2}\right\}. \qquad (38)$$

At high temperatures ($T\gg\mu H_{AE}$, when $\Theta_N<\Theta_D$, or $T\gg\mu H_{AE}(\Theta_D/\Theta_N)^2$, when $\Theta_N>\Theta_D$), the discontinuity in the second sound velocity at the point $H=H_{AE}$ will be absent.

In fields $H\sim H_E$ the expressions for the second

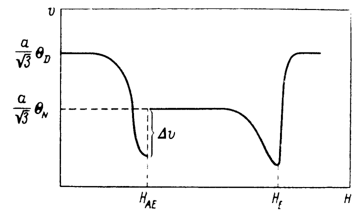

FIG. 3. Dependence of v on H in an EO AFM for the case $\Theta_N<\Theta_D$. For $T\ll\mu H_{AE}$ we have the solid line curve; for $T\gg\mu H_{AE}$ we have the dashed curve for the antiparallel phase and the solid line curve for the other phases.

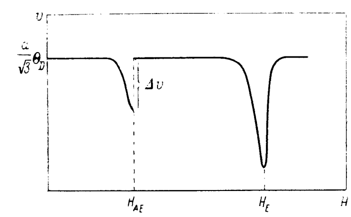

FIG. 4. Dependence of v on H in an EO AFM for $\Theta_N>\Theta_D$ and $T\ll\mu H_{AE}(\Theta_D/\Theta_N)^2$.

sound velocity v and for v_s and Γ_s coincide with those given in formulas (27) and (28).

The case $H=H_E$ is completely identical with the case of a ferromagnetic in a field $H=0$ (see, e.g.,[5]); for $T\ll\Theta_D^2/\Theta_N$ the velocity $v\sim a\sqrt{\Theta_N T}$.

C. The Ferromagnetic Phase ($H>H_E$)

If H exceeds the critical field H_E, the magnetic moments of the sublattices take up a parallel alignment and the antiferromagnetic passes over to a phase with ferromagnetic ordering. The dependence on T and H of the second sound velocity in this phase is no different from the analogous dependence of v in a ferromagnetic. As H increases, the velocity v, for $T\gg\Theta_D^2/\Theta_N$, will change from v_s to v_1, approaching v_1 in accordance with the exponential law $\exp\{-\mu(H-H_E)/T\}$.

The dependence of v on H for an EA AFM is shown schematically in Figs. 3 and 4.

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