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# INTERACTION BETWEEN ELASTIC AND SPIN WAVES IN YTTRIUM IRON GARNET CRYSTALS

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Propagation of 1000 MHz longitudinal and transverse elastic waves in yttrium iron garnet crystals is investigated as a function of the magnetic field strength. The measurements were carried out under conditions of a uniform internal magnetic field. The properties of the propagation of elastic waves are studied for various propagation directions with respect to the crystallographic axes and with respect to the magnetic field. The magnetoelastic constants are determined from the acoustic Faraday and birefringence effects. The values obtained are in agreement with those obtained by other methods. It is shown that the experimental results are in good agreement with the predictions of the phenomenological theory of magnetoelastic interaction.

THE elastic state of a magnetically ordered crystal is related to its magnetic state. Dynamically this relationship manifests itself as the interaction of elastic oscillations with spin waves.[1] As a result of such a relationship the propagation of elastic waves in a magnetically ordered crystal is accompanied by peculiar effects: magnetoelastic resonance, rotation of the plane of polarization, and acoustic birefringence.[1,2]

After the first theoretical papers on the magnetoelastic interaction,[3-5] a series of experiments was carried out investigating the propagation of elastic waves in cubic crystals of yttrium iron garnet.[6-11] However, all these experiments were carried out under conditions of a nonuniform internal magnetic field, and the most detailed of them[7,8] were carried out at relatively low frequencies (200 MHz), i.e., with the magnetic domain structure of the crystals preserved. In addition, only some of the most interesting cases of the propagation of elastic waves were studied in these papers.

In this paper we have set ourselves the following problems.

1. To carry out an investigation of the propagation of longitudinal and transverse elastic waves with frequencies of about 1000 MHz in yttrium iron garnet crystals under the conditions of a uniform internal magnetic field.

2. To study the properties of the propagation of elastic waves as a function of the magnitude of the magnetic field and as a function of the direction of propagation relative to the crystallographic axes and relative to the magnetic field.

3. To compare the predictions of the phenomenological theory of the magnetoelastic interaction with the experimental results.

## 1. EQUATIONS OF MOTION FOR THE ELASTIC DISPLACEMENTS AND FOR THE MAGNETIZATION

The total energy of a magnetically ordered crystal is written in the following form[2]:

$$E = E_z + E_e + E_{me},$$

where $E_z$, $E_e$, and $E_{me}$ are the Zeeman, elastic, and magnetoelastic energies equal to[12,13]

$$E_z = -M_0(\alpha_i H_i), \quad E_e = {}^1/_2 c_{ijkl}\varepsilon_{ij}\varepsilon_{kl}, \quad E_{me} = b_{ijkl}\alpha_i\alpha_j\varepsilon_{kl}.$$

Here $M_0$ is the saturation magnetization, $H_i$ and $\alpha_i$ are the components of the internal magnetic field and of the unit vector of the magnetization, and $\epsilon_{kl}$ are the components of the strain tensor.

For a cubic crystal the fourth-rank tensors $c_{ijkl}$ and $b_{ijkl}$ have three and two independence components respectively[12 13]:

$$c_{xxxx} = c_{11}, \quad c_{xxyy} = c_{12}, \quad c_{xyxy} = c_{44},$$
$$b_{xxxx} - b_{xxyy} = B_1, \qquad 2b_{xyxy} = B_2.$$

The exchange energy has not been included in the expression for the total energy, since at our frequencies its contribution is small.[2]

The equations of motion for the elastic displacements and magnetizations are derived from the expression for the energy[12,13]:

$$\rho\ddot{u}_i = \frac{\partial}{\partial x_k}\left[\frac{1+\delta_{ik}}{2}\frac{\partial E}{\partial \varepsilon_{ik}}\right], \qquad \dot{\alpha} = \frac{\gamma}{M_0}[\alpha, \nabla_\alpha E].$$

The linearized equations of motion which are of interest to us are obtained when one retains only terms of first order in the displacement and magnetization variables. The solutions of these equations are sought in the form of plane waves and the eigenfrequencies and dispersion relations are obtained from the vanishing of the determinant of the system of homogeneous equations for the plane-wave amplitudes.[1,2]

### Longitudinal Waves

In investigating the propagation of longitudinal waves the only question of interest is whether these waves do or do not interact with the spin waves. In order to clarify this question, it is sufficient to obtain the equations of motion for the elastic displacements with allowance for the magnetoelastic coupling.

Making use of the above expression for the energy and the usual formulas for transforming tensor components under rotation of the coordinate axes, one can obtain the equations of motion for elastic waves propagating along various directions. Let us consider the ⟨100⟩, ⟨111⟩, and ⟨110⟩ directions for which we have carried out experimental investigations.

The following general form of the equations for longitudinal elastic displacements is obtained:

$$\rho \ddot{u}_x = c_{\text{eff}} \frac{\partial^2 u_x}{\partial x^2} + \frac{B_{\text{eff}}}{M_0} \sin 2\alpha \frac{\partial M_x'}{\partial x}. \qquad (1)$$

Here x is the direction of propagation; $M_x'$ is the component of the magnetization perpendicular to the magnetic field and lying in the plane containing the magnetic field and the direction of propagation; $\alpha$ is the angle between the magnetic field and the direction of propagation; $c_{eff}$ and $B_{eff}$ are the effective elastic and magnetoelastic constants.

For the ⟨100⟩, ⟨111⟩, and ⟨110⟩ directions the constant $c_{eff}$ is $c_{11}$, $\frac{1}{3}(c_{11} + 4c_{44} + 2c_{12})$, and $\frac{1}{2}(c_{11} + 2c_{44} + c_{12})$ respectively.

The effective magnetoelastic constant is $B_1$ for propagation along the ⟨100⟩ direction and $B_2$ for propagation along the ⟨111⟩. It follows from Eq. (1) that in these cases there is no magnetoelastic coupling for a magnetic field parallel and perpendicular to the direction of propagation, and that it differs from zero for all other directions of the field.

The case in which the wave is directed along the [110] is interesting in that $B_{eff}$ depends on the plane in which the magnetic field lies and in that the magnetoelastic contribution to the equation of motion can differ from zero even for a field perpendicular to the direction of propagation. The effective magnetoelastic constant in Eq. (1) is $(B_1 + B_2)/3$ for a field in the $(1\bar{1}0)$ plane and $B_2$ for a field in the (001) plane.

However, if the magnetic field is directed perpendicular to the [110] direction of propagation, then the equation of motion is of the form

$$\rho \ddot{u}_x = c_{\text{eff}} \frac{\partial^2 u_x}{\partial x^2} + \frac{B_1 - B_2}{2M_0} \sin 2\beta \frac{\partial M_y'}{\partial x}, \qquad (2)$$

where $\beta$ is the angle between the field direction and the [001] direction, and $M_y'$ is the magnetization component perpendicular to the magnetic field and lying in the (110) plane.

Thus, for all the three directions considered the longitudinal waves are in general coupled with the spin waves. This coupling will be specially strong at resonance when the frequency of the elastic waves $\omega$ is equal to the frequency of the spin waves[1]:

$$\omega = \gamma [H(H + 4\pi M_0 \sin^2 \alpha)]^{1/2}.$$

### Transverse Waves

The propagation of transverse waves in the presence of magnetoelastic interaction is accompanied not only by resonance absorption but also by effects connected with the difference in the interaction of the various displacement components in the transverse elastic wave with the spin waves.[1]

As for longitudinal waves, let us consider three directions of propagation: ⟨100⟩, ⟨111⟩, and ⟨110⟩.

In the propagation of elastic waves along the ⟨100⟩, when the magnetic field is directed at an arbitrary angle $\alpha$ to the direction of propagation, the equations of motion obtained for the elastic displacements and magnetizations are the same as the equations cited earlier[14] for the case of magnetoelastic isotropy (in applying them to our case one must set in these equations $b = B_2$). We shall not present these equations here but will restrict ourselves to pointing out the most interesting features which follow from them.

For $\alpha = 0°$, i.e., with the magnetic field parallel to the direction of propagation, the solutions of the equations are in the form of plane waves with circular polarization and with opposite directions of rotation, and with one of these waves practically not interacting with the spin waves.[1,2]

For $\alpha = 45°$ the magnetoelastic coupling exists only for the transverse wave polarized perpendicular to the plane containing the magnetic field and the direction of propagation, and for $\alpha = 90°$—only for the transverse wave polarized in that plane.

For propagation along the [111] we shall consider the case of a field parallel to the direction of propagation. Choosing the coordinate axes x, y, and z along the [111], $[1\bar{1}0]$, and $[11\bar{2}]$, we have the following equations of motion:

$$\dot{M}_y = -M_z\omega_0 - \gamma B_{\text{eff}} \frac{\partial u_z}{\partial x},$$

$$\dot{M}_z = M_y\omega_0 + \gamma B_{\text{eff}} \frac{\partial u_y}{\partial x}$$

$$\ddot{u}_y = v^2 \frac{\partial^2 u_y}{\partial x^2} + \frac{B_{\text{eff}}}{\rho M_0} \frac{\partial M_y}{\partial x},$$

$$\ddot{u}_z = v^2 \frac{\partial^2 u_z}{\partial x^2} + \frac{B_{\text{eff}}}{\rho M_0} \frac{\partial M_z}{\partial x}.$$

Here in accordance with the expression for the energy in the coordinate system under consideration $B_{eff} = (2B_1 + B_2)/3$ and $v^2 = (c_{11} + c_{44} - c_{12})/3\rho$. $\omega_0$ denotes the resonance frequency $\omega_0 = \gamma H$.

Writing the solutions of these equations in the form of circularly polarized plane waves, we obtain the following dispersion relations for components with different directions of rotation:

$$(k^+)^2 = \omega^2 \left[ v^2 - \frac{\gamma B_{\text{eff}}}{\rho M_0(\omega_0 - \omega)} \right]^{-1}, \qquad (3)$$

$$(k^-)^2 = \omega^2 \left[ v^2 - \frac{\gamma B_{\text{eff}}^2}{\rho M_0(\omega_0 + \omega)} \right]^{-1}.$$

Thus, as for the ⟨100⟩ direction, for $\alpha = 0°$ one of the circularly polarized components of the elastic wave turns out practically not to be coupled with the spin waves.

For the [110] direction we consider the cases when the magnetic field is parallel and perpendicular to the direction of propagation. We shall direct the x, y, and z axes along the [110], $[\bar{1}10]$, and [001]. In a field parallel to the direction of propagation the equations of motion are of the form

$$\dot{M}_y = -M_z\omega_0 - \gamma B_2 \frac{\partial u_z}{\partial x},$$

$$\dot{M}_z = M_y\omega_0 + \gamma B_1 \frac{\partial u_y}{\partial x},$$

$$\ddot{u}_y = v_1^2 \frac{\partial^2 u_y}{\partial x^2} + \frac{B_1}{\rho M_0} \frac{\partial M_y}{\partial x},$$

$$\ddot{u}_z = v_2^2 \frac{\partial^2 u_z}{\partial x^2} + \frac{B_2}{\rho M_0} \frac{\partial M_z}{\partial x},$$

where

$$v_1^2 = (c_{11} - c_{12})/2\rho, \quad v_2^2 = c_{44}/\rho, \quad \omega_0 = \gamma H_0.$$

This system leads to the following dispersion equation:

$$(\omega^2 - \omega_0^2)(v_1^2k^2 - \omega^2)(v_2^2k^2 - \omega^2) + \frac{\omega_0\gamma B_1^2}{\rho M_0} k^2 (v_2^2k^2 - \omega^2) + \frac{\omega_0\gamma B_2^2}{\rho M_0} k^2 (v_1^2k^2 - \omega^2) - \frac{\gamma^2 (B_1B_2)^2}{\rho^2 M_0^2} k^4 = 0.$$

Hence, setting $v_1 \approx v_2 = v$, we obtain

$$k \approx \frac{\omega}{v}\left\{1+\frac{\gamma\omega_0}{\omega^2-\omega_0^2}\,\frac{(B_1^2+B_2^2)\pm[(B_1^2-B_2^2)^2+(2B_1B_2\omega/\omega_0)^2]^{1/2}}{4\rho M_0 v^2}\right\} \quad (4)$$

In a field perpendicular to the direction of propagation, we have

$$\dot{M}_x = -\gamma H M_{y'},$$

$$\dot{M}_{y'} = \gamma\left[(H+4\pi M_0)M_x + B_1 \sin\beta\,\frac{\partial u_y}{\partial x} + B_2\cos\beta\,\frac{\partial u_z}{\partial x}\right],$$

$$\ddot{u}_y = v_1^2\frac{\partial^2 u_y}{\partial x^2} + \frac{B_1}{\rho M_0}\sin\beta\,\frac{\partial M_x}{\partial x}, \qquad \ddot{u}_z = v_2^2\frac{\partial^2 u_z}{\partial x^2} + \frac{B_2}{\rho M_0}\cos\beta\,\frac{\partial M_x}{\partial x},$$

where $\beta$ is the angle between the magnetic field and the [001] direction, and the $y'$ axis is perpendicular to the magnetic field and to the direction of propagation.

It follows from the equations of motion that for a field along [001]

$$k_1^2 = \frac{\omega^2}{v_1^2}, \quad k_2^2 = \omega^2\left(v_2^2 + \frac{\gamma B_2^2}{\rho M_0}\,\frac{\gamma H}{\omega^2-\omega_0^2}\right)^{-1}, \quad (5)$$

and for a field along [$\bar{1}$10]

$$k_1^2 = \omega^2\left(v_1^2 + \frac{\gamma B_1^2}{\rho M_0}\,\frac{\gamma H}{\omega^2-\omega_0^2}\right)^{-1}, \quad k_2^2 = \frac{\omega^2}{v_2^2}.$$

In formulas (5) and (6) $\omega_0$ is the resonance frequency

$$\omega_0 = \gamma[H(H+4\pi M_0)]^{1/2}.$$

## 2. EXPERIMENTAL METHOD

The measurements were carried out on the setup used in[15]. Elastic waves with frequencies of about 1000 MHz were excited and recorded by means of the piezoelectric effect with X and AC-cut quartz platelets. The fundamental resonance frequency on the plates was 30 and 20 MHz respectively. The elastic waves were excited by pulses 0.3 to 1.0 $\mu$sec long. The microwave power in the pulse amounted to about 0.5 watt. A superheterodyne receiver was used to record the signals.

The yttrium iron garnet samples were in the form of cylinders 3–4 mm in diameter and 5–15 mm long. One of the samples was prepared in the form of a sphere 8 mm in diameter with a small flat section for attaching the piezoelectric transducer plates. In such a practically spherical sample one obtains a sufficiently uniform internal magnetic field.[15] In order to obtain a uniform magnetic field, cylindrical samples were placed inside spheres of polycrystalline yttrium iron garnet. A part of the measurements was carried out using samples in the form of thin platelets 5 mm in diameter and 0.25 mm thick. In order to resolve the elastic pulses use was made of auxiliary "buffer" rods of topaz and corundum crystals.

The piezoelectric transducer plates were attached to the samples by means of an optical contact or by gluing with ceresin.

In order to obtain transverse waves with circular polarization, used the well-known method of quarter-wave plates.[16,17] As such plates we used yttrium aluminum garnet about 0.5 mm thick, cut perpendicular to the ⟨110⟩ direction.

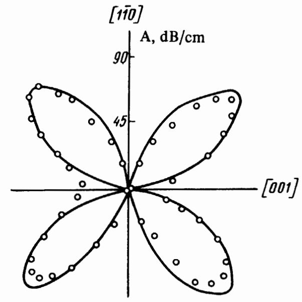


FIG. 1

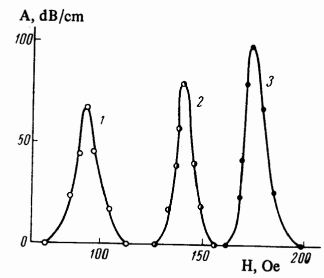


FIG. 2

FIG. 1. Polar diagram of the dependence of resonance absorption of longitudinal elastic waves on the angle $\beta$ between the field and the [001] direction for elastic waves propagating perpendicular to the field along the [110] direction. The frequency was 1470 MHz. The solid lines are drawn under the assumption that the damping is proportional to $\sin^2 2\beta$.

FIG. 2. Resonance absorption lines for longitudinal elastic waves at frequencies of 1110 (curve 1), 1290 (curve 2), and 1470 MHz (curve 3). The conditions are the same as in Fig. 1, $\beta = 45°$.

## 3. EXPERIMENTAL RESULTS AND DISCUSSION

The interaction of elastic and spin waves was investigated for the propagation of elastic waves along the ⟨100⟩, ⟨111⟩, and ⟨110⟩ directions. Investigations of the peculiarities of the interaction as a function of the magnetic field and the angle between the field and the direction of propagation yielded the following results.

Longitudinal waves. In ⟨100⟩ and ⟨111⟩ samples for elastic waves propagating parallel and perpendicular to the field there is no magnetoelastic interaction and the damping of the elastic waves does not depend on the magnetic field. For all other values of the angle between the field and the direction of propagation one observes resonance absorption peaks whose height increases rapidly as the field deviates from the parallel and perpendicular direction. The obtained results are in agreement with formula (1).

For the ⟨110⟩ sample, as the experiment show, the magnetoelastic interaction vanishes for a field parallel to the direction of propagation and, as in the ⟨100⟩ and ⟨111⟩ samples, increases rapidly for a deviation of the field from this direction. However, when the field is perpendicular to the ⟨110⟩ direction of propagation the magnetoelastic interaction differs from zero for definite angles between the field and the crystallographic directions. In Fig. 1 we present a polar diagram of the dependence of the damping of elastic waves at resonance on the angle between the field and the [001] direction. The elastic waves propagate along the [110] perpendicular to the field. In a field parallel to the [001] or [$\bar{1}$10] directions the damping is small and does not depend on the field, i.e., there is no resonance absorption. If, on the other hand, the field is at an angle of 45° to the indicated directions, then the resonance is deepest. The continuous curve in Fig. 1 is plotted under the assumption that the damping at resonance is proportional to $\sin^2 2\beta$, as follows from Eq. (2). It is seen from the Figure that experiment is in good agreement with the theory. We note that we succeeded in measuring such a large damping of the elastic waves by using samples in the form of thin platelets.

The resonance absorption lines of longitudinal waves at three different frequencies are presented in Fig. 2.

Transverse waves. In propagating along the ⟨100⟩ and ⟨111⟩ directions in a field parallel to the direction of propagation, a change in the field leads to oscillations of the amplitude of the elastic pulses when the source and detector of the elastic waves is a usual piezoelectric transducer yielding linearly polarized elastic waves. This phenomenon is explained by the fact that a linearly polarized elastic wave decomposes into two circularly polarized components, one of which interacts with the spin waves. The speed of the interacting component changes with changing field, and as a result of this a definite phase shift is produced between both components; this is equivalent to a rotation of the plane of polarization of the resulting linearly polarized wave. This effect is analogous to the Faraday effect in optics and was observed in yttrium iron garnet for the propagation of elastic waves along the ⟨100⟩ in[6], and under uniform field conditions in[15].

Let us consider the case of propagation along the ⟨111⟩ direction which has not been investigated previously.

In this case, as in the propagation along the ⟨100⟩,[15] we observed periodic variations of the pulse amplitudes when the magnetic field changed; this is connected with the rotation of the plane of polarization whose specific magnitude in rad/cm is $\varphi = (k^+ - k^-)/2$. Using formula (3), we obtain

$$\varphi = \frac{\pi \nu^2 B_{\text{eff}}^2}{\rho M_0 \gamma v^3}\left[H^2 - \left(\frac{\nu}{\gamma}\right)^2\right]^{-1}, \quad B_{\text{eff}} = \frac{2B_1 + B_2}{3}.$$

Here, as in the subsequent formulas, $\gamma = 2.8$ MHz/Oe.

With the aid of this formula we obtained from the experimental magnetic field dependences of the pulse amplitudes the value $B_{\text{eff}} = 4.2 \times 10^6$ erg/cm$^3$.

We thought it interesting to obtain a direct experimental confirmation of the fact that in propagating along the ⟨100⟩ and ⟨111⟩ only one circularly polarized component of the elastic waves with a given sign of the rotation interacts with the spin waves.

In order to produce a circularly polarized elastic wave, we used, as already noted, a plate of aluminum yttrium garnet cut perpendicular to the [110] direction (schematic diagram of Fig. 3). Transverse waves with two directions of polarization [001] and [$\bar{1}$10] were simultaneously excited in this plate. At certain frequencies the plate operates as a quarter-wave plate and transforms the linearly polarized wave produced by the piezoelectric transducer into a circularly polarized wave.

The magnetic field dependence of the pulse amplitude of transverse waves at two frequencies is shown in Fig. 3. At one frequency the resulting wave which enters the yttrium iron garnet sample is linearly polarized (Figs. 3a), at the other—it is circularly polarized. As is seen from the figure, in the first case one observes pulse oscillations (the acoustic Faraday effect), in the second case such oscillations are practically absent and only a decrease in the pulse amplitude takes place at resonance. A reversal of the field direction leads to a change in the direction of precession of the spins. For the linearly polarized wave one observes, as before, pulse oscillations, whereas the pulse amplitude of the circularly polarized wave ceases to depend on the field.

This experiment shows that indeed only the circularly polarized elastic wave with a direction of rotation determined by the direction of spin precession in the magnetic field interacts with the spin waves.

For the ⟨100⟩ sample we also carried out an investigation of the propagation of linearly polarized elastic waves as a function of the angle $\alpha$ between the field and the direction of propagation. The data on the angular dependence of the resonance depth $(A_0 - A_{\text{res}})/A_0$, where $A_0$ and $A_{\text{res}}$ are the amplitudes of the elastic pulses far from resonance and at resonance, presented in Fig. 4, fully confirm the conclusions of the theory on the nature of the interaction of transverse waves polarized perpendicular and parallel

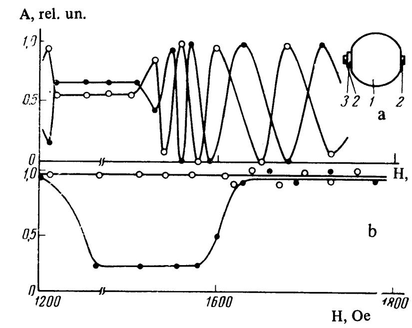


FIG. 3. Dependence of the pulse amplitude of transverse elastic waves propagating along the [100] direction on the magnitude of the magnetic field: a—frequency of 1340 MHz, linearly polarized waves; b—1500 MHz, circularly polarized waves. ○—field parallel to the direction of propagation, ●—field antiparallel to the direction of propagation, 1—sample, 2—piezoelectric transducers, 3—yttrium aluminum garnet plate.

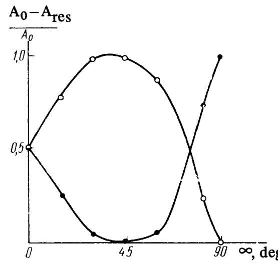


FIG. 4

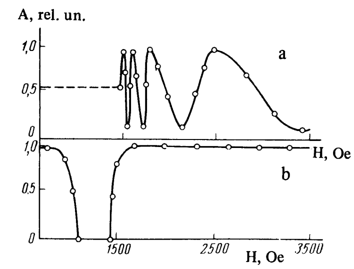


FIG. 5

FIG. 4. Dependence of the resonance depth $(A_0 - A_{\text{res}})/A_0$ where $A_0$ and $A_{\text{res}}$ are the pulse amplitudes far from resonance and at resonance, on the angle between the [100] direction of propagation of the transverse waves and the magnetic field; ●—elastic waves polarized in the plane containing the direction of propagation and the direction of the magnetic field; ○—elastic waves polarized perpendicular to this plane. Frequency of 1100 MHz.

FIG. 5. Dependence of the pulse amplitude of transverse waves propagating along the [110] parallel to the magnetic field on the magnitude of the magnetic field: a—the direction of polarization of the elastic waves makes an angle of 45° with the [001]; b—the direction of polarization is parallel to the [001] direction. Dashes indicate the region of rapid pulse oscillations and the resonance region. Frequency of 1500 MHz.

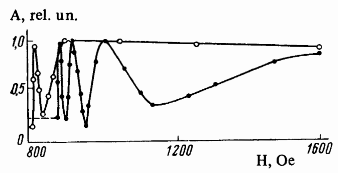


FIG. 6. The same as in Fig. 5a but the magnetic field is perpendicular to the direction of propagation. O—field parallel to the [$\bar{1}$10] direction, ●—field parallel to the [001] direction.

to the plane containing the direction of the magnetic field and the direction of propagation.

For the ⟨110⟩ sample the investigations were carried out in a magnetic field parallel and perpendicular to the direction of propagation.

The experimental results on the magnetic field dependence of the pulse amplitude when the magnetic field is parallel to the [110] direction of propagation are shown in Fig. 5. The oscillations of the pulse amplitudes in the figure are connected with the fact that the simultaneously excited transverse components with the polarization along the [$\bar{1}$10] and [001] directions interact differently with spin waves, and consequently their velocity changes differently as a function of the magnetic field. The phase-shift angle between the components in rad/cm is $\alpha = k_1 - k_2$ which yields with the use of the dispersion relation (4)

$$\alpha = \frac{\pi\nu H}{[H^2-(\nu/\gamma)^2]}\,\frac{\sqrt{(B_1^2-B_2^2)^2+(2B_1B_2\nu/\gamma H)^2}}{\rho M_0 v^3}.$$

From this formula and from the experimental results in Fig. 5 it follows that the best correspondence between theory and experiment is obtained with $B_1 = 3.5$ and $B_2 = 6.5 \times 10^6$ erg/cm$^3$.

When only one of the transverse components is excited the pulse oscillations are, as one would have expected, absent (Fig. 5b).

The dependence of the elastic pulse amplitudes on the field in the case in which the field is perpendicular to the direction of propagation [110] and parallel to the [001] or [$\bar{1}$10] direction is shown in Fig. 6. In these instances oscillations of the pulses are also observed. The corresponding dispersion relations are given by formulas (5) and (6) which lead to the following expression for the phase-shift angle between the components:

$$\alpha = \frac{\pi\nu H}{\rho M_0 v^3} B_{\mathrm{eff}}^2 \left[H(H+4\pi M_0) - \left(\frac{\nu}{\gamma}\right)^2\right]^{-1},$$

where for the field along [001] $B_{\mathrm{eff}} = B_2$, for the field along [$\bar{1}$10] $B_{\mathrm{eff}} = B_1$, and $v_1 \approx v_2 = v$.

Using these formulas and the experimental results of Fig. 6, we find $B_1 = 3.8$ and $B_2 = 6.7 \times 10^6$ erg/cm$^3$.

Thus, the values of the magnetoelastic constants $B_1$ and $B_2$ determined from different effects are in satisfactory agreement with one another, as well as with the values obtained by other methods.[2]

The results of all the investigations carried out in this work show that the predictions of the phenomenological theory of the magnetoelastic interaction are well confirmed by experiments for different directions of propagation of elastic waves with respect to the crystallographic axes and the magnetic field.

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