WAVE FIELDS IN THE OUTSIDE SPACE DURING GRAVITATIONAL COLLAPSE

A. Z. PATASHINSKIĬ and A. A. KHAR'KOV

Novosibirsk State University

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The problem of the wave fields is solved for the space surrounding a collapsing weakly nonspherical mass. The fields in question are the electromagnetic field which has the collapsing object as its source, and the nonspherical part of the metric. Near the gravitational radius there is a wave region for such processes. The nonsingular solutions in this region are derived. By joining these solutions onto the solutions in the region farther out from the gravitational radius we can conclude that during the collapse the external wave fields fall off exponentially with a characteristic time $\tau_0 \sim r_g/c$. For an object with mass of the order of a few times the solar mass this means an extremely rapid ($\tau_0 \sim 10^{-5}$ sec) disappearance of the field in space as the boundary of the body approaches the gravitational radius.

1. STATEMENT OF THE PROBLEM AND INVESTIGATION OF THE SCALAR FIELD

In the space surrounding a collapsing mass there are electromagnetic and gravitational fields connected with the matter. If the collapsing body is nearly spherical in shape the corrections to the metric, which give the deviation from the case of spherical symmetry, can be treated in the linear approximation, and this is also true for a sufficiently weak electromagnetic field. In this case the general form of the equations for the wave fields in empty space is obvious—in an appropriate gauge $\Lambda_{;k}^{;k} = 0$, where Λ is the tensor that describes the field in question. The covariant differentiation is done in the spherically symmetric metric, and therefore the variables can be separated; the solution for Λ can be put in the form of a superposition of generalized spherical harmonics.[1]

In the Schwarzschild coordinates[2] the metric of the space in which the wave is propagated is independent of the time, so that the waves can be resolved into harmonic components. We obtain the complete space of events for the region surrounding the Schwarzschild surface by using the Lemaitre coordinates corresponding to the unperturbed spherically symmetric motion. To connect these reference systems we use the well known formulas[2]

$$t = \tau - 2\sqrt{rr_g} - r_g \ln\left|\frac{\sqrt{r}-\sqrt{r_g}}{\sqrt{r}+\sqrt{r_g}}\right|, \qquad (1.1)$$
$$r = \left[\frac{3}{2}(R-\tau)\right]^{2/3} r_g^{1/3},$$

where R, τ are the Lemaitre coordinates and r, t are the Schwarzschild coordinates.

In a comoving system for the matter the instant of passage inside the gravitational radius is not distinguished in any way.[2,3] Therefore we assume that at the surface of the body near $r = r_g$ the behavior of the fields, which comes into the problem as the condition for matching the solutions, is regular and can be expanded in a series in the distance and the time of the comoving system, measured from the point of intersection of the gravitational radius. A study of the equations for the radial functions then shows that near the gravitational radius r_g, which we hereafter take as unit of length, $r_g = 1$, the nonsingular solutions of Eq. (1.1) are of wave form. As an illustration let us consider the equations in the simplest case, when $\Lambda = \psi$, where ψ is a scalar. The angular functions for the scalar case are the ordinary spherical functions Y_{lm}, and for the radial function $\chi_l(r) = \psi_l(r)[r(r-1)]^{1/2}$ the equation for the ω harmonic in Schwarzschild coordinates takes the form of the Schrödinger equation with the potential (Fig. 1)

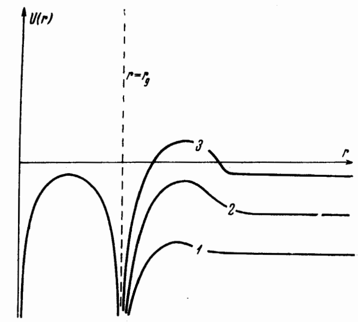

FIG. 1. The potential U(r, ω) for various values of ω. Curve 1 corresponds to the case $\omega r_g \gg 1$, curve 2 to the case $\omega r_g \sim 1$, and curve 3 to the case $\omega r_g \ll 1$.

$$U(r) = \frac{l(l+1)}{r(r-1)} - \frac{4\omega^2 r^4 + 1}{4r^2(r-1)^2}, \qquad (1.2)$$

$$\chi'' - U\chi = 0. \qquad (1.3)$$

Let us examine the behavior of the solutions of Eq. (1.3) near r = 1, where it has the form (y = r − 1)

$$\chi'' + \frac{1+4\omega^2}{4y^2}\chi = 0, \quad \chi_\pm = A_\pm y^{1/2 \pm i\omega} = A_\pm \sqrt{y}\exp(\pm i\omega \ln|y|). \qquad (1.4)$$

Accordingly, in the region $|y| \ll 1$ the field $\psi(y, t)$ is a sum of advanced and retarded waves:

$$\psi(y, t) = F_+(t + \ln|y|) + F_-(t - \ln|y|). \qquad (1.5)$$

In Lemaitre coordinates we find

$$\psi(y, \tau) = F_+(\tau) + F_-(\tau - 2\ln|y|). \qquad (1.6)$$

In order to find the form of the functions $F_\pm$ it is necessary to impose two conditions. The solution must contain only a diverging wave at spatial infinity, and it

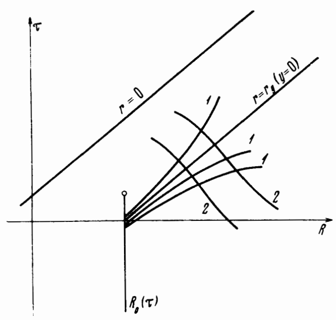

FIG. 2. The straight line $R = R_0(\tau)$ corresponds to the motion of the boundary of the collapsing body. The matter is to the left of the line R_0. The curves 1 and 2 are respectively lines of constant argument for the retarded function F_- and the advanced function F_+.

must satisfy matching conditions at the surface of the collapsing body, which moves according to the law $y_0(\tau)$. Let us first consider the second condition. Let $\tau = 0$ be the instant of proper time when the surface of the body passes inside the gravitational radius: $y(0) = 0$. The Lemaitre coordinate $R_0(\tau)$ of the surface of the body does not change (Fig. 2). With the notation $\psi(y_0(\tau), \tau) = a(\tau)$, we find for $\tau \ll 1$, $|y| \ll 1$

$$a(\tau) = F_+(\tau) + F_-(-2\ln|\tau|),\quad F_-(z) = a[\exp(-z/2)] - F_+[\exp(-z/2)]. \tag{1.7}$$

We note that for $\tau > 0$, $|y| \ll 1$ the argument $z = \tau - 2\ln|y|$ of the function F_- is always large: $z \gg 1$. For $\tau > 0$ the field near the gravitational radius thus has the form

$$\psi(y, \tau) = F_+(\tau) + a[\exp\{-{}^1/_2(\tau - 2\ln|y|)\}] - F_+[\exp\{-{}^1/_2(\tau - 2\ln|y|)\}]. \tag{1.8}$$

For $a(x)$ and $F_+(x)$ with $|x| \ll 1$ it is natural to write

$$a(x) = a + bx + cx^2 + \ldots,\quad F_+(x) = \alpha + \beta x + \gamma x^2 + \ldots, \tag{1.9}$$

which gives for $\psi(y, \tau)$:

$$\psi(y, \tau) = F_+(\tau) + (a - \alpha) + (b - \beta) y e^{-\tau/2} + (c - \gamma) y_2 e^{-\tau} + \ldots. \tag{1.10}$$

For small τ we can also replace the function $F_+(\tau)$ by the series (1.9), and we find that near the surface of the body the field ψ changes slowly as we go away from the matter:

$$\psi(y, \tau) = a + \beta\tau + (b - \beta) y + \ldots. \tag{1.11}$$

Let us consider the region of large τ and $|y| \ll 1$. Here it is convenient to go over to the Schwarzschild time t. The solution for the component of frequency ω is of the form

$$\psi(y, \omega) e^{i\omega t} = C_+ \exp[i\omega(t + \ln|y|)] + C_- \exp[i\omega(t - \ln|y|)]. \tag{1.12}$$

For small ω in the region $-\omega \ln|y| \ll 1$ this solution must go over into the quasistationary solution found in Appendix 1, to assure matching with the solution which contains only a diverging wave at infinity. Near $r = 1$ the quasistationary solution is of the form $\psi \approx C\ln|y|$, where C is proportional to the amplitude of the wave at infinity, $C(\omega)$. We thus find for C_+, C_-, and $C(\omega)$ the values

$$C_+ = -C_-,\quad C = -2i\omega C_-. \tag{1.13}$$

The behavior of $C_-(\omega)$ for small ω follows from the behavior of the function $F_-(z)$ for large z, as given by Eqs. (1.7) and (1.9),

$$F_-(z \gg 1) = (a - \alpha) + (b - \beta)\exp(-z/2) + (c - \gamma)\exp(-z) + \ldots \tag{1.14}$$

Therefore the spectrum $C(\omega)$ has no singularities near $\omega = 0$.[1)] The amplitude $I(\omega)$ of the field at spatial infinity is determined by the quantity $i\omega C_-(\omega)$ and the ratio of the coefficients of the converging and diverging waves $F_\pm$ for the solution at spatial infinity. All of these quantities are regular for small Im (ω), so that the nearest singularities of $I(\omega)$ are located at finite Im $\omega \sim 1$. This means that the field at a finite distance from the gravitational radius falls off exponentially for large times,

$$I(t) \sim \exp(-\lambda t). \tag{1.15}$$

We have given a detailed description of the character of the solution for the scalar field, since the equations for the electromagnetic and gravitational fields are much more complicated, while the general features and the idea of the investigation are the same as in the case we have analyzed.

2. THE ELECTROMAGNETIC FIELD

Let us consider the behavior of the external electromagnetic field during the collapse of a spherically symmetric, electrically neutral body. The body has magnetic or electric multipole moments. The electromagnetic energy of the body is assumed small in comparison with the gravitational energy; i.e., the metric of the external space is the same as the Schwarzschild metric. The possibility of applying such a model to actual astrophysical systems has been discussed in[4].

The general solution of the Maxwell equations for the four-potential A_i near the Schwarzschild surface is found in Appendix 3. For example, the component A_3 is given by (for simplicity we assume that there is no dependence on the angle φ)

$$A_3 = \frac{1}{2}\sum_{l=1} B_-^{(l)}(r, t) P_{01}^l(\cos\theta) = \frac{1}{2}\sum_{l=1} P_{01}^l(\cos\theta)[F_1(t - \ln|y|) + F_2(t + \ln|y|)], \tag{2.1}$$

with $|y| = |r - 1| \ll 1$; F_1, F_2 are arbitrary functions, and P_{01}^1 are associated Legendre polynomials. In the region $r \gg 1$ the solution of the Maxwell equations must describe a diverging wave $r^{-1}F(t - r)$. This condition, together with the matching condition at the surface of the body, completely determines the external electromagnetic field. It is convenient to do the matching in the Lemaitre coordinates (1.1), in which

$$A_3 = \frac{1}{2}\sum_{l=1} P_{01}^l(\cos\theta)[F_1(\tau - 2\ln|y|) + F_2(\tau)].$$

The procedure of constructing a solution satisfying the boundary conditions at the surface of the body and at spatial infinity is given in detail in Sec. 1 for the example of the scalar field. We shall not repeat the developments, and give here the final results.

1)The constants in F_+ and F_- that lead to the appearance of a contribution to $C_\pm(\omega)$ of the form $\delta(\omega)$ can be cancelled out beforehand, or we must set $C_\pm(\omega) = \lim_{\omega \to +0} C_\pm(\omega)$.

a) Near the Schwarzschild surface, immediately after the boundary of the matter has passed inside r_g we have

$$A_3 = \sum_{l=1} P_{01}{}^l(\cos\theta)\{m(0)+[m'(0)-F_2'(0)]y\exp(-\tau/2)+\tau F_2'(0)+\ldots\}, \quad (2.2)$$

with $\tau \ll 1$; $M(\tau)$ is the value of the field component in question at the boundary of the matter.

b) At large times

$$A_3 \sim \exp(-\lambda t). \quad (2.3)$$

Analogous expressions can also be obtained easily for the other components of the four-potential.

In conclusion we make one more remark. The electromagnetic field intensities calculated from (A4.7) have a singularity at r = 1. However, in contrast with the static case, Eqs. (A4.4)—(A4.6), in which the field invariants also have a singularity, the singularity here is of a purely kinematic nature and disappears when we change to a comoving reference system. The field invariants $(\mathbf{E}\cdot\mathbf{H})$ and $(\mathbf{E}^2 - \mathbf{H}^2)$ are finite in the nonstatic case.

3. THE GRAVITATIONAL FIELD

A. Let us consider the collapse of a nonrotating body (with total angular momentum K = 0) with small (axially symmetric) initial deviations from a sphere. It is known[5] that in a comoving reference system small initial perturbations of the matter remain small until the matter has attained very large densities. In this case the characteristics of the outside space also differ little from the case of spherical symmetry everywhere except in a small neighborhood of the singularity. To calculate the small corrections to the spherically symmetric part of the metric in empty space, it is sufficient to solve the linearized Einstein equations for these corrections and satisfy the boundary conditions at the surface of the body and at spatial infinity.

A qualitative analysis of the problem has been given in[6] (cf. also[3]), where it was concluded that the nonspherical corrections to the metric decrease asymptotically outside the matter, and some estimates were made of the rate of this decrease. In this section we make a more detailed analysis of the behavior of the external field.

The symmetry of the problem allows us to look for the metric in the form

$$ds^2 = \left(1-\frac{r_g}{r}\right)(1+d)\,dt^2 - \frac{1+a}{1-r_g/r}dr^2 - r^2(1+b)\,d\theta^2 - r^2\sin^2\theta\,(1+c)\,d\varphi^2. \quad (3.1)$$

A general solution of the linearized Einstein equations near the Schwarzschild surface is found in Appendix 2. It turns out that for $|r-1| = |y| \ll 1$

$$a = d = \sum_{l=2} P_{00}{}^l(\cos\theta)\,(f_+ + f_-),$$

$$c = -b = -2\sum_{l=2}\sqrt{\frac{l(l+1)}{(l-1)(l+2)}}\,P_{02}{}^l(\cos\theta)\,(f_+ + f_-),$$

$$f_+ = f_+(t+\ln|y|), \quad f_- = f_-(t-\ln|y|). \quad (3.2)$$

In the Lemaitre coordinates (1.1) the metric (2.1) takes the form

$$ds^2 = (1+d)\,d\tau^2 - \frac{1+a}{r}dR^2 - r^2(1+b)\,d\theta^2 - r^2\sin^2\theta\,(1+c)\,d\varphi^2. \quad (3.3)$$

All of the functions must be expressed in terms of τ and R. We must have

$$f_+ = f_+(\tau), \quad f_- = f_-(\tau - 2\ln|y|).$$

Satisfying the conditions at the surface of the body and at spatial infinity (where there is an outgoing gravitational wave) in the way illustrated in Sec. 1, we find, for example, for d

$$d(\tau\ll 1,\, y\ll 1) = \Sigma P^l(\cos\theta)\{q(0) + [q'(0) - f_+'(0)]y\exp[-\tau/2] + \tau f_+'(0)\}, \quad (3.4)$$

where $q(\tau)$ is the variable part of the component g_{00} of the metric tensor on the matter;

$$d(\tau \gg 1,\, r) \sim \exp(-\lambda\tau), \quad (3.5)$$

i.e., at times close to the instant $\tau = 0$ when the boundary of the matter goes inside r_g the metric near r_g is the same as on the matter. With increasing time the nonspherical corrections decrease exponentially with time in the entire space.

B. It is not hard to include a weak rotation of the body. It was shown in[6] that the presence of a small rotational angular momentum K does not affect the collapse of the matter. For K ≠ 0 nondiagonal components δg_{03} and δg_{13} of the metric tensor appear in the external metric. For them we get a system of equations independent of (A3.2); it has been studied by many authors,[6,7] and its solution is of the form

$$\delta g_{13} = \psi(r)r^2\sin\theta, \quad \delta g_{03} = \frac{1}{r}\sum_{l=1} a_l P_{01}{}^l(\cos\theta) f_l\left(\frac{1}{r}\right)\sin\theta; \quad (3.6)$$

Here $\psi(r)$ is an arbitrary function, a_l = const, $f_l(x) = x^3u_l(x)\rho(dx/x^4u_l^2(x))$ (sic), and $u_l(x) = F(2+1, 1-1; 4; x)$ is the hypergeometric function. By means of small coordinate transformations which do not change the metric (3.1) and the boundary condition of rigid-body rotation, we can bring (3.6) to the form

$$\delta g_{13} = 0, \quad \delta g_{03} = -\frac{2K}{r}\sin^2\theta. \quad (3.7)$$

Accordingly, in this case the external metric in the region r > 1 will approach the metric of a stationary rotating sphere asymptotically in time, according to the same law (3.5).

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APPENDIX 1

The static equation for a scalar field ψ in the Schwarzschild metric is of the form

$$r(r-1)\psi'' + (2r-1)\psi' - l(l+1)\psi = 0. \quad (A1.1)$$

This is the hypergeometric equation with the parameters

$$a = -l, \quad b = l+1, \quad c = 1.$$

The general solution is given by the expression[8]

$$\psi = C_1 F(-l, l+1, 1, r) + C_2\frac{(-1)^{l+1}}{r^{l+1}}F\left(l+1, l+1, 2l+2, \frac{1}{r}\right). \quad (A1.2)$$

The first solution

$$F(-l, l+1, 1, r) = \frac{1}{l!}\frac{d^l}{dr^l}[r^l(1-r)^l]$$

is regular everywhere except at r = ∞, where it diverges as r^l, $r\to\infty$.

The second linearly independent solution falls off as r^{-l-1} for $r \to \infty$, and behaves like $\ln(r-1)$ for $r \to 1$.

APPENDIX 2

The linearized Einstein equations for the metric (3.1) are (a prime indicates differentiation with respect to r, and a dot, with respect to t)

$$\frac{\partial \dot{a}}{\partial \theta}+\frac{\partial \dot{c}}{\partial \theta}+(\dot{c}-\dot{b})\operatorname{ctg}\theta=0, \tag{A2.1}$$

$$\dot{b}'+\dot{b}\,\frac{2r-3}{2r(r-1)}+\dot{c}'+\dot{c}\,\frac{2r-3}{2r(r-1)}-\frac{2\dot{a}}{r}=0,$$

$$(b'-c')\operatorname{ctg}\theta-\frac{\partial c'}{\partial\theta}-\frac{\partial d'}{\partial\theta}+\frac{2r-3}{2r(r-1)}\frac{\partial d}{d\theta}+\frac{2r-1}{2r(r-1)}\frac{\partial a}{\partial\theta}=0,$$

$$\frac{1}{2r^2}\frac{\partial^2 a}{\partial\theta^2}+\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial a}{\partial\theta}-\frac{r-1}{r^2}a'-\frac{a}{r^2}+\frac{r-1}{2r}b''+\frac{6r-5}{4r^2}b'$$
$$+\frac{b}{r^2}+\frac{r-1}{2r}c''+\frac{6r-5}{4r^2}c'+\frac{1}{2r^2}\frac{\partial^2 c}{\partial\theta^2}+\frac{\operatorname{ctg}\theta}{r^2}\frac{\partial c}{\partial\theta}-\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial b}{\partial\theta}=0,$$

$$\frac{1}{2r^2}\frac{\partial^2 d}{\partial\theta^2}+\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial d}{\partial\theta}+\frac{r-1}{r^2}d'-\frac{r\ddot{b}}{2(r-1)}+\frac{2r-1}{4r^2}b'$$
$$+\frac{b}{r^2}-\frac{a}{r^2}-\frac{r\ddot{c}}{2(r-1)}+\frac{2r-1}{4r^2}c'+\frac{1}{2r^2}\frac{\partial^2 c}{\partial\theta^2}$$
$$+\frac{\operatorname{ctg}\theta}{r^2}\frac{\partial c}{\partial\theta}-\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial b}{\partial\theta}=0,$$

$$\frac{r-1}{2r}c''+\frac{2r-1}{2r^2}c'-\frac{r\ddot{c}}{2(r-1)}-\frac{r\ddot{a}}{2(r-1)}-\frac{2r-1}{4r^2}a'$$
$$+\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial a}{\partial\theta}+\frac{r-1}{2r}d''+\frac{2r+1}{4r^2}d'+\frac{\operatorname{ctg}\theta}{2r^2}\frac{\partial d}{\partial\theta}=0,$$

$$\frac{r-1}{2r}b''+\frac{2r-1}{4r^2}b'-\frac{r\ddot{b}}{2(r-1)}-\frac{r\ddot{a}}{2(r-1)}-\frac{2r-1}{4r^2}a'$$
$$+\frac{1}{2r^2}\frac{\partial^2 a}{\partial\theta^2}+\frac{r-1}{2r}d''+\frac{2r+1}{4r^2}d'+\frac{1}{2r^2}\frac{\partial^2 d}{\partial\theta^2}=0.$$

According to the general rules for invariant decomposition of tensors in terms of generalized spherical functions,[1] we introduce new functions $\chi = c + b$, $\eta = c - b$. Separating the angular variables in the equations, we obtain the following system of equations for a_l, d_l, χ_l, η_l (we omit the indices l here)

$$-\frac{l(l+1)+2}{2r^2}a-\frac{r-1}{r^2}a'+\frac{r-1}{2r}\chi''+\frac{6r-5}{4r^2}\chi'$$
$$-\frac{l(l+1)-2}{4r^2}\chi-\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{4r^2}\eta=0, \tag{A2.1a}$$

$$-\frac{l(l+1)}{2r^2}d+\frac{r-1}{r^2}d'+\frac{2r-1}{4r^2}\chi'-\frac{l(l+1)-2}{4r^2}\chi$$
$$-\frac{r\ddot{\chi}}{2(r-1)}-\frac{a}{r^2}-\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{4r^2}\eta=0, \tag{A2.1b}$$

$$\frac{r-1}{2r}\chi''+\frac{2r-1}{2r^2}\chi'-\frac{r\ddot{\chi}}{2(r-1)}-\frac{r\ddot{a}}{r-1}-\frac{2r-1}{2r^2}a'$$
$$-\frac{l(l+1)}{2r^2}a+\frac{r-1}{r}d''+\frac{2r+1}{2r^2}d'-\frac{l(l+1)}{2r^2}d=0, \tag{A2.1c}$$

$$\frac{r-1}{2r}\eta''+\frac{2r-1}{2r^2}\eta'-\frac{r}{2(r-1)}\ddot{\eta}+\frac{[(l-1)l(l+1)(l+2)]^{1/2}}{2r^2}\cdot$$
$$\cdot(a+d)=0, \tag{A2.1d}$$

$$\left[\frac{(l-1)(l+2)}{l(l+1)}\right]^{1/2}\eta'+\chi'+2d'-\frac{2r-3}{r(r-1)}d-\frac{2r-1}{r(r-1)}a=0, \tag{A2.1e}$$

$$\dot{\chi}'+\frac{2r-3}{2r(r-1)}\dot{\chi}-\frac{2\dot{a}}{r}=0, \tag{A2.1f}$$

$$\dot{a}+\frac{1}{2}\dot{\chi}+\frac{1}{2}\sqrt{\frac{(l-1)(l+2)}{l(l+1)}}\,\dot{\eta}=0. \tag{A2.1g}$$

The solution of the equations (A2.1) near the gravitational radius is of the following form[9]

$$a_l=\frac{1}{2}\chi_l'-\frac{1}{4y}\chi_l,$$

$$d_l=\frac{1}{-1-l(l+1)}\left[\frac{\ddot{\chi}_l}{y}-y\chi_l''-\frac{l(l+1)-1}{2}\chi_l'-\frac{l(l+1)+3}{4y}\chi_l\right],$$

$$\eta_l=-2\sqrt{\frac{l(l+1)}{(l-1)(l+2)}}\left(\frac{1}{2}\chi_l'-\frac{1}{4y}\chi_l\right), \tag{A2.2}$$

$$\chi_l=y[F_+(t+\ln|y|)+F_-(t-\ln|y|)],$$
$$y=r-1.$$

From this we find for the corrections to the Scwharzschild metric the expressions

$$a=d=\sum_{l=2}P_{00}{}^l(\cos\theta)\left[\frac{1}{2}(F_+'-F_-')+\frac{1}{4}(F_++F_-)\right],$$

$$c=-b\approx-2\sum_{l=2}P_{02}{}^l(\cos\theta)\left[\frac{1}{2}(F_+'-F_-')+\frac{1}{4}(F_++F_-)\right]. \tag{A2.3}$$

For the functions $F_\pm$ a prime indicates differentiation with respect to the entire argument.

APPENDIX 3

As is well known,[2] the generally covariant wave equation for the four-potential A_i of the electromagnetic field is

$$g^{km}A_{i;\,k;\,m}=0 \tag{A3.1}$$

with the supplementary condition $g^{km}A_{k;m} = 0$.

Writing these equations out for the case in which g_{ik} is the Schwarzschild metric, and separating the angular variables by means of generalized spherical functions, we get the following system of equations:

$$\frac{r-1}{r}A_0''+\frac{2(r-1)}{r^2}A_0'+\left(\frac{\omega^2 r}{r-1}-\frac{l(l+1)}{r^2}\right)A_0-\frac{i\omega}{r^2}A_1=0,$$

$$\frac{r-1}{r}A_1''+\frac{2}{r}A_1'+\left(\frac{\omega^2 r}{r-1}-\frac{2(r-1)}{r^3}-\frac{l(l+1)}{r^2}\right)A_1$$
$$-\frac{i\omega}{(r-1)^2}A_0-\frac{\sqrt{l(l+1)}}{r^3}B_+=0,$$

$$\frac{r-1}{r}B_-''+\frac{1}{r^2}B_-'+\left(\frac{r\omega^2}{r-1}-\frac{l(l+1)}{r^2}\right)B_-=0, \tag{A3.2}$$

$$\frac{r-1}{r}B_+''+\frac{1}{r^2}B_+'+\left(\frac{r\omega^2}{r-1}-\frac{l(l+1)}{r^2}\right)B_+$$
$$-\frac{4\sqrt{l(l+1)}(r-1)}{r^2}A_1=0,$$

$$\frac{i\omega r}{r-1}A_0+\frac{r-1}{r}A_1'+\frac{2r-1}{r^2}A_1+\frac{\sqrt{l(l+1)}}{2r^2}B_+=0.$$

We have at the same time made a Fourier transformation with respect to the time and introduced the functions

$$B_\pm=A_+\pm A_-;\quad A_+=-\left(\frac{A_3}{\sin\theta}+iA_2\right),\quad A_-=\frac{A_3}{\sin\theta}-iA_2.$$

If there is no dependence on the angle φ (i.e., if the field is axially symmetric), then $T^l_{-10} = T^l_{10}$, and apart from numerical factors the $B_\pm$ are identical with the components A_2 and A_3 of the four-potential A_i.

For $l = 1$ we can find exact static solutions of the system of equations (A3.2). Written for the "physical" components of the four-potential the static solutions, which go over respectively for $r \gg 1$ into the well known expressions for the field of an electric or a magnetic dipole parallel to the z axis, are

$$A_{0\,(\mathrm{phys})} = -\frac{d\cos\theta}{\sqrt{1-1/r}}\left[2-\frac{1}{r}+2(r-1)\ln\frac{r-1}{r}\right] \tag{A3.3}$$

and

$$A_{3\,(\mathrm{phys})} = -6m\sin\theta\left[1+\frac{1}{2r}+r\ln\frac{r-1}{r}\right]. \tag{A3.4}$$

The solution (A3.4) was first found in[4].

Let us find the behavior of the solutions of the system of equations (A3.2) in the region $r \sim 1$. Keeping the main terms of the expansion of the coefficients of the equations in powers of $y = r - 1$, we get the following system of equations:

$$\begin{gathered} yA_0'' + 2yA_0' + \frac{\omega^2}{y}A_0 - i\omega A_1 = 0, \\ yA_1'' + 2A_1' + \frac{\omega^2}{y}A_1 - \frac{i\omega}{y^2}A_0 - [l(l+1)]^{1/2}B_+ = 0, \\ yB_+'' + B_+' + \frac{\omega^2}{y}B_+ - 4\sqrt{l(l+1)}\,yA_1 = 0, \\ yB_-'' + B_-' + \frac{\omega^2}{y}B_- = 0, \\ i\omega y^{-1}A_0 + yA_1' + A_1 + {}^1/_2[l(l+1)]^{1/2}B_+ = 0. \end{gathered} \tag{A3.5}$$

In the static case $\omega = 0$ the solutions near the gravitational radius are of the form

$$B_\pm \approx \mathrm{const}\ln|y|, \qquad A_0 = \mathrm{const}, \quad A_1 \approx \mathrm{const}/y. \tag{A3.6}$$

For $\omega \neq 0$ we have the following asymptotic behavior of the solutions near $r = 1$:

$$B_- = C_1(\omega)y^{i\omega} + C_2(\omega)y^{-i\omega}, \quad B_+ = C_3(\omega)y^{i\omega} + C_4(\omega)y^{-i\omega},$$
$$A_1 = C_+(\omega)y^{-1+i\omega} + C_-(\omega)y^{-i-i\omega}, \quad A_0 = C_+(\omega)y^{i\omega}(\omega) - C_-(\omega)y^{-i\omega},$$

or going over to the t representation:

$$\begin{aligned} B_-(y,t) &= F_1(t+\ln|y|) - F_2(t-\ln|y|), \\ B_+(y,t) &= F_3(t+\ln|y|) + F_4(t-\ln|y|), \\ A_1(y,t) &= \frac{1}{y}[F_+(t+\ln|y|) + F_-(t-\ln|y|)], \\ A_0(y,t) &= F_+(t+\ln|y|) - F_-(t-\ln|y|). \end{aligned} \tag{A3.7}$$

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