NONSTATIONARY EFFECTS IN THE RESISTIVE STATE OF SUPERCONDUCTING FILMS

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The Langevin random-force method is used to investigate the fluctuations of the ordering parameter in a superconducting film in the presence of an electric field at temperatures $T > T_c$ and $T < T_c$. A descending section of the current-voltage characteristic appears in the resistive state below T_c. The fluctuations have in this case characteristic growth times determined by the value of the electric field; this leads to the appearance of preferred emission frequencies. The latter effect may become manifest in a nonmonotonic (oscillating) dependence of the transparency of the film on the frequency of the electromagnetic wave incident on it.

It is well known at present that superconductivity and an electric field are not mutually exclusive concepts. Indeed, in the case of the Josephson effect,[1] superconducting order remains in force also at a nonzero voltage across the tunnel barrier V, and the superconducting current oscillates as a function of the time with frequency $\omega = 2eV/\hbar$. Analogous nonstationary effects, connected with the motion of Abrikosov vortices under the influence of an electric field, arise in the dynamic mixed state of superconductors of the second kind.[2,3] On the whole, it can be stated that if the electric field does not lead to a complete vanishing of the superconductivity, a certain nonstationary picture should be observed, connected with the dynamic behavior of the superconducting-ordering parameter ψ.

The purpose of the present paper is to analyze the nonstationary effect in thin superconducting films in the "resistive" state, i.e., under conditions when the current exceeds the critical value ($j > j_c$). Nonetheless, as shown by experiment, there is no complete vanishing of the superconductivity when $j > j_c$: the voltage across the film remains smaller than in the normal state at the same value of the current. An analogous behavior (sometimes connected with hysteresis) can be observed also when $j < j_c$. A specific feature of this system, compared with the Josephson tunnel junction, is the fact that the electric field influences not only the phase of the ordering parameter, but also the modulus of ψ. It can be shown in this case that at equilibrium, in the presence of even a weak constant electric field E, we have $\psi \to 0$.[4,5] This is connected with the fact that when $E \neq 0$ the momentum of the Cooper pair increases without limit, whereas it is known that pairs with large values of the momentum are unstable and are consequently destroyed, i.e., ψ vanishes. Therefore the experimentally observed nonzero value of ψ should be connected with a deviation from equilibrium, i.e., with fluctuations. The picture of the resistivity at $E \neq 0$ and $T < T_c$ thus turns out to be close to the picture of the fluctuation conductivity of films at $T > T_c$, first considered by Aslamazov and Larkin.[6]

In the present paper we analyze the dynamic behavior of the superconducting-ordering parameter ψ in a film

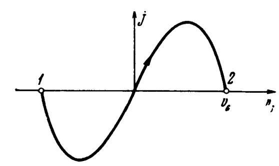

FIG. 1. Dependence of the current on the superfluid velocity at $T \to T_c$.

in the presence of an electric field at temperatures both higher and lower than T_c.[1)] It turns out here that although the fluctuation spectrum contains all the frequencies, nonetheless certain frequencies are singled out. In a weak electric field E_0 at $T < T_c$, singularities appear in different characteristics of the film at frequencies $\omega = n\omega_0$, where ω_0 is defined by the relation (compare with [4])

$$\hbar\omega_0 = 2\pi e E_0 \xi(T) \tag{1}$$

($\xi(T)$ is the temperature-dependent coherence radius of the superconductor).

The reason for it can be understood from Fig. 1, which shows the dependence of the current in a superconducting channel on the "superfluid velocity" parameter v_s (see also [9]). The critical velocity v_c in Fig. 1 is equal to $v_c = \hbar/m\xi(T)$. The time dependence of v_c is determined by the equation $m dv_s/dt = 2eE$, whence $v_s = v_{s0} + 2eEt/m$. Therefore the time required for the passage of the particle between points 1 and 2 in Fig. 1 is equal to

$$\tau_0 = \frac{mv_c}{eE_0} = \frac{\hbar}{eE_0\xi}, \tag{2}$$

corresponding to an oscillation frequency $\omega_0 = 2\pi/\tau_0$, the expression for which is given above. Thus, the superconducting nucleus produced at the point 1 reaches after a time τ_0 the point 2, after which it is destroyed within a time τ_1, which is small compared with τ_0 if the electric field E_0 tends to zero. Of course, nuclei can be

1)We consider the fluctuation conductivity of films in the spirit of the theory of Aslamazov and Larkin, and do not take into account the corrections introduced into their expression by Maki [7] and Thompson [8]. These corrections are small very close to T_c, or else in the case of strongly inelastic scattering. Qualitatively, at any rate, they do not influence the effect considered below.

produced also at other points, but at any rate the time interval τ_0 is singled out (extremal). As a result, singularities appear in the high-frequency conductivity of the film at frequencies $\omega = n\omega_0$. Owing to the random character of the fluctuations, the singularities have not a resonant but an oscillatory character, i.e., they appear in the form of a nonmonotonic (periodic) variation of the impedance of the film as a function of the frequency, with a period ω_0. We present below a quantitative theory confirming these qualitative considerations.

Proceeding to a calculation of the fluctuation conductivity of the films, we shall use the method of random forces, the so-called Langevin equation in the theory of random processes (see, for example, [10]).[2)] According to this method, it is necessary to include in the equation describing the dynamic behavior of the investigated system a random "force" S(t), the amplitude of which is determined from a condition according to which the rms fluctuating quantity should coincide with its value calculated statistically on the basis of the equipartition law. In the case of interest to us, the role of the fluctuating quantity is played by ψ, and the dynamic equation describing its time evolution, is of the Abrahams-Tsuneto type:[12]

$$\frac{\partial\psi}{\partial t}+\Gamma\psi-D\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)^2\psi=0, \quad (3)$$

where

$$\Gamma=8\pi^{-1}(T-T_c), \quad D={}^1/_3 v_0 l \quad (4)$$

(v_0 is the Fermi velocity and l is the mean free path of the electrons, $l \ll \xi_0 \sim v_0/T_C$).

It follows from an analysis performed by a number of authors particularly by Gor'kov and Éliashberg[13] and by Schmid,[14] that the character of the relaxation of the Cooper pairs in the superconductor depends on the type of the "ground state," i.e., the value of $\psi = \psi_0$ in the absence of fluctuations. As already discussed, $\psi_0 = 0$ in the presence of an electric field and in the presence of equilibrium at temperatures higher or lower than T_C. We shall call such a state "normal." In this case the role of the relaxation time τ_1 of the Cooper pairs is played by the reciprocal of the parameter Γ introduced above: $\tau_1 \sim 1/\Gamma$. Indeed, in the normal state the gap in the spectrum is equal to zero, making it possible to derive an equation of the type (3) (see [15]), describing precisely such a character of the relaxation. Another type of the ground state corresponds to a value of ψ_0 different from zero (superconducting state). In this case, owing to the presence of a gap in the spectrum, the relaxation time of the Cooper pairs becomes of the order of $\tau_2 \sim \tau_\epsilon\sqrt{T_C/(T_C - T)}$, and in this case, generally speaking, $\tau_2 \gg \tau_1$. Here $\tau_\epsilon \approx 10^{-8}$ sec is the time of energy dissipation in a normal metal as a result of electron-phonon and electron-electron collisions.[14,15]

We shall consider below the "normal" ground state described by an equation of the type (3) or by an analogous equation for superconductors with paramagnetic impurities, obtained by Gor'kov and Éliashberg.[15]

2) An analogous approach at $T > T_C$ was already used earlier by Schmid [11].

Introducing in (3) the random forces, we obtain the Langevin equation

$$\frac{\partial\psi}{\partial t}+\Gamma\psi-D\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)^2\psi=S(\mathbf{r},t). \quad (5)$$

The stochastic term S(r, t) in (5) has the following correlation properties (we consider the case of a film with thickness $d \ll \xi(t)$):

$$\overline{S(\mathbf{r},t)}=0, \quad \overline{S(\mathbf{r},t)S(\mathbf{r}',t')}=0,$$
$$\overline{S(\mathbf{r},t)S^*(\mathbf{r}',t')}=4mTDd^{-1}\delta(\mathbf{r}-\mathbf{r}')\delta(t-t'). \quad (6)$$

We can write analogously an equation of the type (5) for a thin superconducting filament. In this case we obtain in lieu of (6)

$$\overline{S(x,t)S^*(x',t')}=\frac{4mTD}{d_1 d_2}\delta(x-x')\delta(t-t'), \quad (6')$$

where d_1 and d_2 are the transverse dimensions of the filament (small compared with $\xi(T)$).

It can be easily verified that relations (6) and (6′) are obtained from the requirement that the correlation function of the ordering parameter $\overline{\psi(\mathbf{r},t)\psi^*(\mathbf{r}',t)}$ at $T > T_C$, calculated on the basis of Eq. (5) coincides with the corresponding quantity obtained by directly averaging $\psi\psi^*$ with the Gibbs factor $\exp\{-\beta F(\psi)\}$, where $\beta = 1/T$ and $F(\psi)$ is a functional of the Ginzburg-Landau free energy, equal in this case ($\hbar = 1$) to

$$F=\frac{d}{2m}\int d^2r\left[\left|\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right|^2+\frac{1}{\xi^2}|\psi|^2\operatorname{sign}\Gamma\right]. \quad (7)$$

$\xi(T)$ is the temperature-dependent coherence radius, which we define for $T < T_C$ and $T > T_C$ by the relation

$$\xi^2(T)=D/|\Gamma(T)|. \quad (8)$$

Equation (5) is gauge-invariant, since multiplication of S by an arbitrary phase factor $\exp[i\chi(\mathbf{r},t)]$ does not violate relations (6) and (6′). From the physical point of view (5) describes the motion of a macroscopic system—a condensate under the influence of random "molecular" jolts, having a δ-correlated spectrum—which is analogous to the motion of a Brownian particle in a liquid.

With the aid of (5) and (6) it is easy to calculate the fluctuation current resulting from an arbitrarily time-varying electric field $\mathbf{E} = \mathbf{E}(t)$ (E lies in the plane of the film). In the Ginzburg-Landau theory, the current is expressed by the relation

$$\mathbf{j}=\frac{2e}{m}\operatorname{Im}\left[\psi^*\left(\nabla-\frac{2ie}{c}\mathbf{A}\right)\psi\right]. \quad (9)$$

Changing over to the Fourier representation, we obtain for the mean value of $\mathbf{j}$:

$$\bar{\mathbf{j}}(t)=\frac{2e}{m}\sum_{\mathbf{k}}\left(\mathbf{k}-\frac{2e}{c}\mathbf{A}(t)\right)\overline{|\psi_{\mathbf{k}}(t)|^2},$$
$$\mathbf{A}(t)=-c\int_0^t\mathbf{E}(t')dt'. \quad (10)$$

The solution of (5) in the **k**-representation is

$$\psi_{\mathbf{k}}(t)=\exp\{-p_{\mathbf{k}}(t)\}\int_{-\infty}^t\exp\{p_{\mathbf{k}}(t')\}S_{\mathbf{k}}(t')dt', \quad (11)$$

where $S_k(t)$ are the Fourier components of the function S(r, t) and satisfy the relation (see (6))

$$\overline{S_{\mathbf{k}}(t)S_{\mathbf{k}'}^{*}(t')}=\frac{4mTD}{dL_1L_2}\delta_{\mathbf{k}\mathbf{k}'}\delta(t-t') \quad (12)$$

(L_1 and L_2 are the dimensions of the film in the x and y directions). The quantity $p_k(t)$ in (11) is given by

$$p_{\mathbf{k}}(t)=\int_0^t\left[\Gamma+D\left(\mathbf{k}-\frac{2e}{c}\mathbf{A}(t')\right)^2\right]dt'. \quad (13)$$

Taking (11) into account, we can calculate explicitly the sum over **k** in (10):

$$\sum_{\mathbf{k}}=\frac{L_1L_2}{(2\pi)^2}\int d^2k.$$

Omitting the simple derivations, we present the expression obtained in this manner for the current:

$$\bar{\mathbf{j}}(t)=-\frac{2e^2T}{\pi cd}\int_{-\infty}^{t}\frac{d\tau}{t-\tau}\left[\mathbf{A}(t)-\frac{1}{t-\tau}\int_{\tau}^{t}\mathbf{A}(t')dt'\right]e^{-2\Gamma(t-\tau)}$$
$$\times\exp\left\{-\frac{8e^2D}{c^2}\left[\int_{\tau}^{t}\mathbf{A}^2(t')dt'-\frac{1}{t-\tau}\left(\int_{\tau}^{t}\mathbf{A}(t')dt'\right)^2\right]\right\}. \quad (14)$$

The obtained formula is a general expression for the current produced in the film when an arbitrarily time-varying but spatially homogeneous electric field **E**(t) is applied. This formula holds both above and below T_c, and its applicability is not limited by the condition that **E** be small. The results obtained by other authors are limiting cases of expression (14) (see below).

1. We consider first the case when **E** is constant. Then **A**(t) = − c**E**t. The current **j** will not depend on the time, and the expression for its value can be reduced, on the basis of (14), to the form

$$\bar{\mathbf{j}}=\frac{e^2T\mathbf{E}}{2\pi d|\Gamma|}\int_0^{\infty}\exp\left(-\lambda\,\mathrm{sign}\,\Gamma-\frac{\varepsilon^2\lambda^3}{3}\right)d\lambda, \quad (15)$$

where ϵ denotes the dimensionless parameter

$$\varepsilon=eE\xi/2|\Gamma|. \quad (16)$$

Above T_c in a weak field ($\epsilon \ll 1$), formula (15) yields exactly the Aslamazov and Larkin expression for the fluctuation conductivity:[6]

$$\sigma=\frac{j}{E}=\frac{e^2T}{2\pi d\Gamma}=\frac{e^2}{16d}\frac{T}{T-T_c}. \quad (17)$$

From (15) it is possible to find the dependence of j(E) in a strong field, when the current-voltage curve of the fluctuation current is already nonlinear. This question was considered in recent papers.[11,16] The strong-field criterion $\epsilon \gtrsim 1$ or $eE\xi \gtrsim \Gamma$ denotes that the energy acquired by the electron over the coherence length is compared with the reciprocal relaxation time Γ of the Cooper pairs. In a strong field ($\epsilon \gg 1$), we have $j \propto E^{1/3}$, and this dependence remains valid up to very large values of the electric field intensity E.

Below T_c we have negative Γ (see (4)). Therefore the integral (15) converges (the conductivity remains finite) only because of the second term in the argument of the exponential (15). This is precisely the term that accounts for the acceleration of the pair by the electric field and the resultant destruction of the pair. Calculating the integral (15) asymptotically as $\epsilon \to 0$, we get

$$j\approx\frac{eT}{d\xi}\left(\frac{eE\xi}{2\pi|\Gamma|}\right)^{1/3}\exp\left\{\frac{4|\Gamma|}{3eE\xi}\right\}. \quad (18)$$

This formula was obtained earlier by Gor'kov[5] with the aid of a microscopic calculation, and independently (prior to the publication of [5]) by us.

Of course, expression (18) is valid if the fields are not too weak, so long as the fluctuation current $\bar{j}$ is smaller than the critical unpairing current j_c. When $\bar{j} \sim j_c$ the fluctuations can no longer be regarded as linear (5).

It is important that, in accordance with formula (18), the current $\bar{j}$ increases with decreasing electric field E. Therefore, if the fluctuation component of the conductivity is sufficiently large, the current-voltage characteristic of the film (with allowance for the normal current $j_n = \sigma_n E$) will have a decreasing section, i.e., a negative resistance, leading to instability. Apparently there are grounds for hoping to explain in this manner the generation of high-frequency oscillations by superconducting films in the resistive regime, an effect recently observed by Churilov, Dmitriev, and Beskorskiĭ.[17] An investigation of the character of the resultant instability, which calls, however, for an analysis of the nonlinear fluctuations, is a very complicated matter and will be the subject of further investigations.

2. At temperatures above T_c it is easy to obtain in general form an expression for the linear response of the system **j**(t) to an arbitrary (weak) alternating electric field **E**(t). Linearizing in formula (14), we arrive at an integral connection between **j** and **E**:

$$\mathbf{j}(t)=\sigma_{AL}\int_{-\infty}^{t}K(t-t')\mathbf{E}(t')dt'; \quad (19)$$

here σ_{AL} denotes the static fluctuation conductivity of the film in accordance with Aslamazov and Larkin

$$\sigma_{AL}=e^2T/2\pi\Gamma d, \quad (20)$$

and the kernel K(t) is given by

$$K(t)=4\Gamma e^{-2\Gamma t}\int_0^{\infty}\frac{\tau}{(t+\tau)^2}e^{-2\Gamma\tau}d\tau, \quad (21)$$

The function K(t) has a characteristic radius $\Delta t \sim 1/\Gamma$. Taking the Fourier component of K(t), we obtain the frequency dependence of the fluctuation conductivity $\sigma(\omega)$. This is easiest to do by considering directly the response to a monochromatic signal in formula (14). This leads to the expression

$$\sigma(\omega)=\sigma_{AL}K(\omega), \quad (22)$$

where

$$K(\omega)=2\Gamma\int_0^{\infty}\frac{2(1+i\omega\tau-e^{i\omega\tau})}{(\omega\tau)^2}e^{-2\Gamma\tau}d\tau. \quad (23)$$

K is equal to unity when $\omega \to 0$, K(0) = 1, as it should. Calculating the integral (21), we obtained for the real and imaginary parts of K the explicit expressions

$$\mathrm{Re}\,K=\frac{2}{\Omega}\,\mathrm{arctg}\,\Omega-\frac{1}{\Omega^2}\ln(1+\Omega^2),\ \Omega=\frac{\omega}{2\Gamma}, \quad (24)$$

$$\mathrm{Im}\,K=-\frac{2}{\Omega}+\frac{2}{\Omega^2}\,\mathrm{arctg}\,\Omega+\frac{1}{\Omega}\ln(1+\Omega^2). \quad (25)$$

Formula (24) for the real part of $K(\omega)$ was obtained earlier by Schmidt[18] with the aid of the fluctuation-dissipation theorem. Formula (25) describes the frequency dependence of the imaginary part of $K(\omega)$. As follows from the latter expression, at small ω the value of Im $\sigma(\omega)$ is proportional to ω:

$$\operatorname{Im}\sigma(\omega) \approx \frac{\omega}{6\Gamma}\sigma_{AL}, \quad \omega \ll \Gamma, \tag{26}$$

corresponding to the capacitive character of the fluctuation conductivity.

3. Finally, let us consider the region of temperatures below T_c, assuming that the film is simultaneously under the influence of a constant electric field $\mathbf{E}_0$ and a weak alternating field $\mathbf{E}_1$:

$$\mathbf{E}(t) = \mathbf{E}_0 + \mathbf{E}_1 \cos\omega t. \tag{27}$$

Linearizing in (14) with respect to $\mathbf{E}_1$, we obtain

$$\mathbf{j}(t) = \mathbf{j}_0 + \mathbf{j}_1(t), \tag{28}$$

where $\mathbf{j}_0$ has already been calculated above (formula (15) at $\mathbf{E} = \mathbf{E}_0$), and for $\mathbf{j}_1(t)$ we have

$$\mathbf{j}_1(t) = \operatorname{Re}\{[\sigma_\perp(\omega)\mathbf{E}_1 + \sigma_\parallel(\omega)\mathbf{n}(\mathbf{n}\mathbf{E}_1)]e^{-i\omega t}\}; \tag{29}$$

$\mathbf{n}$ is a unit vector along $\mathbf{E}_0$: $\mathbf{n} = \mathbf{E}_0/E_0$. The quantities $\sigma_\perp(\omega)$ and $\sigma_1(\omega)$ in (29) are determined by the expressions

$$\sigma_\perp(\omega) = \frac{2e^2T}{\pi d}\int_0 d\tau\, \frac{1 + i\omega\tau - e^{i\omega\tau}}{(\omega\tau)^2}\cdot\exp\left\{-2\Gamma\tau - \frac{2}{3}De^2E_0{}^2\tau^3\right\},$$

$$\sigma_\parallel(\omega) = -\frac{8e^2T}{\pi d}\int_0^\infty d\tau\, De^2E_0{}^2\tau^3\,\frac{\omega\tau(e^{i\omega\tau}+1) + 2i(e^{i\omega\tau}-1)}{(\omega\tau)^3} \tag{30}$$

$$\times\exp\left\{-2\Gamma\tau - \frac{2}{3}De^2E_0{}^2\tau^3\right\}. \tag{31}$$

Inasmuch as $\Gamma < 0$, the convergence of the integrals is ensured only as a result of the second term, proportional to τ^3, in the arguments of the exponential (compare with (15)).

At small values of E_0, the integrals written out above can be calculated by the saddle-point method. The saddle points $\tau = \tau_0$ is determined from the equation

$$\frac{d}{d\tau}\left[2|\Gamma|\tau - \frac{2}{3}De^2E_0{}^2\tau^3\right] = 0, \tag{32}$$

whence we obtain, using (8), $\tau_0 = 1/eE_0\xi$, which agrees with formula (2) and corresponds to the picture discussed at the beginning of the article. In the limit $\tau_0|\Gamma| \gg 1$, we obtain asymptotic expressions for the conductivities $\sigma_\perp$ and $\sigma_\parallel$:

$$\sigma_\perp(\omega) \approx \tilde{\sigma}_{AL}F_\perp(\omega\tau_0)\,(2\pi|\Gamma|\tau_0)^{1/2}\cdot\exp\left(\frac{4}{3}|\Gamma|\tau_0\right), \tag{33}$$

$$\sigma_\parallel(\omega) \approx -\tilde{\sigma}_{AL}F_\parallel(\omega\tau_0)\frac{2}{3\pi}(2\pi|\Gamma|\tau_0)^{3/2}\cdot\exp\left(\frac{4}{3}|\Gamma|\tau_0\right), \tag{34}$$

where $\tilde{\sigma}_{AL}$ is expressed in analogy with the fluctuation conductivity of the film at $T > T_c$ (formula (20))

$$\tilde{\sigma}_{AL} = e^2T/2\pi|\Gamma|d, \tag{35}$$

and the functions $F_\perp(z)$ and $F_\parallel(z)$ are given by

$$F_\perp(z) = 2z^{-2}(1 + iz - e^{iz}), \tag{36}$$

$$F_\parallel(z) = 6z^{-3}[2i(1 - e^{iz}) - z(1 + e^{iz})]. \tag{37}$$

They are normalized in such a way that at the point $z = 0$ we have $F_\perp(0) = F_\parallel(0) = 1$. Figure 2 shows plots of the real and imaginary parts of $F_\perp$ and $F_\parallel$ as functions of the parameter $z = \omega\tau_0$. As seen from the curves, $F_\perp$ and $F_\parallel$ oscillate as functions of their argument and the amplitude of the oscillations decreases with increasing z.

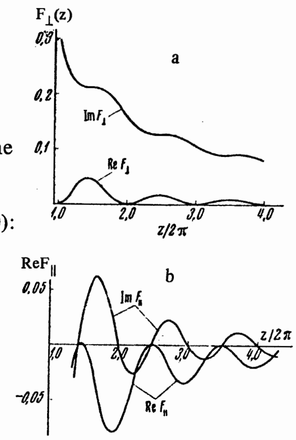

FIG. 2. Frequency dependence of the conductivities $\sigma_\perp(\omega)$ and $\sigma_\parallel(\omega)$: a—plot of the function $F(z) = \sigma_\perp(\omega)/\sigma_\perp(0)$, b—plot of the function $F_\parallel(z) = \sigma_\parallel(\omega)/\sigma_\parallel(0)$: the parameter is $z = \omega\tau_0$.

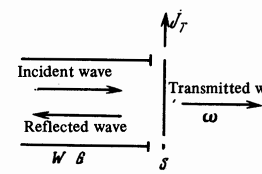

FIG. 3. Scheme of experiment for observation of oscillations of the impedance in the resistive state.

Thus, the conductivities $\sigma_\perp(\omega)$ and $\sigma_\parallel(\omega)$ are nonmonotonic (oscillating) functions of the alternating-signal frequency ω. The period of the oscillations is equal to $\Delta\omega = \omega_0 = 2\pi/\tau_0$, where the quantity ω_0 corresponds exactly to formula (1). We note that the parallel conductivity $\sigma_\parallel(\omega)$ is negative at low frequencies, and it greatly exceeds $\sigma_\perp(\omega)$ by virtue of the condition $\tau_0|\Gamma| \gg 1$ (see (33) and (34)). If the direction of the high-frequency field $\mathbf{E}_1$ makes an angle θ with a constant field $\mathbf{E}_0$ then, in accordance with (29), the total conductivity is

$$\sigma(\theta, \omega) = \sigma_\perp(\omega) + \sigma_\parallel(\omega)\cos^2\theta. \tag{38}$$

At all values of the angles with the exception of θ close to $\pi/2$, we have amplification of the incident signal. This is understandable, since we are on the decreasing section of the current-voltage characteristic.[3)]

Homogeneous current oscillations corresponding to formulas (33) and (34) can be observed in a film by means of the scheme shown in Fig. 3. The superconducting film S, which is made resistive by the transport current j_T passing through it, covers a section of a waveguide WG. In this case the intensity of the radiation passing through the film is an oscillating function of the frequency ω. We note that according to (33) and (34) the oscillations should take place also at a fixed frequency as a function of the field intensity E_0 or the coherence length (actually, the temperature) ξ, since τ_0 depends on these parameters. In the latter case, however, there occurs also an essential monotonic dependence of σ, described by the exponential terms in

[3)]It is assumed that the film is connected in an external circuit with a given voltage. As usual, to this end it is necessary that the internal resistance of the source be much smaller than the dynamic resistance of the film. In addition, we ignore the possibility of formation of an inhomogeneous situation similar to Gunn's moving domains in semiconductors with a descending N-shaped characteristic (see [19]). When the inhomogeneity is taken into account, the character of the resultant oscillations is different. Analysis of this question calls for allowance for the nonlinear fluctuations (see the remark made at the end of Sec. 1).

formulas (33) and (34). Nonetheless, if the frequency of the radiation incident on the film is sufficiently large, $\omega\tau_0 \gg 1$, then the change of τ_0, needed to observe one period of the oscillations, may be small, and the monotonic part of the dependence of σ on τ_0 will be weak. It is easy to see that to this end it is necessary that the frequency be high in comparison with $|\Gamma|$, namely $\omega > {}^8/_3\pi|\Gamma|$. According to (4), at $T_c - T = 10^{-2}$ °K this corresponds to $\omega \gtrsim 3\times10^{10}$ sec^{-1}.

In conclusion, I am grateful to L. P. Gor'kov for a discussion of the work and for supplying a preprint of his article [5]. I am also grateful to I. M. Dmitrenko for constant interest in the problem and for useful discussions.

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