MODIFICATION OF THE GLAUBER FORMULA FOR VIOLATION OF THE POMERANCHUK THEOREM

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It is pointed out that the usual form of the Glauber formula for total deuteron cross sections breaks down if the Pomeranchuk theorem is violated, and a simple modification of it is derived for this case. Assuming the validity of the Okun-Pomeranchuk theorem, under general conditions the Glauber correction reduces asymptotically to $\delta\sigma = 2\sigma_{el}$, where σ_{el} is the total elastic cross section for scattering by nucleons.

THE total cross section for scattering of a given particle by neutrons is usually extracted from the proton and deuteron scattering data by means of the Glauber formula[1]

$$\sigma_d = \sigma_p + \sigma_n + \delta\sigma, \tag{1}$$

$$\delta\sigma = -\frac{\langle r^{-2}\rangle}{4\pi}\sigma_p\sigma_n(1-\alpha_p\alpha_n), \tag{2}$$

where a_N is the ratio of the real to imaginary part of the forward scattering amplitude f_N ($N = n, p$) and $\langle r^{-2}\rangle$ is the mean inverse square radius of the deuteron. The additional terms which can appear in eq. (2) to take charge-exchange processes into account[2] do not affect the considerations of this paper and are therefore omitted for simplicity.

There have been recent speculations concerning a possible violation of the Pomeranchuk theorem[3] for certain processes[4-6], although the evidence for the violation has also been questioned[7,8]. Such a violation would imply an asymptotic behavior[3] $a_N = C_N \ln k$ for some constant C_N, where k is the c.m. momentum. Under this condition eq. (2) would lead to an unphysical logarithmic growth of $\delta\sigma$. Let us consider why this formula breaks down in this case.

Equation (2) is derived from the more general Glauber formula[1]

$$\delta\sigma = \frac{2}{k^2}\,\mathrm{Re}\int S(q)f_n(q)f_p(q)\,d^2q, \tag{3}$$

where $S(q)$ is the deuteron form factor for momentum transfer q, by assuming that $f_N(q)$ is slowly varying within the peak of $S(q)$ at $q = 0$. However, a logarithmic growth of a_N implies a shrinkage of the diffraction peak in $d\sigma/dt$[5], since $\sigma_{el} = \int(d\sigma/dt)dt \le \sigma$. Such a shrinkage would invalidate the derivation of eq. (2) at sufficiently high energies. In this case, on the other hand, as $k \to \infty$ eq. (3) reduces instead to

$$\delta\sigma = \frac{2}{k^2}\int \mathrm{Re}\,f_n(q)\,\mathrm{Re}\,f_p(q)\,d^2q. \tag{4}$$

using $S(0) = 1$ and assuming no variation of the phase of f_N within the diffraction peak.

For definiteness, let us assume that the diffraction peak remains exponential in form even at asymptotic energies, so that $d\sigma_N/dt = g_N(k)\exp[-a_N(k)t]$. From the above considerations it follows that asymptotically

$$g_N(k) = \frac{C_N^2\sigma_N^2}{16\pi}\ln^2 k, \qquad a_N(k) = \frac{\gamma_N C_N^2\sigma_N}{16\pi}\ln^2 k, \tag{5}$$

where $\gamma_N^{-1} = \sigma_{el}/\sigma$. Substituting eqs. (5) into (4), we obtain

$$\delta\sigma = \frac{4C_pC_n\sigma_p\sigma_n}{\gamma_pC_p^2\sigma_p + \gamma_nC_n^2\sigma_n}[1 + O(k^{-2})]. \tag{6}$$

If we assume the Okun-Pomeranchuk theorem[9] concerning the asymptotic equality $f_p = f_n$, which is already well satisfied at present accelerator energies, then eq. (6) simplifies asymptotically to

$$\delta\sigma = 2\sigma/\gamma = 2\sigma_{el}. \tag{7}$$

We note that in (7) $\delta\sigma > 0$, in contrast with the usual result $\delta\sigma < 0$. This difference is due to the fact that in eq. (3) each amplitude f_N is mainly real in our case, but mainly imaginary in the usual case. Under the more restrictive assumption of the dominance of f_N by the exchange of the Pomeranchuk trajectory, and hence of the validity of the Pomeranchuk theorem, the asymptotic behavior $\delta\sigma \to 0^-$ was established in[10] independently of $\gamma(k)$. In our case, on the other hand, the asymptotic limit of $\delta\sigma$ is positive, since $\lim \sigma_{el} \ne 0$[11,12].

If the Pomeranchuk theorem is violated, the logarithmic growth of a_N, and consequently the applicability of eq. (6), becomes effective only at energies very much higher than those of existing accelerators[6]. Thus, although one would expect to observe a practically constant value of σ_d in the large energy region in which σ_N is already constant and eq. (2) is valid, σ_d would attain its asymptotic value only at much higher energies.

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