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ABSORPTION OF LIGHT BY FREE CARRIERS IN A THIN FILM WITH NON-IDEAL SURFACE

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We determine the spectrum of absorption of electromagnetic waves by free carriers in scattering by the surface of a thin semiconductor or semimetal film. The analysis is carried out in the approximation of weak non-specularity, using the method proposed by us earlier[1].

N the present paper we consider the absorption of electromagnetic waves by free carriers in a thin (quantizing) semiconductor or semimetal film. It is assumed that the volume mechanisms of dissipation are negligible, i.e., the electron mean free path is determined only by collisions with the surface of the film. Thus, we investigate the influence of the non-ideal character of the surface on the absorption of high-frequency radiation. The geometric imperfection of the film boundary is taken into account within the framework of the method developed by the authors in[1].

The Green's function of the electron obeys a boundary condition containing the random function u(x, y), where x and y are the coordinates of the point on the surface. To average over the ensemble of the functions u(x, y), a diagram technique is developed. The discussion of the proposed model and the development of the diagram technique are contained in[1].

As will be shown below, absorption under these conditions is weak in the sense that the absorption coefficient is much smaller than the reciprocal film thickness. As to the nondissipative damping of the electromagnetic wave in the collisionless carrier plasma, it likewise does not play an important role. Indeed, we consider a situation in which the concentration of the electrons is such that at a temperature of absolute zero only the first sub-bands of the transverse quantization are populated, i.e., $n \sim (1-10)\, a^{-3}$, where a is the film thickness. The frequency of the radiation will be assumed to be of the order of the characteristic electron energy $\omega \sim (1-10)\hbar/m^* a^2$, where m* is the effective mass. Then the attenuation length of the field is

$$l \sim c/\omega \left(1 - \frac{4\pi n e^2}{\varepsilon m^* \omega^2}\right)^{-1/2} \sim a\sqrt{\frac{\varepsilon m^* a c^2}{4\pi e^2}},$$

where ϵ is the dielectric constant.

Consequently, $l \gg a$, so that even when $m^* \sim 0.1\, m_l$ and a ~ 100 Å the quantity under the square root amounts to $\sim 5 \times 10^4$. Finally, the film thickness is certainly smaller than the wavelength of the radiation of the indicated frequency, owing to the nonrelativistic character of electron motion (dipole approximation). The circumstances listed above make it possible to regard the wave field as practically homogeneous within the volume of the film.

The energy dissipated in the film per unit time is expressed in known fashion in terms of the Green's function of the electron:

$$Q(\omega) = -2\pi e^2 \int \langle \nabla_\alpha G_E(\mathbf{r}, \mathbf{r}')\, \nabla_\beta G_{E+\omega}(\mathbf{r}', \mathbf{r}) \rangle \times \frac{f(E+\omega) - f(E)}{\omega}\, d\mathbf{r}\, d\mathbf{r}'\, dE\, \overline{\varepsilon_\alpha(t)\, \varepsilon_\beta(t)}. \tag{1}$$

Here f(E) is the electron distribution function, and $\epsilon_{\alpha,\beta}(t)$ of the components of the electric vector of the wave. The angle brackets denote averaging over the ensemble of the random functions u(x, y), and the bar denotes averaging over the period of the wave.

Let us consider separately the cases of polarization of the wave along the film and perpendicular to it.

In the case of polarization parallel to the film, the absorption is possible only because the boundary is not ideal. Indeed, in an ideal film the operator of the electromagnetic perturbation is diagonal in the quantum numbers of the transverse motion. Therefore the problem reduces to an interaction of an electromagnetic field with a two-dimensional free electron gas. In the absence of collisions, the absorption is forbidden by the energy and momentum conservation laws. Thus, the intensity of absorption of a wave polarized along the film is directly connected with the degree of non-specularity of the surface.

Let us write (1) in the form ($R_E(p) \equiv \sqrt{2E - p^2}$), where A is the volume of the film, and $\sigma_{\alpha\beta}(\omega)$ is the high-frequency conductivity averaged over the volume. Changing over in the causal Green's functions G_E and $G_{E+\omega}$ to Fourier components with respect to x and y, and assuming the film to be isotropic in its plane, we obtain

$$\sigma_{xx} = \sigma_{yy} \equiv \sigma_{\|},$$

$$\sigma_{\|}(\omega) = -\frac{\pi e^2}{V} \int \langle G_E(\mathbf{p}, \mathbf{p}', z, z')\, G_{E+\omega}(\mathbf{p}', \mathbf{p}, z', z) \rangle (\mathbf{p}\mathbf{p}') \times \frac{f(E+\omega) - f(E)}{\omega}\, \frac{d\mathbf{p}\, d\mathbf{p}'\, dz\, dz'}{(2\pi)^4}\, dE. \tag{2}$$

Here **p** and **p′** are the two-dimensional momenta; we chose a system of units in which ħ = m* = 1. The function $G_E(\mathbf{p}, \mathbf{p}', z, z')$ is determined in the following manner

$$G_E(\mathbf{p}, \mathbf{p}', z, z') = \frac{4\pi^2 \sin R_E(p) z \sin R_E(p') z'}{R_E(p) R_E(p') \cos R_E(p) a \cos R_E(p') a} [\nu_A(\mathbf{p}, \mathbf{p}') - \nu_R(\mathbf{p}, \mathbf{p}')], \tag{3}$$

with ν_A and ν_R satisfying the equation

$$\frac{\operatorname{tg}[R \pm i\delta] a}{R \pm i\delta} \nu_{R,A}(\mathbf{p}, \mathbf{p}') + (2\pi)^{-2} \int u(\mathbf{p} - \mathbf{q}) \nu_{R,A}(q, \mathbf{p}')\, d\mathbf{q} = -\delta(\mathbf{p} - \mathbf{p}'). \tag{4}$$

The upper sign corresponds to ν_R and the lower to ν_A; $\delta \to +0$.

We shall denote by a tilde quantities whose energy variable is $E + \omega$, for example, $f(E) = f$ but $f(E + \omega) = \tilde{f}$. After substituting (3) in (2) and integrating with respect to z and z′, we obtain

$$\sigma_{\|}(\omega) = -\frac{\pi e^2}{V}\int \frac{\tilde{f} - f}{\omega}\langle [\nu_R(\mathbf{p}, \mathbf{p}') - \nu_A(\mathbf{p}, \mathbf{p}')][\tilde{\nu}_R(\mathbf{p}', \mathbf{p}) - \tilde{\nu}_A(\mathbf{p}', \mathbf{p})]\rangle \times (\mathbf{p}\mathbf{p}')\varphi(p)\varphi(p')\,d\mathbf{p}\,d\mathbf{p}'\,dE, \tag{5}$$

where

$$\varphi(p) = \frac{1}{2\omega}\left[\frac{\operatorname{tg}\tilde{R}a}{\tilde{R}} - \frac{\operatorname{tg}Ra}{R}\right], \quad R = R(p).$$

In the weak non-specularity approximation ($W \ll a^4$, where $W(p)$ is the Fourier transform of the correlation function $W(\mathbf{r}_1 - \mathbf{r}_2) \equiv \langle u(\mathbf{r}_1)u(\mathbf{r}_2)\rangle$) the calculation of the average product of two Green's functions reduces to summation of a ladder sequence of diagrams (see[1]). The integral equation for the quantity

$$\Delta(\mathbf{p}, \mathbf{p}') \equiv 4\pi^2 S^{-1}\langle [\nu_R - \nu_A][\tilde{\nu}_R - \tilde{\nu}_A]\rangle,$$

where S is the area of the film, takes the form

$$\Delta(\mathbf{p}, \mathbf{p}') = g(p)\tilde{g}(p')[\delta(\mathbf{p} - \mathbf{p}') + (2\pi)^{-2}\int W(\mathbf{p} - \mathbf{q})\Delta(\mathbf{q}, \mathbf{p}')\,d\mathbf{q}], \tag{6}$$

$$g(p) = \operatorname{Im}\left[\frac{\operatorname{tg}R(p)a}{R(p)} - \Sigma(p)\right]^{-1}, \tag{6a}$$

where $\Sigma(p)$ is the self-energy part. The calculation of $\Sigma(p)$ and the derivation of formulas (6) and (6a) can be found in[1].

Multiplying (6) by $\varphi(p')(\mathbf{p}\mathbf{p}')$ and integrating with respect to $\mathbf{p}'$, we obtain an equation for the auxiliary quantity

$$M(p) \equiv [g\tilde{g}]^{-1}\int \Delta(\mathbf{p}, \mathbf{p}')(\mathbf{p}, \mathbf{p}')\varphi(p')\,d\mathbf{p}',$$

in terms of which $\sigma_{\|}(\omega)$ is expressed:

$$M(p) = p^2\varphi(p) + (2\pi)^{-2}\int \frac{\mathbf{p}\mathbf{q}}{q^2} g(q)\tilde{g}(q)W(\mathbf{p} - \mathbf{q})M(q)\,d\mathbf{q}, \tag{7}$$

$$\sigma_{\|}(\omega) = -\frac{e^2}{4\pi a\omega}\int (\tilde{f} - f)\varphi(p)g(p)\tilde{g}(p)M(p)\,d\mathbf{p}\,dE. \tag{8}$$

In this approximation, the quantity $R\Sigma$ is much smaller than unity, and therefore the functions $g(p)$ and $\tilde{g}(p)$ have sharp maxima when $R(p)a/\pi$ and $\overline{R}(p)a/\pi$ respectively are close to integers. The maxima of g and $\tilde{g}$ coincide, besides in the trivial case $\omega = 0$, at frequencies corresponding to "vertical" resonant transitions, i.e., transitions between the sub-bands at an invariant longitudinal momentum. These frequencies, as seen from (6a), are determined by the system of equations:

$$\operatorname{tg}\sqrt{2E - p^2}\,a = \sqrt{2E - p^2}\operatorname{Re}\Sigma_E(p),$$
$$\operatorname{tg}\sqrt{2E + 2\omega - p^2}\,a = \sqrt{2E + 2\omega - p^2}\operatorname{Re}\Sigma_{E+\omega}(p).$$

Since $R(p)\Sigma \ll 1$, the resonant frequencies are close to the values corresponding to a film with ideal boundary: $\omega^0_{mn} = (\pi^2/2a^2)(m^2 - n^2)$, where m and n are integers. We shall denote the resonant frequencies of a non-ideal film by ω_{mn}. At $\omega = \omega_{mn}$, only one pair of maxima coincides, i.e., the n-th maximum of g(p) coincides with the m-th maximum of g(p).

We confine ourselves henceforth to the case of absolute zero temperature and to an electron density smaller than $3\pi/2a^3$, so that only the first sub-band is populated. The factor $\tilde{f} - f$ in (8) limits the region of integration with respect to E from above to the value $E_{max} = \mu$, where μ is the Fermi level: $\mu = \pi^2/2a^2 + \pi na$ (see[1]), and from below to the value $E_{min} = \mu - \omega$, under the condition that $\omega < \mu$. On the other hand, if $\omega > \mu$, the integral with respect to E is cut off at $E = 0$, owing to the function g(p), since the latter is proportional to the density of states and vanishes identically at $E < 0$. Thus, in the situation in question, at any value of E from the integration interval, the function g(p) has only one maximum at the point $p = p_1 \approx \sqrt{2E - \pi^2/a^2}[1 + O(R\Sigma)]$, at which $\tan R(p_1)a = R(p_1)\operatorname{Re}\Sigma_E(p_1)$. Outside a small vicinity of resonant frequencies ω_{m_1}, which will be estimated below, the integral in the right side of (7) can be omitted. Indeed, writing $\Sigma = \Sigma' + i\Sigma''$, we obtain from (6a)

$$g(p) = \frac{R^2(p)\Sigma''(p)}{(\operatorname{tg}Ra - R\Sigma')^2 + (R\Sigma'')^2}, \tag{9}$$

where Σ'' is of the order of W/a^3 (see[1]).

Let us estimate the contributions of different regions of integration with respect to q in Eq. (7). The intervals between the maxima of g(q) and $\tilde{g}(q)$ make a contribution proportional to W^3; the region of the maximum of g or $\tilde{g}$ at the nonresonant frequency has a q dimension proportional to $R\Sigma''$, and its contribution is of the order of $(W/a^4)^2$. Finally, for frequencies close to resonance, $|\omega - \omega_{m_1}| \sim (\pi^2/a^2)R\Sigma'' \sim (\pi^2/a^2)\cdot(W/a^4)$, in the interval (7) only the region of the double maximum at $q = p_1 \approx \sqrt{2E - \pi^2/a^2}$ is significant. In this region $g\tilde{g} \sim (\Sigma'')^{-2}$ and the integral makes an appreciable contribution. Thus, outside a narrow vicinity of the resonant frequencies, we can put $M(p) = p^2\varphi(p)$, and we obtain for $\sigma_{\|}$ the expression

$$\sigma_{\|}(\omega) = -\frac{e^2}{4\pi a\omega}\int_\eta^\mu p^2\varphi^2(p)g(p)\tilde{g}(p)\,d\mathbf{p}\,dE. \tag{10}$$

The lower limit of the integral with respect to E in formula (10) is in any case larger than zero. In the approximation $R\Sigma \ll 1$ employed by us, the imaginary part of $\Sigma_E(p)$ vanishes when $E < \pi^2/2a^2$ (see[1]). Therefore $\eta = \max(\mu - \omega, \pi^2/2a^2)$. It is easy to see that only the maxima of g and $\tilde{g}$ are important for the integration in (10), since the intervals between them make a contribution on the order of $(R\Sigma'')^2$. Taking this into account and using expression (9) as well as the formula for $\Sigma_E(p)$ from[1], we obtain

$$\Sigma_E''(p) = -\sum_{n=1}^{N(E)}\frac{(\pi n)^2}{2q^3}\overline{W}(p, p_n),$$

$$\sigma_{\|}(\omega) = \frac{\pi^5 e^2}{16\omega^3 a^7}\int_\eta^\mu \sum_{n=1}^{\tilde{N}} n^2\overline{W}(p_1, \tilde{p}_n)(p_1^2 + \tilde{p}_n^2)\,dE. \tag{11}$$

Here $p_n = \sqrt{2E - (\pi n/a)^2}$, $\tilde{p}_n = \sqrt{2(E + \omega) - (\pi n/a)^2}$, N(E) is the integer part of the number $\sqrt{2Ea^2}/\pi$, and $\overline{W}$ is the result of averaging over the angles:

$$\overline{W}(p, q) = (2\pi)^{-1}\int_0^{2\pi} W(\sqrt{p^2 + q^2 - 2pq\cos\varphi})\,d\varphi.$$

We have assumed that the correlation function W(p) depends only on the modulus of p. It is seen from (11) that the absorption vanishes for an ideal film when $W \to 0$.

To find $\sigma_{\|}(\omega)$ near ω_{m_1} it is necessary to solve Eq.

(7). In order not to complicate the calculations, we shall assume that $\omega = \omega_{m1}$ (exact resonance). Taking the foregoing estimates into account, we get

$$M(p) = p^2\varphi(p) + \frac{p(m\pi)^2}{2p_1a^3}\frac{\widetilde{W}(p, p_1)M(p_1)}{|\Sigma''(p_1)| + |\widetilde{\Sigma}''(p_1)|}, \tag{12}$$

i.e., M(p) is expressed in terms of its value at the point p_1. Putting $p = p_1$, we get $M(p_1)$ and obtain ultimately

$$M(p) = p^2\varphi(p) + \frac{pp_1(m\pi)^2\varphi(p_1)\widetilde{W}(p, p_1)}{2a^3(|\Sigma''(p_1)| + |\widetilde{\Sigma}''(p_1)|) - (\pi m)^2\widetilde{W}(p_1, p_1)}. \tag{13}$$

In (12) and (13) we have for $\widetilde{W}(p, p_1)$

$$\widetilde{W}(p, p_1) = (2\pi)^{-1}\int_0^{2\pi} W(\sqrt{p^2 + p_1^2 - 2pp_1\cos\varphi})\cos\varphi\, d\varphi.$$

In calculating $\sigma_{\|}(\omega)$, the principal role is again played by the vicinities of the maxima of g(p) and $\tilde{g}(p)$. The only maximum of g(p) at $p = p_1$ is a double maximum, and its contribution should be calculated by using M(p) from (13). The remaining maxima of $\tilde{g}(p)$ at the points $p = p_n$ make the same contribution as in the nonresonant region at frequencies, since $\varphi(\tilde{p}_n) \gg \varphi(p_1) = (^1\!/_2\omega_{m1})(\widetilde{\Sigma}'(p_1) - \Sigma'(p_1))$, and we can assume that $M(\tilde{p}_n) = \tilde{p}_n^2\varphi(\tilde{p}_n)$. The result is (for $m \neq 1$, i.e., if the resonant frequency is not equal to zero):

$$\sigma_{\|}(\omega = \omega_{m1}) = \frac{\pi^3e^2m^2}{4a\omega_{m1}^3}\int_\eta^\mu \frac{(2E - \pi^2/a^2)[\widetilde{\Sigma}'(p_1) - \Sigma^1(p_1)]^2\,dE}{2a^3[|\Sigma''(p_1)| + |\widetilde{\Sigma}''(p_1)|] - (m\pi)^2\widetilde{W}(p_1, p_1)}$$
$$+ \frac{\pi^5e^2}{16a^7\omega_{m1}^3}\int_\eta^\mu \mathop{S}_{n=1}^{\widetilde{N}} n^2\overline{W}(p_1, \tilde{p}_n)\tilde{p}_n^2\,dE. \tag{14}$$

We note that the quantities Σ' and Σ'', being both small in the sense of $\Sigma' \ll a$ and $\Sigma'' \ll a$, can in general have different orders of magnitude. For example, if the correlation function is

$\langle u(\mathbf{r}_1)u(\mathbf{r}_2)\rangle = H^2\exp[-\mathbf{r}_1 - \mathbf{r}_2)^2/4L^2]$, then $W(p) = \pi H^2L^2\exp(-p^2L^2)$.

The parameters H and L have the meaning of the characteristic height and width of the roughnesses of the surface. In the case $L \ll a$, we obtain in the region $E \sim a^{-2}$ and $pa \sim 1$

$$\Sigma_E''(p) = \frac{\pi^3}{12a^3}[2N(E) + 1][N(E) + 1]N(E)H^2L^2; \quad \Sigma_E'(p) = \sqrt{\pi}H^2L^{-1},$$

i.e., $\Sigma'/\Sigma'' \sim a^3/L^3 \gg 1$. It is still possible to satisfy here the condition $H^2L^{-1} \ll a$, i.e., $|\Sigma| \ll a$.

Thus, the absorption of a wave polarized along the film is proportional to the degree of nonspecularity of the surface also in the case of resonance. The ratio of the energy observed in the film to the incident energy is of the order of $\sigma_{\|}a/c \sim \alpha W/a^4$, where α is the fine-structure constant.

Let us proceed now to the case of a wave polarized perpendicular to the film. For the component σ_{zz}, which we shall denote by $\sigma_\perp$, we obtain the following expression:

$$\sigma_\perp = -\frac{e^2}{4\pi a\omega}\int(\tilde{f} - f)\psi(p)\psi_1(p)\Delta_1(\mathbf{p}, \mathbf{p}')\,d\mathbf{p}\,d\mathbf{p}'\,dE,$$
$$\psi(p) \equiv \frac{1}{2\omega}\left(1 + \frac{R}{\widetilde{R}}\operatorname{tg}Ra\operatorname{tg}\widetilde{R}a - \sec Ra\sec\widetilde{R}a\right), \tag{15}$$
$$\psi_1(p) \equiv \frac{1}{2\omega}\left(1 + \frac{\widetilde{R}}{R}\operatorname{tg}Ra\operatorname{tg}\widetilde{R}a - \sec Ra\sec\widetilde{R}a\right),$$

where $\Delta_1(\mathbf{p}, \mathbf{p}')$ satisfies the equation

$$\Delta_1(\mathbf{p}, \mathbf{p}') = g(p)\tilde{g}(p)\left[\delta(\mathbf{p} - \mathbf{p}') + (2\pi)^{-2}\int W(\mathbf{p} - \mathbf{q})\Delta_1(\mathbf{q}, \mathbf{p}')\,d\mathbf{q}\right]. \tag{16}$$

Performing calculations similar to those made for $\sigma_{\|}$, we find that $\sigma_\perp(\omega)$ is determined by the system of equations

$$\sigma_\perp(\omega) = \frac{-e^2}{4\pi a\omega}\int_\eta^\mu \psi(p)L(p)g(p)\tilde{g}(p)\,d\mathbf{p}\,dE, \tag{17}$$

$$L(p) = \psi_1(p) + (2\pi)^{-1}\int\overline{W}(p, q)L(q)g(q)\tilde{g}(q)q\,dq. \tag{18}$$

We note that in the considered case of perpendicular polarization there is no factor (pq) under the integral sign (compare with (7)). Therefore the result contains only the quantity $\overline{W}$ and not $\widetilde{W}$, i.e., it contains the "nontransport" scattering cross section. The solution of Eq. (18) is analogous to the derivation of formula (13), and the result is of the form

$$L(p) = \psi_1(p) \text{ for } |\omega - \omega_{m1}|/\omega_{m1} \gg W/a^4;$$
$$L(p) = \psi_1(p) + \frac{(m\pi)^2\psi_1(p_1)\overline{W}(p, p_1)}{2a^3[|\Sigma''(p_1)| + |\widetilde{\Sigma}''(p_1)|] - (m\pi)^2\overline{W}(p_1, p_1)} \tag{19}$$

near the m-th resonance. The second term in (19) is significant at all values of m if $p \to p_1$, for in this case both terms in (19) are of the same order, and also at arbitrary p, if m is even.

The latter circumstance is connected with the selection rules for optical transitions in an ideal film. In the dipole approximation, transitions are possible between the sub-bands whose numbers have different parity. In our case (only the first sub-band is populated) the allowed transitions correspond to even m and in this case $\psi_1(p_1) = \omega^{-1}[1 + O(\Sigma^2/a^2)]$. Near the resonant frequencies corresponding to the forbidden transitions, i.e., to odd m, we can assume $L(p) = \psi_1(p)$, just as far from resonance, for in this case

$$\psi_1(p_1) = \pi^2(2\omega a^2)^{-1}[m^2\Sigma'\widetilde{\Sigma}' - {}^1\!/_2(\Sigma')^2 - {}^1\!/_2m^2(\widetilde{\Sigma}')^2].$$

In the regions between the resonances we obtain

$$\sigma_1(\omega) = \frac{\pi^5e^2}{4\omega^3a^7}\int_\eta^\mu \mathop{S}_{n=1}^{\widetilde{N}} n^2\overline{W}(p_1, \tilde{p}_n)\cdot\{\operatorname{ctg}^2{}^1\!/_2\sqrt{\pi^2 + 2\omega a^2}\,(2\omega + \pi^2/a^2)$$
$$+ a^{-2}[(n\pi)^2 - 2\omega a^2]\operatorname{cosec}^2\sqrt{n^2\pi^2 - 2\omega a^2}\cdot[\cos\sqrt{n^2\pi^2 - 2\omega a^2} - (-1)^n]^2\}\,dE. \tag{20}$$

The second term in the square brackets of (20) has no singularities at the points $2\omega a^2 = (n\pi)^2$.

Near the resonances corresponding to allowed transitions, the main contribution to (17) is made by the vicinity of the double maximum at $p = p_1$. In this case the absorption turns out to be large ($\sigma_\perp \sim (e^2/a)(a^4/W)$):

$$\sigma_\perp(\omega = \omega_{m1}) = \frac{\pi^3m^2e^2}{a\omega^3}\int_\eta^\mu[2a^3(|\Sigma''(p_1)| + |\widetilde{\Sigma}''(p_1)|) - (m\pi)^2\overline{W}(p_1, p_1)]^{-1}\,dE. \tag{21}$$

Finally, for forbidden resonant transitions the contribution from the maximum to the point p_1 is negligibly small, since $\psi(p_1)\psi(p_1) \sim \Sigma^4/\omega^2a^4$. Therefore the result coincides with (20) when $\omega \to \omega_{m1}$, where m is odd:

$$\sigma_\perp(\omega_{m1}) = \frac{\pi^5e^2}{4\omega_{m1}^3a^7}\int_\eta^\mu \mathop{S}_{n=1}^{\widetilde{N}} n^2\overline{W}(p_1, \tilde{p}_n)\cdot$$
$$\times\frac{[(n\pi/a)^2 - 2\omega_{m1}][\cos\sqrt{n^2\pi^2 - 2\omega_{m1}a^2} - (-1)^n]^2}{\sin^2\sqrt{(n\pi)^2 - 2\omega_{m1}a^2}}\,dE. \tag{22}$$

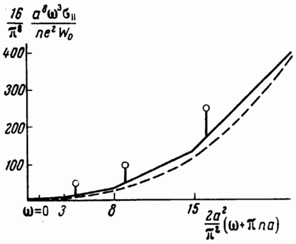

FIG. 1. The function $\omega^2\sigma_{\parallel}(\omega)$. The light circles denote the maxima of the solid curve $\omega^3\sigma_{\parallel}(\omega)$ at the resonance points $\omega = \omega_{ml}$. In the employed scale, the finite widths of the maxima and the kink at $\omega = \pi na$ are hardly noticeable. The dashed curve represents the function $^2/_{15}[2a^2\pi^{-2}(\omega + \pi na)]^{5/2}$.

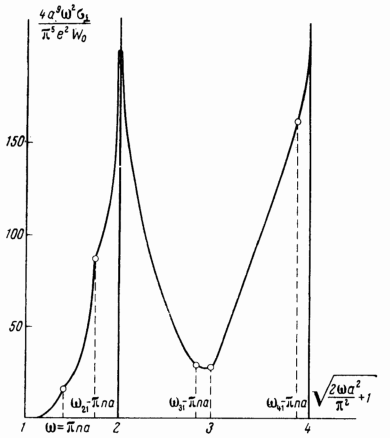

FIG. 2. The function $\sigma_{\perp}(\omega)$. The circles denote the points at which the curve has breaks. The maxima at the points 2 and 4 are shown in a reduced scale. The thick vertical lines represent the δ-like absorption spectrum of an ideal film.

Thus, if the electromagnetic-wave polarization is perpendicular to the film the scattering of the electrons by the surface leads to a "smearing" of the δ-like absorption spectrum of the ideal film.

Let us illustrate the character of the functions $\sigma_{\parallel}(\omega)$ and $\sigma_{\perp}(\omega)$, using as an example a correlator of the simplest type, $W(p) = W_0 = \text{const}$ ("white noise"). The ω scale is broken up into regions with different frequency dependences of the absorption coefficient. We assume that the first sub-band of transverse quantization is less than half filled, i.e., $\pi na < 3\pi^2/4a^2$, where n is the electron density.

In the region $\omega < \pi na$ we obtain $\sigma_{\parallel}(\omega) = \pi^6e^2W_0n/4a^6\omega^2$. This formula corresponds to the well known limiting law for absorption by free carriers at low frequencies. The proportionalities of $\sigma_{\parallel}$ to ω^{-2} in the region of small ω is retained regardless of the correlator employed. If ω becomes smaller than π^2W_0/a^6, it is necessary to take into account the overlap of the maxima of g and $\mathfrak{g}$; then $\sigma_{\parallel}(\omega)$ tends to the value of the static conductivity as $\omega \to 0$ (see[1]).

When $\omega > \pi na$ we get

$$\sigma_{\parallel}(\omega) = \frac{\pi^6e^2W_0n}{8a^6\omega^3}\sum_{m=1}^{\infty} m^2(\omega + \pi na - \omega_{m1})\,\theta(\omega + \pi na - \omega_{m1}),$$

where $\theta(x) = 1$ for $x > 0$ and $\theta(x) = 0$ for $x < 0$. Thus, the function $\omega^3\sigma_{\parallel}(\omega)$ is a broken straight line with breaks at the points $\omega = \omega_{m1} - \pi na$, i.e., at quantum energies such that the electrons are cast from the Fermi level to the bottom of the m-th sub-band. "In the mean," the $\sigma_{\parallel}(\omega)$ curve is close to the plot of $\omega^{-1/2}$. Besides the breaks at $\omega = \omega_{m1} - \pi na$, the function $\omega^3\sigma_{\parallel}(\omega)$ has narrow maxima in the vicinity of the resonant frequencies ω_{m1}. The width of the maxima is of the order of π^2W_0/a^6, and the amplitude of the m-th maximum exceeds the ordinate of the smooth $\sigma_{\parallel}(\omega)$ curve at the point ω_{m1} by approximately $\ln^2[2\mu a^2/\pi^2(m^2 - 1)]$ times. Figure 1 shows the function $\omega^3\sigma_{\parallel}(\omega)$.

More complicated expressions are obtained for $\sigma_{\perp}(\omega)$. We present here formulas pertaining to the first few sections of the spectrum:

$$\omega < \pi na:\quad \sigma_{\perp} = \sigma_0[C(\omega) + C(-\omega)],$$

$$C(\omega) = (\pi^2 + 2\omega a^2)\operatorname{ctg}^2\frac{1}{2}\sqrt{\pi^2 + 2\omega a^2},\qquad \sigma_0 \equiv \frac{\pi^5e^2W_0}{4a^9\omega^2};$$

$$\pi na < \omega < \omega_{21} - \pi na:\quad \sigma_{\perp} = \sigma_0[C(\omega) + C(-\omega)]\frac{\pi na}{\omega};$$

$$\omega_{21} - \pi na < \omega < \omega_{21}:\quad \sigma_{\perp} = \sigma_0\Big[(C(\omega) + C(-\omega))\frac{\pi na}{\omega} + \frac{4}{\omega}(\omega + \pi na - \omega_{21})(C(\omega) + T(-\omega))\Big],$$

$$T(\omega) = (4\pi^2 + 2\omega a^2)\operatorname{tg}^2\frac{1}{2}\sqrt{4\pi^2 + 2\omega a^2};$$

$$\omega_{21} < \omega < \omega_{31} - \pi na:\quad \sigma_{\perp} = \sigma_0\Big[(C(\omega) + C(-\omega))\frac{\pi na}{\omega} + \frac{4\pi na}{\omega}(C(\omega) + T(-\omega))\Big].$$

These formulas are valid outside the vicinity $\Delta\omega \sim \pi^2W_0/a^6$ of the point ω_{21} (allowed resonance). Inside the indicated vicinity; the absorption is determined by formula (21): $\sigma_{\perp}(\omega_{21}) = 2\pi^2e^2n/W_0\omega_{21}^3$, i.e., there is a sharp maximum. Similar maxima occur for all the remaining resonant frequencies corresponding to allowed transitions (see Fig. 2).

Of course, all the sharp kinks of the curves $\sigma_{\parallel}(\omega)$ and $\sigma_{\perp}(\omega)$ become smoothed out in the higher approximations of the parameter RΣ. The region in which the smoothing takes place is small compared with the distance between the breaks. The relative order of this smallness is W_0/a^4.

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[1] A. V. Chaplik and M. V. Éntin, Zh. Eksp. Teor. Fiz. 55, 908 (1968) [Sov. Phys.-JETP 28, 514 (1969)].

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