NATURAL FLUCTUATIONS IN SOLID-STATE LASERS

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We investigate the natural fluctuations of the amplitude and phase of the field in solid-state lasers. We obtain general expressions for the spectral densities of the fluctuations of solid-state and gas lasers, and also of a molecular generator. The behavior of the spectral density of the field-amplitude fluctuations of the solid-state laser is investigated. The question of the width of the emission line is considered.

1. INTRODUCTION

THE character of the amplitude and phase fluctuations of laser radiation depends significantly on the ratio of the characteristic time parameters of the working medium and of the field. If we denote by $1/\gamma_a$ and $1/\gamma_b$ the relaxation times of the populations of levels a and b (a is the upper level), by $1/\nu_{ab}$ the polarization relaxation time, and by $1/\Delta\omega_r$ ($\Delta\omega_r = \omega_0/Q$) the relaxation time of the field in an empty resonator, then we can distinguish between the cases:

1) He–Ne laser

$$\gamma_a \sim \gamma_b \sim \gamma_{ab} \gg \Delta\omega_r, \tag{1.1}$$

2) molecular generator

$$\gamma_a \sim \gamma_b \sim \gamma_{ab} \ll \Delta\omega_r; \tag{1.2}$$

3) solid-state laser

$$\gamma_a \sim \gamma_b \ll \Delta\omega_r \ll \gamma_{ab}. \tag{1.3}$$

In the He–Ne laser, the field fluctuations are slow compared with the polarization fluctuations. Consequently, the field can be regarded as given in the calculation of the polarization fluctuations. This makes it possible to reduce the problem of calculating the fluctuations in the He–Ne laser to a solution of a system of equations for the amplitudes and phases of the electromagnetic waves.

The fluctuations in He–Ne lasers were calculated by Lamb[1], Lamb and Scully[2], Lax[3,4], Haken[5], Willis[6], Fleck[7], Kazantsev and Surdutovich[8], Klimontovich and Landa[9], Landa[10], and Klimontovich[11].

The fluctuations in a molecular generator are described, in fact, on the basis of the same equations for the elements of the density matrix ρ_a, ρ_b, and ρ_{ab} and of the field as in the semiclassical laser theory (see Oraevskiĭ[12] and Malakhov[13]). The calculations performed in the second part of[8] are actually also valid only for a molecular generator.

The calculation of the fluctuations in solid-state lasers has been carried out in a number of papers, but the most general results were apparently obtained by Lax[4]. In particular, Lax presents expressions for the fluctuations of the intensities (number of photons) of the radiation, and the radiation line width.

The present paper is devoted to a calculation of the fluctuations of the amplitude and phase of radiation from a solid-state laser. To calculate the polarization noise, we use the method described in[9,11]. This makes it possible to obtain relatively simple expressions for the spectral densities of the fluctuations of the amplitude and the field intensity for arbitrary excesses over threshold (within the framework of the correlation approximation). A detailed investigation has been made of the behavior of the spectral densities of the amplitude fluctuations. The correspondence of the obtained results with the results of Lax[4], Lamb[2], and Kazantsev and Surdutovich[8] is considered.

We consider also the question of the line width of the solid-state laser radiation.

2. INITIAL EQUATIONS

As the initial equations for the calculation of the fluctuations we use the equations for the density matrix elements ρ_a, ρ_b, and ρ_{ab}, on which the semiclassical theory of the laser is based:

$$\left(\frac{d}{dt} + i\omega_{ab} + \gamma_{ab}\right)\rho_{ab} = -\frac{ie}{\hbar} r_{ab} E D, \tag{2.1}$$

$$\frac{dD}{dt} = \frac{2ie}{\hbar}(r_{ab}\rho_{ba} - \rho_{ab}r_{ba})E - \frac{\gamma_a+\gamma_b}{2}(D - D^{(0)}) - \frac{\gamma_a-\gamma_b}{2}(R - R^{(0)}), \tag{2.2}$$

$$\frac{dR}{dt} = -\frac{\gamma_a-\gamma_b}{2}(D - D^{(0)}) - \frac{\gamma_a+\gamma_b}{2}(R - R^{(0)}), \tag{2.3}$$

where

$$D = \rho_a - \rho_b, \quad R = \rho_a + \rho_b.$$

The equation for the field is written in the form

$$\frac{\partial^2\mathscr{E}}{\partial t^2} + \frac{\omega_0}{Q}\frac{\partial\mathscr{E}}{\partial t} - c^2\frac{\partial^2\mathscr{E}}{\partial x^2} = -4\pi\frac{\partial^2 P}{\partial t^2} + \omega_0^2 E^{(T)}. \tag{2.4}$$

Here P is the polarization and $E^{(T)}$ is the source of the thermal noise. We seek $\mathscr{E}$ in the form

$$\mathscr{E} = {}^1/_2[E\exp\{-i(\omega_0 t + \varphi - k_0 R)\} + \text{c.c.}]. \tag{2.5}$$

We introduce also the slow functions

$$P^s = P\sin(\omega_0 t + \varphi - k_0 R) = -{}^1/_2 ien(r_{ba}\tilde\rho_{ab} - \tilde\rho_{ba}r_{ab}), \tag{2.6}$$

$$P^c = P\cos(\omega_0 t + \varphi - k_0 R) = {}^1/_2 en(r_{ba}\tilde\rho_{ab} + \tilde\rho_{ba}r_{ab}), \tag{2.7}$$

where

$$\rho_{ab} = \tilde\rho_{ab}\exp\{-i(\omega_0 t + \varphi - k_0 R)\},$$
$$\rho_{ba} = \tilde\rho_{ba}\exp\{i(\omega_0 t + \varphi - k_0 R)\}.$$

Substituting (2.5)–(2.7) into the system of equations for the elements of the density matrix and the field, we obtain the complete system of abbreviated equations

$$\left(\frac{d}{dt}+\frac{\omega_0}{2Q}\right)E=-4\pi\omega_0P^s-\omega_0E^{\tau}\sin(\omega_0t+\varphi-k_0R), \quad (2.8)$$

$$\frac{d\varphi}{dt}=-\frac{4\pi\omega_0}{E}P^c-\frac{\omega_0}{E}E^{(\tau)}\cos(\omega_0t+\varphi-k_0R), \quad (2.9)$$

$$\left(\frac{d}{dt}+\gamma_{ab}\right)P^s=-\frac{ne^2|r_{ab}|^2}{6\hbar}ED, \quad (2.10)$$

$$\left(\frac{d}{dt}+\gamma_{ab}\right)P^c=0, \quad (2.11)$$

$$\frac{dD}{dt}+\gamma_+(D-D^{(0)})=\frac{2}{\hbar n}P^sE-\gamma_-(R-R^{(0)}), \quad (2.12)$$

$$\frac{dR}{dt}+\gamma_+(R-R^{(0)})=-\gamma_-(D-D^{(0)}). \quad (2.13)$$

It is assumed here that the frequency deviation for the line center is equal to zero,

$$\gamma_+=(\gamma_a+\gamma_b)/2,\quad \gamma_-=(\gamma_a-\gamma_b)/2.$$

To find the noise we represent the functions P, R, D, and E in the form

$$P=\bar{P}+\delta P,\ D=\bar{D}+\delta D,\ R=\bar{R}+\delta R,\ E=\bar{E}+\delta E.$$

At not too small an excess over threshold, Eqs. (2.8)–(2.13) can be linearized and the fluctuations can be calculated by using the correlation approximation. In this case we obtain the system

$$\left(\frac{d}{dt}+\frac{\omega_0}{2Q}\right)\delta E=-4\pi\omega_0\delta P^s-\omega_0E^{(\tau)}\sin(\omega_0t+\varphi-k_0R), \quad (2.14)$$

$$\frac{d\delta\varphi}{dt}=-\frac{4\pi\omega_0}{\bar{E}}\delta P^c-\frac{\omega_0}{\bar{E}}E^{(\tau)}\cos(\omega_0t+\varphi-k_0R), \quad (2.15)$$

$$\left(\frac{d}{dt}+\gamma_{ab}\right)\delta P^s=-\frac{ne^2|r_{ab}|^2}{6\hbar}(\bar{D}\delta E+\bar{E}\delta D), \quad (2.16)$$

$$\left(\frac{d}{dt}+\gamma_{ab}\right)\delta P^c=0, \quad (2.17)$$

$$\left(\frac{d}{dt}+\gamma_+\right)\delta D=\frac{2}{\hbar n}(\bar{E}\delta P^s+\bar{P}^s\delta E)-\gamma_-\delta R \quad (2.18)$$

$$\left(\frac{d}{dt}+\gamma_+\right)\delta R=-\gamma_-\delta D. \quad (2.19)$$

3. AMPLITUDE FLUCTUATIONS

In the case of a solid-state laser, the fluctuations of the polarization break up into two parts: the noise part of the fluctuation, δP_n, which does not depend on the field, and the induced part δP_i of the polarization fluctuations, due to the field fluctuations. In gas He–Ne lasers, the latter fluctuations are missing, since the field does not have time to change during the relaxation time of the atom. On the other hand, in solid-state lasers δP_i does account for the influence of the field fluctuations on the polarizability of the medium.

To find δP_i^S we use the system of equations (2.16), (2.18), and (2.19). Taking the Fourier transform of these equations and recognizing that $\gamma_{ab}\gg\gamma_a,\gamma_b,\omega_0/Q$, and also recognizing that in the stationary state we have

$$4\pi\bar{P}^s=-\frac{1}{2Q}\bar{E},\quad \frac{4\pi e^2|r_{ab}|^2n}{3\hbar\gamma_{ab}}\bar{D}=\frac{1}{Q},$$

we obtain an expression for $(\delta P_i^S)_\omega$ as a function of $(\delta E)_\omega$:

$$4\pi(\delta P_i{}^s)_\omega=-\frac{1}{2Q}\left[1-2\frac{\gamma_a\gamma_b}{\gamma_+}a\bar{E}^2\cdot\right. \quad (3.1)$$

$$\left.\times\left(-i\omega+\gamma_+-\frac{\gamma_-^2}{-i\omega+\gamma_+}+\frac{\gamma_a\gamma_b}{\gamma_+}a\bar{E}^2\right)^{-1}\right](\delta E)_\omega,$$

where

$$a=e^2|r_{ab}|^2\gamma^+/3\hbar^2\gamma_{ab}\gamma_a\gamma_b.$$

The noise part of the polarization fluctuations does not depend on the field fluctuations. We therefore get for the noise part from the system (2.16), (2.18), and (2.19)

$$\left(\frac{d}{dt}+\gamma_{ab}\right)\delta P_n{}^s=-\frac{ne^2|r_{ab}|^2}{6\hbar}\bar{E}\delta D, \quad (3.2)$$

$$\left(\frac{d}{dt}+\gamma_+\right)\delta D=\frac{2}{\hbar n}\bar{E}\delta P_n{}^s-\gamma_-\delta R, \quad (3.3)$$

$$\left(\frac{d}{dt}+\gamma_+\right)\delta R=-\gamma_-\delta D. \quad (3.4)$$

In order to determine from this system the spectral density of the polarization fluctuations $(\delta P_p^S\delta P_n^S)_\omega$, it is necessary to have the initial conditions, i.e., the simultaneous mean values $\langle\delta P_n^S\delta P_n^S\rangle$, $\langle\delta P_n^S\delta D\rangle$, and $\langle\delta P_n^S\delta R\rangle$. They are obtained from the formula for the second moments of the elements of the density matrix

$$\langle\delta\rho_{nm}\delta\rho_{m'n'}\rangle=\frac{1}{2nV}[\delta_{nn'}\bar{\rho}_{m'm}+\delta_{mm'}\bar{\rho}_{nn'}]. \quad (3.5)$$

Using this formula, we get

$$\langle\delta D\delta P_n{}^s\rangle=0, \quad (3.6)$$

$$\langle\delta R\delta P_n{}^s\rangle=\frac{\bar{P}^s}{nV}, \quad (3.7)$$

$$\langle\delta P_n{}^s\delta P_n{}^s\rangle=\frac{1}{4}\frac{ne^2|r_{ab}|^2}{3V}(\rho_a+\rho_b). \quad (3.8)$$

Using the initial conditions (3.6)–(3.8), we get from the system (3.2)–(3.4)

$$(\delta P_n{}^s\delta P_n{}^s)_{\omega=0}=\frac{\hbar}{8\pi QV}\frac{1}{1+a\bar{E}^2}\left[\frac{\rho_a+\rho_b}{\rho_a-\rho_b}-\frac{\gamma_-}{\gamma_+}a\bar{E}^2\right]. \quad (3.9)$$

Since the spectrum $(\delta P_n^S\delta P_n^S)_\omega$ is broad—on the order of $\gamma_{ab}\gg\gamma_a,\gamma_b,\omega_0/Q$, we need know only $(\delta P_n^S\delta P_{n'}^S)_{\omega=0}$.

Substituting $\delta P^S=\delta P_n^S+\delta P_i^S$ in the equation for the field (2.14), we get

$$(\delta E)_\omega=\omega_0(\xi)_{\omega=0}/Z_\omega,$$

$$Z_\omega=-i\omega+\frac{\omega_0}{Q}\frac{\gamma_a\gamma_b}{\gamma_+}a\bar{E}^2$$

$$\times\left(-i\omega+\gamma_+-\frac{\gamma_-^2}{-i\omega+\gamma_+}+\frac{\gamma_a\gamma_b}{\gamma_+}a\bar{E}^2\right)^{-1}, \quad (3.10)$$

$$(\xi)_{\omega=0}=-4\pi(\delta P_m{}^s)_{\omega=0}-(E^{(T)})_{\omega=0}.$$

Using the well known formula for the thermal noise in a resonator

$$(E^{(\tau)}E^{(\tau)})_{\omega=0}=\frac{4\pi\hbar}{VQ}\left(\bar{n}+\frac{1}{2}\right),\qquad \bar{n}=(e^{\hbar\omega/kT}-1)^{-1}$$

and expression (3.9), we obtain for the spectral density of the noise source the expression

$$(N)_{\omega=0}=(\xi\xi)_{\omega=0}=$$

$$=\frac{4\pi\hbar}{VQ}\left[\bar{n}+\frac{1}{2}+\frac{1}{2}\frac{1}{1+a\bar{E}^2}\left(\frac{\rho_a+\rho_b}{\rho_a-\rho_b}-\frac{\gamma_-}{\gamma_+}a\bar{E}^2\right)\right]. \quad (3.11)$$

Let us consider now, for simplicity, the case $\gamma_a=\gamma_b=\gamma$. In this case the expression for the spectral density of the field-amplitude fluctuations takes the form

$$(\delta E)_\omega^2=\omega_0^2(N)_{\omega=0}/A_\omega, \quad (3.12)$$

$$A_\omega=\omega^2\left[1-\frac{\omega_0}{Q}\frac{\gamma a\bar{E}^2}{\omega^2+\gamma^2(1+a\bar{E}^2)^2}\right]^2+\left[\frac{\omega_0}{Q}\frac{\gamma^2a\bar{E}^2(1+a\bar{E}^2)}{\omega^2+\gamma^2(1+a\bar{E}^2)^2}\right]^2.$$

From this expression we derive immediately two cases:

1. $\omega_0/Q \gg \gamma$. Since $\gamma_{ab} \gg \omega_0/Q$, this is the case of a solid-state laser. In this case, the spectral density of the field-amplitude fluctuations can be regarded approximately as consisting of two lines: a broad line

$$(\delta E)_\omega{}^2 = \omega_0{}^2(N)_{\omega=0} \Big/ \left[\omega^2 + \left(\frac{\omega_0}{Q}\frac{a\bar{E}^2}{1+a\bar{E}^2}\right)^2\right] \tag{3.13}$$

on which there is a narrow peak at the frequency

$$\omega_{\max} \cong \sqrt{\omega_0\gamma a\bar{E}^2/Q}. \tag{3.14}$$

The width of this peak at half-height is

$$\Delta\omega \cong \gamma(1 + a\bar{E}^2). \tag{3.15}$$

The spectral density of the fluctuations of the field at the maximum amounts to

$$(\delta E)^2_{\omega=\omega_{\max}} = \omega_0{}^2(N)_{\omega=0}/\gamma^2(1 + a\bar{E}^2)^2. \tag{3.16}$$

Such a peak on the curve of the spectral density of the amplitude fluctuations of a solid-state laser was predicted by McCumber[14] and observed experimentally in[15].

Using (3.12), we can find the mean-squared fluctuations of the field amplitude:

$$\langle\Delta E^2\rangle = \frac{1}{2}\omega_0{}^2(N)_{\omega=0}\left[\frac{1+a\bar{E}^2}{\omega_0 a\bar{E}^2/Q} + \frac{1}{\gamma}\frac{1}{1+a\bar{E}^2}\right]. \tag{3.17}$$

2. $\omega_0/Q \ll \gamma$. This is the case of a gas laser γ_{ab}, $\gamma \gg \omega_0/Q$. In this case

$$(\delta E^2)_\omega = \omega_0(N)_{\omega=0} \Big/ \left[\omega^2 + \left(\frac{\omega_0}{Q}\frac{a\bar{E}^2}{1+a\bar{E}^2}\right)^2\right]. \tag{3.18}$$

This is the usual expression for the spectral density of field-amplitude fluctuations of an He—Ne laser, with

$$\langle\Delta E^2\rangle = \frac{1}{2}\omega_0{}^2(N)_{\omega=0}\frac{1+a\bar{E}^2}{\omega_0 a\bar{E}^2/Q}. \tag{3.19}$$

In analogy with the derivation of (3.12) for the case ω_0/Q, $\gamma \ll \gamma_{ab}$, we can obtain the corresponding expression also for the case $\omega_0/Q \gg \gamma_{ab}, \gamma$, corresponding to a molecular generator. Under the condition $a\bar{E}^2 \ll 1$, this expression takes the form

$$(\delta E)_\omega{}^2 = \frac{\omega_0{}^2(N)_{\omega=0}}{\omega^2 + (\omega_0/2Q)^2}\frac{(2a\bar{E}^2\gamma^4 + \omega^4 - \omega^2\gamma^2)^2 + \omega^2\gamma^2(\omega^2+\gamma^2)^2}{[(2a\bar{E}^2\gamma^2)^2 + \omega^2(\omega^2+\gamma^2)]^2}. \tag{3.20}$$

From this we obtain the mean-squared field fluctuations:

$$\langle\Delta E^2\rangle = \frac{1}{2}\omega_0{}^2(N)_{\omega=0}\left[\left(\frac{\omega_0}{2Q}\right)^{-1} + \frac{1}{2}\frac{\gamma}{(\omega_0/2Q)^2 a\bar{E}^2}\right]. \tag{3.21}$$

In the correlation approximation, it is easy to go over from the field fluctuations to the fluctuations of the number of photons in the resonator. Indeed

$$\frac{dn_{\text{ph}}}{dt} = \frac{V}{4\pi\hbar\omega_0}E\frac{dE}{dt}. \tag{3.22}$$

In the correlation approximation, i.e., in the case of a large number of photons in the resonator, we neglect the terms $\delta E\,\delta P^S$ and $E^{(T)}\delta E$. In this case the equation for δn_{ph} takes the form

$$\frac{d\delta n_{\text{ph}}}{dt} = -\frac{\omega_0}{2Q}\delta n_{\text{ph}} - \left(\frac{V\omega_0\langle n_{\text{ph}}\rangle}{2\pi\hbar}\right)^{1/2}[4\pi\delta P^s - E^{(T)}], \tag{3.23}$$

where $\langle n_{\text{ph}}\rangle$ is the mean number of photons in the resonator. From this equation and from (2.16), (2.18), and (2.19) it follows that in the correlation approximation the spectral functions of the fluctuations of the number of photons in the resonator and the spectral functions of the fluctuations of the field amplitudes are related by

$$(\delta n_{\text{ph}})_\omega{}^2 = \frac{V\langle n_{\text{ph}}\rangle}{2\pi\hbar\omega_0}(\delta E)_\omega{}^2. \tag{3.24}$$

Thus, we obtain expressions for $\langle\Delta n^2_{\text{ph}}\rangle$ in the following three cases:

1) Solid-state laser, $\gamma_{ab} \gg \omega_0/Q \gg \gamma$. Taking (3.17) and (3.24) into account, we get

$$\langle\Delta n_{\text{ph}}{}^2\rangle \cong (\bar{n}+1)\langle n_{\text{ph}}\rangle\left[\frac{\eta}{\eta-1} + \frac{\omega_0}{Q\gamma}\frac{1}{\eta}\right], \tag{3.25}$$

where $\eta = a\bar{E}^2 + 1$ is the excess over threshold. This expression agrees with the expression obtained by Lax[4] for $\langle n^2_{\text{ph}}\rangle$.

2) Gas laser, $\gamma_{ab}, \gamma \gg \omega_0/Q$. In this case

$$\langle\Delta n_{\text{ph}}{}^2\rangle \cong (\bar{n}+1)\langle n_{\text{ph}}\rangle\frac{\eta}{\eta-1}. \tag{3.26}$$

This agrees with the expression obtained by Lamb for a gas laser[2].

3) Molecular generator, $\gamma = \gamma_{ab} \ll \omega_0/Q$. Here

$$\langle\Delta n_{\text{ph}}{}^2\rangle \cong (\bar{n}+1)\langle n_{\text{ph}}\rangle\left[1 + \frac{\gamma}{\omega_0(\eta-1)/Q}\right]. \tag{3.27}$$

The last formula coincides with the expression obtained for this case by Kazantsev and Surdutovich[8].

4. PHASE FLUCTUATIONS. LINE WIDTH

The phase fluctuations in a solid-state laser at zero deviation from the line center are described by Eqs. (2.15) and (2.17). We see that at zero detuning the phase fluctuations do not depend on the amplitude fluctuations.

From (2.17) we get the spectral density of the noise source. The initial condition, calculated from formula (3.5), is

$$\langle\delta P_n{}^c\,\delta P_n{}^c\rangle = \frac{ne^2|r_{ab}|^2}{12V}(\rho_a + \rho_b). \tag{4.1}$$

In this case from (2.17), taking into account the stationarity condition

$$\frac{4\pi ne^2|r_{ab}|^2}{3\hbar\gamma_{ab}}(\rho_a - \rho_b) = \frac{1}{Q},$$

we obtain

$$(\delta P_n{}^c\,\delta P_n{}^c)_{\omega=0} = \frac{\hbar}{8\pi QV}\frac{\rho_a+\rho_b}{\rho_a-\rho_b}. \tag{4.2}$$

From (2.15) we get

$$\frac{d\delta\varphi}{dt} = \frac{\omega_0}{E}\xi, \tag{4.3}$$

$$(\xi\xi)_{\omega=0} = 16\pi^2(\delta P_n{}^c\,\delta P_n{}^c)_{\omega=0} + (E^{(T)}E^{(T)})_{\omega=0}. \tag{4.4}$$

Thus

$$(\xi\xi)_{\omega=0} = \frac{4\pi\hbar}{VQ}\left(\bar{n} + \frac{1}{2} + \frac{1}{2}\frac{\rho_a+\rho_b}{\rho_a-\rho_b}\right). \tag{4.5}$$

Consequently, the natural width of the emission line of a solid-state laser is written in the form

$$\Delta\omega = \frac{2\pi\hbar\,\omega_0{}^2}{VQ\bar{E}^2}\left(\bar{n} + \frac{1}{2} + \frac{1}{2}\frac{\rho_a+\rho_b}{\rho_a-\rho_b}\right), \tag{4.6}$$

i.e., it is determined by the same formulas as in a gas laser.

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