INSTABILITY OF ENTROPY WAVES DUE TO THE GYRORELAXATION EFFECTS IN A PLASMA OF FINITE PRESSURE

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Submitted February 6, 1970

Zh. Eksp. Teor. Fiz. 59, 524–532 (August, 1970)

We investigate the influence of the gyrorelaxation effect on the stability of entropy waves in a plasma of finite pressure. We show that the entropy waves are unstable, owing to the gyrorelaxation effect in a certain interval of positive $\partial \ln n_0/\partial \ln p_0$, particularly at $\nabla T_0 = 0$, and the unstable disturbances are those whose phase velocity has the same order of magnitude and the same direction as the velocity of the magnetic drift of the electrons. Since this process cannot be investigated correctly with the aid of the known systems of hydrodynamic equations, a new system of macroscopic equations for a plasma is derived to describe this process on the basis of a solution of the Boltzmann kinetic equation.

1. INTRODUCTION

ENTROPY waves in an inhomogeneous collision plasma of finite pressure ($\beta \equiv 8\pi p/B^2 \gtrsim 1$, p—plasma pressure, B—magnetic field intensity) were investigated by Kadomtsev[1] and Mikhaĭlovskaya.[2] They were discussed also by Ware[3] in connection with the problem of the stability of the Zeta apparatus. These waves have sufficiently small longitudinal wave numbers ($k_\parallel \to 0$), and their transverse phase velocity is essentially connected with the drift velocity of the particles under the influence of the inhomogeneity of the magnetic field.

An important property of these waves is the equality, with opposite sign, of the perturbed electron and ion temperatures ($T_e' = -T_i'$), and the associated absence of perturbations of the magnetic field and of the total plasma pressure ($B' = 0$, $p' = 0$).

As was shown for the case of force lines in the form of concentric circles[1] and for the case of a field with straight force lines,[2] entropy waves may build up if the temperature gradient is not too small in comparison with the pressure gradient:

$$\frac{\partial \ln T}{\partial \ln p} > \frac{7}{10} + \frac{\beta}{4}. \tag{1.1}$$

This result is obtained in the zeroth approximation with respect to the ratio of the oscillation frequency to the frequency of the ion-ion collisions ν_i, $\omega/\nu_i \to 0$. The purpose of the present paper is to clarify the role of effects of order ω/ν_i in the buildup of entropy waves under conditions when the inequality (1.1) is not satisfied. Physically, the terms of order of ω/ν_i at $k_\parallel = 0$ correspond to the so-called gyrorelaxation effect,[4] wherein a change of the transverse particle pressure brings about also a change in their longitudinal pressure, owing to collisions between the particles (in this case between the ions). (The gyrorelaxation effect was discussed earlier by Budker[5] and by Schluter[6] in connection with the problem of plasma heating by an alternating magnetic field.)

In the hydrodynamic description of a plasma, effects of order ω/ν_i are determined by the viscosity tensor. The viscosity tensor for a plasma in a magnetic field was calculated in a number of papers (see, for example, Braginskiĭ[4] and Kaufman[7]). However, as shown by one of the authors,[8] the approximations used in these papers do not correspond to the approximations of the theory of gradient instabilities. We therefore cannot use in our problem the expression for the viscosity tensor in the form given by Braginskiĭ[4] or by Kaufman.[7] In Sec. 2 below we follow the method of [8] and solve the Boltzmann equation in the required approximation, obtaining a system of macroscopic equations needed for the investigation of the role of the gyrorelaxation effect in entropy waves, under the assumption that the magnetic force lines are straight. The investigation of the entropy waves is carried out in Sec. 3, and the results are discussed in Sec. 4.

2. DRIFT KINETIC EQUATION

1. Transformation of the Kinetic Equation

Bearing in mind that we must investigate perturbations of a plasma with $\mathbf{B}_0 \parallel z$ ($\mathbf{B}_0$—unperturbed magnetic field), $k_z = 0$ (k_z—wave number along the magnetic field), $\mathbf{B}_\perp' = 0$, and $\mathbf{E}_z' = 0$, we assume in the nonlinearized Boltzmann equation $\mathbf{B} = B\mathbf{e}_z$, $\partial/\partial z = 0$, and $E_z = 0$. This equation can then be represented in the form

$$\frac{\partial f}{\partial t} + \mathbf{v}_\perp \nabla f + \frac{dv_\perp}{dt}\frac{\partial f}{\partial v_\perp} + \frac{d\theta}{dt}\frac{\partial f}{\partial \theta} = C. \tag{2.1}$$

Here the transverse velocity is $\mathbf{v}_\perp \equiv \mathbf{v} - \mathbf{e}_z v_z = v_\perp (\mathbf{e}_x \cos + \mathbf{e}_y \sin\theta)$, $\mathbf{e}_x$ and $\mathbf{e}_y$ are unit vectors, C is the collision term, and the derivatives $dv_\perp/dt$ and $d\theta/dt$, as follows from the equations of motion of the particles, are determined by the relations

$$\begin{aligned} \frac{dv_\perp}{dt} &= \frac{e}{m}(E_x \cos\theta + E_y \sin\theta), \\ \frac{d\theta}{dt} &= -\omega_B - \frac{e}{mv_\perp}(E_x \sin\theta - E_y \cos\theta), \end{aligned} \tag{2.2}$$

$\omega_B = eB/mc$; e and m—charge and mass of the particles. Using (2.2), we can write (2.1) in the form:

$$\left\{\frac{\partial}{\partial t} + \cos\theta\left(D_1 + A_1\frac{\partial}{\partial\theta}\right) + \sin\theta\left(D_2 + A_2\frac{\partial}{\partial\theta}\right)\right\} f = \omega_B \frac{\partial f}{\partial\theta} + C, \tag{2.3}$$

where

$$D_1 = v_\perp\frac{\partial}{\partial x} + \frac{e}{m}E_x\frac{\partial}{\partial v_\perp}; \quad D_2 = v_\perp\frac{\partial}{\partial y} + \frac{e}{m}E_y\frac{\partial}{\partial v_\perp},$$

$$A_1 = \frac{eE_y}{mv_\perp}, \quad A_2 = -\frac{eE_x}{mv_\perp}.$$

We now represent the distribution function in the form

$$f = \bar{f} + \tilde{f}; \tag{2.4}$$

$\bar{f}$ and $\tilde{f}$ are respectively the parts of the distribution function averaged over the angles θ and oscillating with θ. Let us average (2.3) over the angles:

$$\frac{\partial}{\partial t}\bar{f} + D_1\langle\tilde{f}\cos\theta\rangle + D_2\langle\tilde{f}\sin\theta\rangle + A_1\langle\tilde{f}\sin\theta\rangle - A_2\langle\tilde{f}\cos\theta\rangle = \bar{C}, \tag{2.5}$$

where $\langle\ldots\rangle$ is the averaging symbol. We then subtract (2.5) from (2.3):

$$\frac{\partial}{\partial t}\tilde{f} + D_1\bar{f}\cos\theta + D_1(\tilde{f}\cos\theta - \langle\tilde{f}\cos\theta\rangle)$$
$$+ D_2\bar{f}\sin\theta + D_2(\tilde{f}\sin\theta - \langle\tilde{f}\sin\theta\rangle) + A_1\left(\frac{\partial\tilde{f}}{\partial\theta}\cos\theta - \langle\tilde{f}\sin\theta\rangle\right)$$
$$+ A_2\left(\frac{\partial\tilde{f}}{\partial\theta}\sin\theta + \langle\tilde{f}\cos\theta\rangle\right) = C - \bar{C} + \omega_B\frac{\partial\tilde{f}}{\partial\theta}. \tag{2.6}$$

We have thus obtained a system of kinetic equations (2.5) and (2.6) for the averaged and oscillating distribution functions. This system can be readily reduced to a single equation for the averaged distribution function, by assuming that the cyclotron frequency of rotation of the particles ω_B is much larger than the reciprocal times $\partial/\partial t$ and $\nabla\sqrt{T/m}$ (T is the particle temperature) and the frequency of collisions between the particles. We can then obtain from (2.6)

$$\tilde{f} = \frac{1}{\omega_B}(D_1\sin\theta - D_2\cos\theta)\bar{f}. \tag{2.7}$$

After substituting (2.7) in (2.5) and performing simple calculations, we obtain the drift kinetic equation for the collision plasma:

$$\frac{\partial\bar{f}}{\partial t} + (\mathbf{V}_E\nabla)\bar{f} - \varepsilon_\perp \operatorname{div}\mathbf{V}_E\frac{\partial\bar{f}}{\partial\varepsilon_\perp} + \varepsilon_\perp(\mathbf{b}\nabla)\bar{f} = \bar{C}, \tag{2.8}$$

where*

$$\mathbf{V}_E = \frac{c}{B}[\mathbf{E}\mathbf{e}_z], \quad \varepsilon_\perp = \frac{v_\perp^2}{2}, \quad \mathbf{b}_\alpha = \left[\nabla\frac{1}{\omega_{B\alpha}}\mathbf{e}_z\right].$$

The drift kinetic equation for a collisionless plasma was obtained earlier by Rudakov and Sagdeev.[9]

2. Solution of Eq. (2.8)

We see the function $\bar{f}$ in the form

$$\bar{f} = F + f_1, \tag{2.9}$$

where F is the Maxwellian distribution function

$$F = n\left(\frac{m}{2\pi T}\right)^{3/2}\exp\left(-\frac{mv^2}{2T}\right), \tag{2.10}$$

and satisfies, in the case when the electron and ions have the same temperature, the condition

$$C(F) = 0. \tag{2.11}$$

After substituting (2.9) and (2.10) in (2.8) and taking (2.11) into account, we arrive at the following equation f_1:

$$\left\{-\frac{1}{2}X_0\left(\xi_z^2 - \frac{\xi^2}{3}\right) + \frac{1}{2}X_1\left(\xi_z^2 - \frac{\xi^2}{3}\right)L_1^{(5/2)}\left(\frac{\xi^2}{2}\right)\right.$$
$$\left.+\frac{4}{3}X_1L_2^{(1/2)}\left(\frac{\xi^2}{2}\right)\right\}F = \bar{C}(f_1), \tag{2.12}$$

where $L_m^{(p)}(x)$ are Sonine-Laguerre polynomials (see [4]),

$$X_0 = \operatorname{div}\mathbf{V}_E + \frac{(\mathbf{b}\nabla)p}{mn} + \frac{(\mathbf{b}\nabla)T}{m}, \quad X_1 = \frac{(\mathbf{b}\nabla)T}{m},$$
$$\xi^2 = \frac{mv^2}{T}, \quad \xi_z^2 = \frac{mv_z^2}{T}.$$

We are interested only in the ion equation (2.12), since f_1 is much smaller for electrons than ions, owing to the larger frequency of electron collisions. In this case C denotes the ion-ion collision integral. It was determined by the well known Landau integro-differential formula (see, for example, [4]).

The solution of (2.12) can be represented in the form of the following sum of two infinite series (compare with [4]):

$$\Phi \equiv \frac{f_1}{F} = \frac{1}{p}\sum_{m=2}^{\infty}a_m^{(1)}L_m^{(1/2)}\left(\frac{\xi^2}{2}\right)$$
$$+\frac{1}{2p}\left(\xi_z^2 - \frac{\xi_\perp^2}{2}\right)\sum_{m=0}^{\infty}a_m^{(2)}L_m^{(5/2)}\left(\frac{\xi^2}{2}\right). \tag{2.13}$$

The expansion of the spherically-symmetrical part of the function Φ begins with m = 2, since we assume that n and T are the density and the temperature of the particles, and therefore are determined completely by the function F.

Following Braginskiĭ,[4] we confine ourselves in the solution of (2.12) to allowance for only the first two terms of the series in each of the sums (2.13), since allowance for a larger number of terms does not result in an appreciable increase in the accuracy of the results.

Multiplying (2.12) successively by

$$L_2^{(1/2)}\left(\frac{\xi^2}{2}\right), \quad L_3^{(1/2)}\left(\frac{\xi^2}{2}\right), \quad \xi_z^2 - \frac{\xi^2}{3}, \quad \left(\xi_z^2 - \frac{\xi^2}{3}\right)L_1^{(5/2)}\left(\frac{\xi^2}{2}\right)$$

and integrating with respect to the velocities, we can obtain (see, for example, [4]) the following systems of equations for the coefficients $a_m^{(p)}$:

$$a_2^{(1)} + \frac{31}{12}a_3^{(1)} = 0, \quad a_2^{(1)} + \frac{3}{4}a_3^{(1)} = -\frac{5}{3}\frac{p}{\nu_i}X_1; \tag{2.14}$$

$$a_0^{(2)} + \frac{3}{4}a_1^{(2)} = \frac{5}{9}\frac{p}{\nu_i}X_0, \quad a_0^{(2)} + \frac{205}{36}a_1^{(2)} = -\frac{70}{27}\frac{p}{\nu_i}X_1. \tag{2.15}$$

Here p = nT, $\nu_i \equiv \tau_i^{-1}$ is the frequency of the ion-ion collisions (an expression for τ_i is given in [4]). From (2.14) and (2.15) we get

$$a_2^{(1)} = -\frac{155}{66}\frac{p}{\nu_i}X_1, \quad a_3^{(1)} = \frac{10}{11}\frac{p}{\nu_i}X_1; \tag{2.16}$$

$$a_0^{(2)} = \frac{2}{3}\frac{p}{\nu_i}\left(\frac{1025}{1068}X_0 + \frac{105}{178}X_1\right), \quad a_1^{(2)} = -\frac{2}{3}\frac{p}{\nu_i}\left(\frac{15}{89}X_0 + \frac{70}{89}X_1\right). \tag{2.17}$$

We have thus obtained the averaged distribution function for the ions. Knowledge of this function enables us to obtain macroscopic equations for the particle density and for the pressure.

3. Macroscopic Equations

Using (2.10) and (2.13), and integrating (2.8) with respect to the velocities with weights 1 and $mv^2/2$ respectively, we obtain the following equations for the density and for the pressure:

$$\frac{\partial n}{\partial t} + (\mathbf{V}_E\nabla)n + n\operatorname{div}\mathbf{V}_E + (\mathbf{b}\nabla)\frac{p}{m} - \frac{1}{2}(\mathbf{b}\nabla)\frac{a_0^{(2)}}{m} = 0,$$

$$\frac{3}{2}\left(\frac{\partial p}{\partial t} + \mathbf{V}_E\nabla p\right) + \frac{5}{2}p\operatorname{div}\mathbf{V}_E + \frac{5}{2}(\mathbf{b}\nabla)\frac{pT}{m} + \tag{2.18}$$

*$[\mathbf{E}\mathbf{e}_z] \equiv \mathbf{E}\times\mathbf{e}_z$.

$$+(\mathbf{b}\nabla)\frac{T}{m}\left(\frac{5}{2}a_2^{(1)}-\frac{7}{4}a_0^{(2)}+\frac{7}{4}a_1^{(2)}\right)-\frac{1}{2}a_0^{(2)}\operatorname{div}\mathbf{V}_E=0. \quad (2.19)$$

After substituting here the expressions for $a_m^{(p)}$ from (2.16) and (2.17), we arrive at the sought result

$$\frac{\partial n}{\partial t}+(\mathbf{V}_E\nabla)n+n\operatorname{div}\mathbf{V}_E+(\mathbf{b}\nabla)\frac{p}{m}$$
$$-\frac{1}{3}(\mathbf{b}\nabla)\frac{p}{m\nu_i}\left\{0.96\operatorname{div}\mathbf{V}_E+0.96(\mathbf{b}\nabla)\frac{p}{mn}+1.55(\mathbf{b}\nabla)\frac{T}{m}\right\}=0,$$
$$\frac{3}{2}\left(\frac{\partial p}{\partial t}+\mathbf{V}_E\nabla p\right)+\frac{5}{2}p\operatorname{div}\mathbf{V}_E+\frac{5}{2}(\mathbf{b}\nabla)\frac{pT}{m}- \quad (2.20)$$
$$-\frac{7}{6}(\mathbf{b}\nabla)\frac{pT}{m\nu_i}\left\{1.13\operatorname{div}\mathbf{V}_E+1.13(\mathbf{b}\nabla)\frac{p}{mn}+7.54(\mathbf{b}\nabla)\frac{T}{m}\right\}$$
$$-\frac{p}{3\nu_i}\operatorname{div}\mathbf{V}_E\left\{0.96\operatorname{div}\mathbf{V}_E+0.96(\mathbf{b}\nabla)\frac{p}{mn}+1.55(\mathbf{b}\nabla)\frac{T}{m}\right\}=0. \quad (2.21)$$

Besides the equations for the density of the pressure, we obtain also an equation for the current

$$\mathbf{j}=\sum_{\alpha=i,e}e_\alpha n_\alpha\mathbf{V}_\alpha=\sum_{\alpha=i,e}e_\alpha\int\mathbf{v}_\alpha f_\alpha\,d\mathbf{v}_\alpha. \quad (2.22)$$

After calculating the integrals in (2.22), using for this purpose (2.7), (2.9), (2.10), and (2.13), we get

$$\mathbf{j}=\frac{c}{B}[\mathbf{e}_z\nabla(p_i+p_e)]-\frac{1}{3}\frac{c}{B}\left[\mathbf{e}_z\nabla\frac{p_i}{\nu_i}\{0.96\operatorname{div}\mathbf{V}_E+0.96(\mathbf{b}_i\nabla)p_i/m_in_i+1.55(\mathbf{b}_i\nabla)T_i/m_i\}\right]. \quad (2.23)$$

3. ENTROPY WAVE INSTABILITY DUE TO THE GYRORELAXATION EFFECT

In conjunction with Maxwell's equations, Eqs. (2.20), (2.21), and (2.23) form a closed system. (The electron equations for the density and pressure are obtained from (2.20) and (2.21), by neglecting in the latter the terms containing $1/\nu_i$ and by assigning the index e to the quantities T, M, m, p, and ω_B). Let us investigate with the aid of the resultant equations the stability of the entropy wave, taking the gyrorelaxation effect into account.

1. The Approximation $\omega/\nu_i \to 0$

We shall show that in this approximation we obtain the results of Mikhaĭlovskaya's investigations of entropy waves,[2] and Kadomtsev's criterion for the instability of these waves.[1] From (2.20), (2.21), and (2.23) we obtain for the perturbed quantities the system of equations

$$-i\omega\frac{n'}{n_0}+i\frac{p'}{p_0}\omega_B{}^*+\frac{cE_y'}{B_0}(\varkappa_n-\varkappa_B)+\frac{iB_z'}{B_0}(\omega-\omega_p{}^*)=0,$$
$$-i\frac{p'}{p_0}(\omega-2\gamma\omega_B{}^*)-\frac{in'}{n_0}\gamma\omega_B{}^*+\frac{cE_y'}{B_0}(\varkappa_p-\gamma\varkappa_B)$$
$$+i\gamma\frac{B_z'}{B_0}(\omega-2\omega_p{}^*+\omega_n{}^*)=0,$$
$$B_z'B_0+4\pi(p_i'+p_e')=0, \quad (3.1)$$

where the prime pertains to the perturbed quantities,

$$\omega_A{}^*=\frac{k_yp_0}{m\omega_Bn_0}\varkappa_A, \qquad \varkappa_A=\frac{\partial\ln A_0}{\partial x},$$

$A_0 \equiv (n_0, B_0, p_0)$, the first two equations of (3.1) pertain to both types of particles, and $\gamma = 5/3$.

Following [2], we write down the system (3.1) in terms of the variables

$$X=\frac{p_i'-p_e'}{p_0}, \qquad Y=\frac{\omega}{\varkappa_p}\frac{p_i'+p_e'}{p_0}+ic\frac{E_y'}{B_0},$$

$$Z=\frac{n'}{n_0}-\frac{\varkappa_n}{\varkappa_p}\frac{p_i'+p_e'}{p_0}, \quad (3.2)$$

where $p_0 = p_{0i} + p_{0e}$ and $T_{0i} = T_{0e}$. In terms of these variables, the system (3.1) takes the form

$$\omega X+\gamma\omega_{Bi}{}^*Z=0, \qquad -2\gamma\omega_{Bi}{}^*X+(\varkappa_p-\gamma\varkappa_B)Y=0,$$
$$\omega_{Bi}{}^*X+(\varkappa_B-\varkappa_n)Y-\omega Z=0. \quad (3.3)$$

The same result is obtained by putting $p_e' + p_i' = 0$ and $B_Z' = 0$ in (3.1) and (3.2); this indicates that the perturbations have a force-free character.

From (3.3) we obtain the dispersion equation for the entropy waves (in Kadomtsev's terminology[1])

$$\omega^2+\gamma\omega_{Bi}^{*2}\left(1+2\gamma\frac{\varkappa_B-\varkappa_n}{\varkappa_p-\gamma\varkappa_B}\right)=0 \quad (3.4)$$

and the criterion for the instability of these waves, which was written out also in Sec. 1,

$$\frac{\varkappa_T}{\varkappa_p}>1-\frac{1}{2\gamma}+\frac{\beta}{4}. \quad (3.5)$$

The results (3.4) and (3.5) coincide with the corresponding results of [1,2].

2. Allowance for Terms of Order ω/ν_i

Assume that the instability criterion (3.5) is not satisfied. Then a decisive role in the stability of the entropy waves will be played by terms of order ω/ν_i, corresponding to the gyrorelaxation effect. Let us take these terms into account in the perturbed equations (2.20), (2.21), and (2.23). We then obtain in lieu of the system of the ion equation (3.1)

$$-i\omega\frac{n'}{n_0}+(\varkappa_n-\varkappa_B)\frac{cE_y'}{B_0}+i(\omega-\omega_{pi}{}^*)\frac{B_z'}{B_0}+i\omega_{Bi}{}^*\frac{p_i'}{p_{0i}}$$
$$+i\frac{0{,}96}{5}\frac{\omega_{Bi}{}^*}{\nu_i}\left\{-i\omega+\frac{p_i'}{p_{0i}}+\frac{cE_y'}{B_0}\varkappa_p+\frac{42}{41}i\left(\omega_{Ti}{}^*\frac{B_z'}{B_0}-\omega_{Bi}{}^*\frac{T_i'}{T_{0i}}\right)\right\}=0,$$
$$-i(\omega-2\gamma\omega_{Bi}{}^*)\frac{p_i'}{p_{0i}}+(\varkappa_p-\gamma\varkappa_B)\frac{cE_y'}{B_0}-i\gamma\omega_{Bi}{}^*\frac{n'}{n_0}$$
$$+i\gamma\frac{B_z'}{B_0}(\omega-2\omega_{pi}{}^*+\omega_{ni}{}^*)+\frac{7}{15}\cdot 1{,}13\frac{i\omega_{Bi}{}^*}{\nu_i}\left(-i\omega\frac{p_i'}{p_{0i}}\right.$$
$$\left.+\varkappa_p\frac{cE_y'}{B_0}\right)+\frac{7}{9}\cdot 6.41\frac{\omega_{Bi}{}^*}{\nu_i}\left(\omega_{Bi}{}^*\frac{T_i'}{T_{0i}}-\omega_{Ti}{}^*\frac{B_z'}{B_0}\right)=0,$$
$$\frac{B_z'}{B_0}=-\frac{\beta}{2}\frac{(p_{i\perp}'+p_e')}{p_0}, \quad (3.6)$$

where

$$p_{i\perp}'=p_i'+\frac{0{,}96}{5}\frac{p_{0i}}{\nu_i}\left[-i\omega\frac{p_i'}{p_{0i}}+\varkappa_p\frac{cE_y'}{B_0}+\frac{42}{41}i\left(\omega_{Ti}{}^*\frac{B_z'}{B_0}-\omega_{Bi}{}^*\frac{T_i'}{T_{0i}}\right)\right].$$

The electron density and the electron pressure are determined as before by Eqs. (3.1).

It is convenient to rewrite the system (3.6) in terms of the variables

$$X'=\frac{p_{i\perp}'-p_e'}{p_0}, \qquad Y'=\frac{\omega}{\varkappa_p}\frac{p_{i\perp}'+p_e'}{p_0}+ic\frac{E_y'}{B_0},$$
$$Z'=\frac{n'}{n_0}-\frac{\varkappa_n}{\varkappa_p}\frac{p_{i\perp}'+p_e'}{p_0}. \quad (3.7)$$

In terms of these variables, the system (3.6) takes the form

$$\{\omega+\varepsilon(A+D)\}X'+\frac{\varkappa_p}{\omega}\varepsilon AY'+\{\gamma\omega_{Bi}{}^*-\varepsilon D\}Z'=0,$$

$$\{-2\gamma\omega_{Bi}^{*}+\varepsilon(A+D)\}X'+\left\{\varkappa_p-\gamma\varkappa_B+\frac{\varkappa_p}{\omega}\varepsilon A\right\}Y'-\varepsilon DZ'=0,$$

$$\omega_{Bi}^{*}X'+(\varkappa_B-\varkappa_n)Y'-\omega Z'=0. \qquad (3.8)$$

Here

$$\varepsilon=i\frac{0{,}96}{10}\frac{\omega}{\nu_i}, \qquad A=\omega-0{,}59_{Bi}^{*},$$

$$D=\frac{42}{41}\frac{\omega_{Bi}^{*}}{\omega}(\omega+22{,}02\omega_{Bi}^{*}).$$

The determinant of the system (3.8) yields the dispersion equation. When $\beta < 1$, this equation takes the form

$$\omega^2-\gamma\omega_{Bi}^{*2}\left(2\gamma\frac{\varkappa_n}{\varkappa_p}-1\right)+\varepsilon\left\{(A+D)\left(\omega+\gamma\omega_{Bi}^{*}\frac{\varkappa_n}{\varkappa_p}\right)\right.$$

$$\left.+A\left(2\gamma\omega_{Bi}^{*}+\omega+\frac{\gamma\omega_{Bi}^{*2}}{\omega}\right)+D\left(2\gamma\omega_{Bi}^{*}\frac{\varkappa_n}{\varkappa_p}-\omega_{Bi}^{*}+\omega\frac{\varkappa_n}{\varkappa_p}\right)\right\}=0. \qquad (3.9)$$

Here ϵ is a small parameter. In the zeroth approximation in ϵ we obtain from (3.9) the frequencies of the two branches of the oscillations:

$$\omega^{(\pm)}=\pm\,\omega_{Bi}^{*}\sqrt{\gamma\left(2\gamma\frac{\varkappa_n}{\varkappa_p}-1\right)}, \qquad (3.10)$$

which are real if the condition (3.5) is satisfied. The terms of order ϵ yield the increments of these branches of the oscillations:

$$\operatorname{Im}\omega^{(\pm)}=-\frac{0{,}96}{20}\frac{\omega_{Bi}^{*2}}{\nu_i}\left\{37.8\frac{\varkappa_n}{\varkappa_p}+17{,}89\right.$$

$$\pm\sqrt{\gamma(2\gamma\varkappa_n/\varkappa_p-1)}\,[2.69\varkappa_n/\varkappa_p+16{,}71+(22{,}55\varkappa_n/\varkappa_p$$

$$\left.-0{,}59)(2\gamma\varkappa_n/\varkappa_p-1)^{-1}]\right\}. \qquad (3.11)$$

In particular when $\nabla T_0 = 0$ ($\kappa_n/\kappa_p = 1$), it follows from (3.11) that the branch $\omega^{(-)}$ builds up with an increment

$$\operatorname{Im}\omega^{(-)}=\frac{1}{20}\frac{\omega_{Bi}^{*2}}{\nu_i}. \qquad (3.12)$$

The instability takes place also in a certain region of positive κ_n'/κ_p. In particular, when $\kappa_n/\kappa_p \gg 1$ we obtain from (3.11)

$$\operatorname{Im}\omega^{(\pm)}=\mp 0.3\left(\frac{\varkappa_n}{\varkappa_p}\right)^{3/2}\frac{\omega_{Bi}^{*2}}{\nu_i}, \qquad (3.13)$$

i.e., as above, the branch $\omega^{(-)}$ is unstable.

4. DISCUSSION OF RESULTS

We have shown that when ion-ion collisions are taken into account the region of instability of entropy waves increases. Instability occurs in this case even if there is no gradient of the equilibrium, temperature, $\nabla T_0 = 0$. The instability is connected with a buildup of perturbations whose phase velocity is of the order of the velocity of the magnetic drift of the particles,

$$\omega / k_y \approx (v_\perp^2 / 2\omega_B)(\partial \ln B_0 / \partial x),$$

and the waves travel in the direction of the magnetic drift of the electrons (the unstable branch with the minus sign, see formula (3.10)). Such perturbations are particularly significant when $\beta \approx 1$, and when the gradients of the magnetic field and of the pressure are of the same order, $\partial \ln B_0/\partial x \approx \partial \ln p_0/\partial x$.

The growth increment of the perturbations is smaller by one order of magnitude at $\nabla T_0 = 0$ than the expression ω_{Bi}^{*2}/ν_i that follows from qualitative estimates (see formula (3.12)). Therefore it follows from an estimate of the coefficient of turbulent diffusion $D_\perp \approx (\operatorname{Im}\omega)^2/k_\perp^2 \operatorname{Re}\omega)$ (see [10]) that the instability investigated above should not lead to catastrophic consequences in the case of containment of a collision plasma with finite pressure. Under real conditions, owing to this instability, there should be observed, however, azimuthal waves with velocity on the order of the magnetic drift of the particles.

A plasma with parameters satisfying our assumptions was realized in the experiments of Bodin and Newton.[11] As noted in [11], the velocity of transverse diffusion does not exceed in this case the classical velocity, and the latter is smaller by only one order of magnitude than the Bohm velocity. Under these conditions, the instability considered by us leads only to a slight increase of the plasma diffusion. The appearance of instability under the conditions of [11] could be noted by investigating the low-frequency azimuthal noise; this, however, was not done as yet.

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Translated by J. G. Adashko
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