

FLUCTUATION RESISTANCE OF A TUNNEL JUNCTION AT TEMPERATURE ABOVE CRITICAL

I. O. KULIK

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted April 8, 1970

Zh. Eksp. Teor. Fiz. 59, 937-944 (September, 1970)

We analyze the effect of the fluctuating conductivity in superconducting tunnel junctions at temperatures above T_c . We show that the fluctuation component of the tunnel current reveals quantum interference effects in a high-frequency field, namely: 1) generation of radiation at the Josephson frequency with a spectral width determined by the reciprocal relaxation time of the Cooper pairs Γ ; 2) appearance of singularities on the current-voltage characteristic when the junction is exposed to high-frequency power radiation; the position of the singularity is determined by the formula $V_n = n\hbar\Omega/2e$ (Ω —irradiation frequency, $\Omega \gg \Gamma$); 3) an oscillating dependence of the slope of the current-voltage characteristic at zero on the amplitude of the high-frequency signal.

In an earlier paper^[1], the author pointed out the existence of an oscillating current of fluctuation origin in the junction between two superconductors at a temperature exceeding the critical temperature of the superconducting transition T_c . The present paper contains a more detailed theory of this phenomenon and an analysis of the form of the current-voltage characteristic of the junction in a circuit with a given current. In the latter case, a change takes place in the effective resistance as a result of the "fluctuation pairing," in analogy with the Aslamazov-Larkin effect (fluctuation conductivity of films at temperatures $T > T_c$)^[2].

The behavior of a Josephson tunnel junction differs in a circuit with a given current from that with a given voltage. According to the well known relations, the Josephson current is equal to

$$J = J_0 \sin(\alpha_1 - \alpha_2 + \varphi(t)), \quad \varphi = \frac{2e}{\hbar} \int_0^t V(t) dt, \quad (1)$$

where $V(t)$ is the voltage across the barrier, $\alpha = \alpha_1 - \alpha_2$ is the relative phase shift of the ordering parameters of the two superconductors.

In the case of a given current J , the coherent phase α is determined by the current in accordance with (1), and the potential difference across the barrier is $V = 0$ (if $J < J_0$). At a fixed voltage, to the contrary, $\varphi = 2eVt/\hbar$, and the thermodynamic averaging over the phase α gives $\langle J(t) \rangle = 0$ at any instant of time. However, the current correlator $\langle J(t)J(t') \rangle$ no longer vanishes after averaging with respect to α :

$$\langle J(t)J(t') \rangle = J_0^2 \langle \sin(\alpha + \omega_0 t) \sin(\alpha + \omega_0 t') \rangle = \frac{1}{2} J_0^2 \cos \omega_0(t - t'). \quad (2)$$

Its value oscillates as a function of the time with frequency $\omega_0 = 2eV/\hbar$, and this is a reflection of the coherence of the phase in the Josephson junction at non-zero voltage V across the barrier. Thus, setting the current I or the junction voltage V corresponds to different physical situations. A similar situation obtains also at temperatures exceeding T_c .

As shown in^[1], the fluctuation ("Josephson") component of the tunnel junction near T_c is given by ($eV \ll T_c$)

$$J = A \text{Im} [\psi_1(t) \psi_2^*(t) e^{i\varphi}], \quad \frac{\partial \varphi}{\partial t} = \frac{2e}{\hbar} V, \quad (3)$$

where $\psi_i(t)$ are the ordering parameters of the superconductors 1 and 2 at $T > T_c$, and A is a constant equal to

$$A = \frac{2\pi^3}{7\zeta(3)} \frac{T_c}{NeR} \quad (4)$$

(N is the electron density and R the resistance of the junction in the normal state).

Formula (3) pertains to the case of a "point" junction, whose characteristic dimensions are small compared with the coherence length $\xi(T) \sim v_0/\sqrt{(T - T_c)T_c}$. For a junction of arbitrary geometry it is necessary to take into account the spatial dependence of ψ . In this the current is obtained by integrating (3) over the cross section of the contact (S is the cross area):

$$J = A \int \frac{d^2x}{S} \text{Im} [\psi_1(x, t) \psi_2^*(x, t) e^{i\varphi}]. \quad (5)$$

The constant A is determined as before by expression (4), in which R has the meaning of the total resistance of the normal state. We note that the phase φ in (5) is independent of the coordinates, since the barrier voltage $V = \hbar\varphi/2e$ cannot vary over distances on the order of $\xi \sim 10^{-4}$ cm.

The equations satisfied by the ordering parameters of the superconductors $\psi_i(x, t)$ are the Langevin equations^[3]

$$\frac{\partial \psi_i}{\partial t} + \Gamma(1 - \xi^2 \nabla^2) \psi_i = S_i(x, t), \quad (6)$$

where Γ is the reciprocal relaxation time of the Cooper pair^[4], and the random "forces" $S_i(x, t)$ satisfy the correlation properties:

$$\langle S_i \rangle = 0, \quad \langle S_i S_j \rangle = 0, \quad (7)$$

$$\langle S_i(x, t) S_j^*(x', t') \rangle = \frac{4mT\Gamma\xi^2}{\hbar^2 d_i} \delta^2(x - x') \delta(t - t') \delta_{ij}$$

The thicknesses of the films forming the tunnel junction (Fig. 1) will be denoted d_i , and will be assumed to be small compared with the coherence length $\xi(T)$: $d_{1,2} \ll \xi$. This makes it possible to disregard the dependence of ψ_i on the coordinate normal to the junction plane.

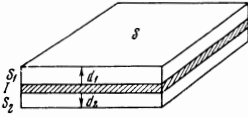


FIG. 1. Diagram of tunnel junction.

1. We consider first the case when the voltage V across the barrier is specified. We have in the k -representation

$$J(t) = A \sum_{\mathbf{k}} \text{Im}[\psi_{\mathbf{k}}^{\dagger}(t) \psi_{\mathbf{k}}^{*2}(t) e^{i\omega_0 t}], \quad \omega_0 = \frac{2eV}{\hbar}.$$

The quantities $\psi_{\mathbf{k}}^{\dagger}(t)$ are obtained by directly integrating (6):

$$\psi_{\mathbf{k}}^{\dagger}(t) = e^{-\Gamma_{\mathbf{k}} t} \int_{-\infty}^t e^{\Gamma_{\mathbf{k}} \tau} S_{\mathbf{k}}^{\dagger}(\tau) d\tau, \quad (8)$$

where $\Gamma_{\mathbf{k}} = \Gamma(1 + k^2 \xi^2)$, and the $S_{\mathbf{k}}$ satisfied by virtue of (7) the relations

$$\langle S_{\mathbf{k}}^{\dagger}(t) S_{\mathbf{k}'}^{*2}(t') \rangle = \frac{4mT\xi^2\Gamma}{\hbar^2 S d_1} \delta_{\mathbf{k}\mathbf{k}'} \delta(t-t') \delta_{ij}. \quad (9)$$

It is seen from (8) that $\langle \psi_{\mathbf{k}}^{\dagger}(t) \rangle = \langle \psi_{\mathbf{k}}^2(t) \rangle = 0$, from which it follows that the mean value of the Josephson current $J(t)$ also vanishes: $\langle J(t) \rangle = 0$. The current correlator is equal to

$$\langle J(t)J(t') \rangle = \frac{1}{2} A^2 \text{Re} \sum_{\mathbf{k}} \{ \langle \psi_{\mathbf{k}}^{\dagger}(t) \psi_{\mathbf{k}}^{*2}(t') \rangle \langle \psi_{\mathbf{k}}^{*2}(t) \psi_{\mathbf{k}}^{\dagger}(t') \rangle e^{i\omega_0(t-t')} \}. \quad (10)$$

On the basis of (8) and (9) we obtain for the correlation function of the ordering parameter

$$\langle \psi_{\mathbf{k}}^{\dagger}(t) \psi_{\mathbf{k}}^{*2}(t') \rangle = \frac{2mT\xi^2}{S d_1 (1 + k^2 \xi^2) \hbar^2} e^{-\Gamma_{\mathbf{k}} |t-t'|}. \quad (11)$$

Substituting (11) in (10) we get

$$\langle J(t)J(t') \rangle = \frac{1}{2} J_1^2 \sum_{\mathbf{k}} \frac{\exp\{-2\Gamma(1 + k^2 \xi^2)|t-t'|\}}{(1 + k^2 \xi^2)^2} \cos \omega_0(t-t'), \quad (12)$$

where

$$J_1 = A \frac{2mT\xi^2}{\hbar^2 S \sqrt{d_1 d_2}}. \quad (13)$$

If the characteristic dimensions of the junction are small compared with ξ , we can retain in the sum (12) only the term with $k = 0$:

$$\langle J(t)J(t') \rangle \approx \frac{1}{2} J_1^2 \exp\{-2\Gamma|t-t'|\} \cos \omega_0(t-t'), \quad (14)$$

corresponding to the following radiation spectrum^[1]

$$K(\omega) = J_1^2 \left[\frac{\Gamma}{(\omega - \omega_0)^2 + 4\Gamma^2} + \frac{\Gamma}{(\omega + \omega_0)^2 + 4\Gamma^2} \right] \quad (15)$$

($K(\omega)$ is the Fourier component of the function $\langle J(t)J(t') \rangle$).

We see therefore that the spectrum has a Lorentz form with a width determined by the reciprocal pair relaxation time Γ , where $\Gamma = 8(T - T_C)/\pi \hbar$ ^[4]. The quantity J_1 plays the role of the amplitude of the Josephson current at temperatures exceeding T_C .

For a junction of large area, to the contrary, we can replace the summation with respect to k in (12) by integration ($\sum_{\mathbf{k}} = (2\pi)^{-2} S \int d^2 k$), which yields

$$\langle J(t)J(t') \rangle = J_1^2 \frac{S}{4\pi} \int_0^{\infty} \frac{k dk}{(1 + k^2 \xi^2)^2} \exp\{-2\Gamma(1 + k^2 \xi^2)|t-t'|\} \cos \omega_0(t-t'). \quad (16)$$

From this we obtain for the form of the spectrum at $S \gg \xi^2$

$$K(\omega) = J_1^2 \frac{S}{16\pi \xi^2 \Gamma} \left[f\left(\frac{\omega - \omega_0}{2\Gamma}\right) + f\left(\frac{\omega + \omega_0}{2\Gamma}\right) \right], \quad (17)$$

where

$$f(x) = \frac{1}{2x^2} \ln(1 + x^2). \quad (18)$$

The role of the "amplitude" of the Josephson current in wide junctions is played by the quantity $J_M = \frac{1}{4} J_1 \xi^{-1} \sqrt{S/\pi}$, which now is much larger than J_1 . The value of J_1 does not depend on the junction area (since the product RS is constant), and therefore J_M is proportional to $S^{1/2}$.

2. We now proceed to consider the situation with a given current J through the junction. In this case the barrier voltage is not constant and is determined from the equation for the conservation of the total current, which consists of the "superconducting" current (5) and the normal current $J_N = V/R = \hbar \varphi / 2eR$. We write this equation in the form

$$A \int \frac{d^2 x}{S} \text{Im}[\psi_1(x, t) \psi_2^*(x, t) e^{i\varphi}] + \frac{\hbar}{2eR} \varphi + i(t) = J + J_{\sim} \sin \Omega t, \quad (19)$$

where the right-hand side represents the given current $J(t)$, equal to the sum of the dc component J and the alternating part $J_{\sim} \sin \Omega t$, corresponding to oscillations with amplitude J_{\sim} and frequency Ω . In addition, the fluctuation current $i(t)$ has been added to the right side of (19); this current is connected with the thermal noise of the resistance R . In accordance with the Nyquist theorem we have

$$\langle i(t) \rangle = 0, \quad \langle i(t)i(t') \rangle = 2T\delta(t-t')/R. \quad (20)$$

Solving (19), we get the dependence of the phase of the time, $\varphi = \varphi(t)$, and with it the junction voltage $V = \hbar \varphi / 2e$. It is then necessary to find the mean value of the voltage $\langle V \rangle$ as a function of the current J , i.e., the current-voltage characteristic of the tunnel junction. J_{\sim} is regarded in this case as a parameter on which the form of the current-voltage characteristic depends. Introduction of the term $J_{\sim} \sin \Omega t$ characterizes the action of the HF irradiation on the tunnel current.

Proceeding to the solution of (19), we note that above T_C the Josephson current represents a small correction, and therefore can be sought in the form of an expansion in powers of its amplitude: $\varphi = \varphi_0(t) + \varphi_1(t) + \dots$. The first term $\varphi_0(t)$ of the expansion satisfies equation (19) with $A = 0$, $\varphi_1 \sim A$, etc.

The zeroth approximation to the solution of the equation can be found in trivial manner:

$$\varphi_0(t) = \omega t - \frac{\omega_{\sim}}{\Omega} \cos \Omega t - \frac{2e}{\hbar} R \int_{t_0}^t i(t) dt. \quad (21)$$

Here

$$\omega = \frac{2e}{\hbar} R J_1, \quad \omega_{\sim} = \frac{2e}{\hbar} R J_{\sim}. \quad (22)$$

Similarly, for $\varphi_1(t)$ we obtain

$$\varphi_1(t) = -\frac{2e}{\hbar} R A \int_{-\infty}^t \text{Im}[Z(\tau) e^{i\varphi_0(\tau)}] d\tau. \quad (23)$$

$Z(t)$ stands here for the quantity

$$Z(t) = \sum_k \psi_k^1(t) \psi_k^{*2}(t) = \sum_k e^{-2i\Gamma_k t} \int_{-\infty}^t dt_1 \int_{-\infty}^t dt_2 e^{i\Gamma_k(t_1+t_2)} S_k^1(t_1) S_k^{*2}(t_2). \quad (24)$$

The average (statistical) value of the voltage is determined on the basis of (19) by means of the formula

$$\langle V(t) \rangle = RJ + RJ_{\sim} \sin \Omega t - AR \operatorname{Re} \langle Z(t) e^{i\omega(t)} \varphi_1(t) \rangle, \quad (25)$$

in which we have retained only the lowest-order term of the expansion in the amplitude of the Josephson current and have discarded terms at higher order. Substituting (25) in (23), we obtain

$$\begin{aligned} \langle \omega(t) \rangle &= 2e \langle V(t) \rangle / \hbar = \omega + \omega_{\sim} \sin \Omega t \\ &- \frac{1}{2} \left(\frac{2e}{\hbar} RA \right)^2 \int_{-\infty}^t d\tau \operatorname{Im} \{ \langle Z(t) Z^*(\tau) \rangle \langle e^{i\omega(t)} e^{-i\omega(\tau)} \rangle \}. \end{aligned} \quad (26)$$

We have taken into account here the fact that the fluctuations of $Z(t)$ and $i(t)$ are statistically independent.

All the correlators in (26) can be easily calculated. Omitting the simple intermediate steps, we present the final expression for the average voltage across the barrier at the instant of time t . For the quantity

$$\langle \omega(t) \rangle = 2e \langle V(t) \rangle / \hbar$$

we have

$$\begin{aligned} \langle \omega(t) \rangle &= \omega + \omega_{\sim} \sin \Omega t - \frac{1}{2} \omega_1^2 \operatorname{Im} \int_{-\infty}^t d\tau \sum_k \frac{1}{\Gamma_k^2} \exp\{-(2\Gamma_k + \gamma)(t - \tau)\} \\ &\times \exp i\omega(t - \tau) \exp \left\{ -\frac{i\omega_{\sim}}{\Omega} (\cos \Omega t - \cos \Omega \tau) \right\}, \end{aligned} \quad (27)$$

where $\omega_1 = 2eRJ_1/\hbar$, and $\gamma = (2e/\hbar)^2 RT$ is the line width of the Josephson radiation and is connected with the thermal fluctuations of the voltage^[5].

Calculating the integral with respect to τ in (27) and averaging $\langle \omega(t) \rangle$ over the time, we obtain for the average junction voltage $\bar{V} = \hbar \bar{\omega} / 2e = \hbar \langle \bar{\omega}(t) \rangle / 2e$ ($J_n(x)$ are Bessel functions)

$$\bar{\omega} = \omega - \frac{1}{2} \omega_1^2 \operatorname{Im} \sum_k \frac{1}{(1 + k^2 \xi^2)^2} \sum_{n=-\infty}^{\infty} \frac{J_n^2(\omega_{\sim}/\Omega)}{2\Gamma_k + \gamma - i(\omega - n\Omega)}. \quad (28)$$

Formula (28) solves our problem completely. It determines the form of the current-voltage characteristic of the tunnel junction when account is taken of the fluctuation current J_1 , and also the character of the variation of the current-voltage curve under the influence of the HF voltage $v(t) = v_{\sim} \sin \Omega t$, with amplitude $v_{\sim} = RJ_{\sim}$.

3. Proceeding to analyze expression (28), let us consider first the case when there is no external radiation: $\omega_{\sim} = 2eV_{\sim}/\hbar = 0$. The current-voltage characteristic is in this case the form (we recall that $\bar{\omega}$ is proportional to the average voltage \bar{V} , and ω is proportional to the current J through the junction: $\bar{\omega} = 2eV/\hbar$ and $\omega = 2eRJ/\hbar$)

$$\bar{\omega} = \omega - \frac{\omega_1^2 S}{4\pi} \int_0^k \frac{k dk}{(1 + k^2 \xi^2)^2} \frac{\omega}{\omega^2 + [2\Gamma(1 + k^2 \xi^2) + \gamma]^2}. \quad (29)$$

The integral can be evaluated, but the corresponding expression is quite cumbersome. We therefore consider separately cases when the principal role is played by pair relaxation ($\Gamma \gg \gamma$) or by thermal fluctuations in the junction ($\gamma \gg \Gamma$).

In the case $\gamma \gg \Gamma$ we obtain

$$\bar{\omega} = \omega - \frac{S\omega_1^2}{8\pi\xi^2} \frac{\omega}{\omega^2 + \gamma^2}, \quad \omega_1 = \frac{2e}{\hbar} RJ. \quad (30)$$

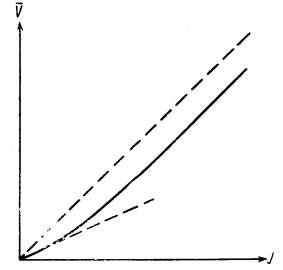


FIG. 2. Current-voltage curve of tunnel junction with account taken of the fluctuation pairing effects (the dashed line shows the current-voltage characteristic of the normal state).

To the contrary, when $\Gamma \ll \gamma$, the corresponding relation is

$$\bar{\omega} = \omega - \frac{S\omega_1^2}{8\pi\xi^2} \frac{1}{\omega} \left(1 - \frac{2\Gamma}{\omega} \operatorname{arctg} \frac{\omega}{2\Gamma} \right). \quad (31)$$

In both cases, the current-voltage characteristic of the tunnel current is nonlinear (Fig. 2), and the transition to the nonlinear region corresponds to voltages $2eV/\hbar$ of the order of γ or Γ , respectively.

At low voltages, the action of the fluctuations on the tunnel current reduces to a change (decrease) of the resultant junction resistance, and is analogous to the influence of the fluctuation pairing on the conductivity of thin films^[2]. On the basis of (29) we obtain in the linear region

$$\left(\frac{d\bar{V}}{dJ} \right)_{J=0} = R \left\{ 1 - \frac{S\omega_1^2}{8\pi\xi^2} \frac{1}{\gamma^2} \left[1 + \frac{1}{1 + \gamma/2\Gamma} - \frac{4\Gamma}{\gamma} \ln \left(1 + \frac{\gamma}{2\Gamma} \right) \right] \right\}. \quad (32)$$

When $\gamma \gg \Gamma$ (not too close to T_C) the fluctuation increment to the conductivity is proportional to $1/\Gamma \sim 1/(T - T_C)$ ($\xi^2 = D/\Gamma$, where D is the diffusion coefficient), i.e., the fluctuation conductivity varies with the temperature in exactly the same manner as in the Aslamazov-Larkin theory^[2]. Not too close to T_C , when Γ becomes larger than γ , the fluctuation part of the conductivity begins to decrease much more rapidly, like $1/\Gamma^3 \sim 1/(T - T_C)^{3.1}$. We note by way of an example that when $R = 1$ ohm the temperature region in which Γ becomes comparable with γ is $T - T_C \sim 10^{-3}$ °K.

4. We now analyze the case when the contact is exposed to a HF field with amplitude v_{\sim} . We consider by way of an illustration the case of small dimensions ($S \ll \xi^2$). Then, according to (28) we obtain, retaining only the term with $k = 0$,

$$\bar{\omega} = \omega - \frac{1}{2} \omega_1^2 \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{\omega_{\sim}}{\Omega} \right) \frac{\omega - n\Omega}{(\omega - n\Omega)^2 + (2\Gamma + \gamma)^2}. \quad (33)$$

As seen from this expression, the current-voltage curve has a number of singularities that are equidistantly distributed along the voltage axis²⁾, and have the form shown in Fig. 3 (we assume that $\Omega \gg \Gamma, \gamma$). The values of these values $V_n = n\hbar\Omega/2e$ correspond to the position of the so-called "Shapiro steps" in Josephson tunnel junctions exposed to external monochromatic

¹⁾ All the foregoing conclusions pertain to the case of superconductors with identical T_C . It is easy to use the same method to analyze a situation in which only one of the metals is normal ($T_{C1} < T$), and the other is superconducting ($T_{C2} > T$).

²⁾ Since the second term in the right side of (33) constitutes a small increment proportional to the square of the amplitude of the fluctuation current, we need not differentiate between ω and $\bar{\omega}$ in the resonant denominators of formula (33).

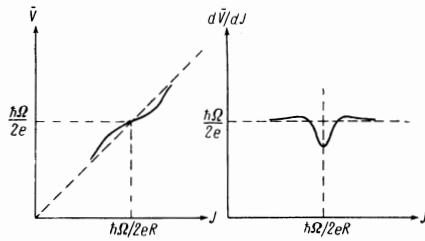


FIG. 3. Form of singularity of the current-voltage characteristic of the fluctuation current when the junction is exposed to monochromatic radiation of frequency $\Omega \gg \Gamma, \gamma$.

fields of frequency Ω ^[6]. In the case considered by us, owing to the presence of strong fluctuations, these singularities have the form not of steps but of smoother curves with a Lorentz line shape³⁾.

Equation (33) reveals also another feature of the influence of irradiation on the form of the J-V curve of the fluctuation current, namely oscillations with amplitude of the microwave signal $v \sim$ in the slope of the current-voltage characteristic at zero, and also in the slopes of the "steplike" singularities that take place at voltages $V_n = n\hbar\Omega/2e$. For the singularity numbered n , these oscillations are described by the square of the Bessel function $J_n^2(2ev \sim / \hbar\Omega)$. Obviously, the latter phenomenon is analogous to the oscillations of the height of the Shapiro steps with the amplitude of the alternating field in the Josephson effect below T_C ^[6].

In the case of "broad" tunnel junctions ($S \gg \xi^2$) we obtain for the form of the current-voltage characteristic the following expression (we invert the $\bar{\omega}(\omega)$ relation with account taken of the remark made in footnote²⁾)

$$\omega = \bar{\omega} + \frac{S\omega_1^2}{8\pi\xi^2} \sum_{n=-\infty}^{\infty} J_n^2\left(\frac{\omega \sim}{\Omega}\right) \int_1^{\infty} \frac{dx}{x^2} \frac{\bar{\omega} - n\Omega}{(\bar{\omega} - n\Omega)^2 + (2\Gamma x + \gamma)^2}. \quad (34)$$

If $\Omega \gg 2\Gamma + \gamma$, the slope at the center of the n -th "step," $\bar{\omega} = n\Omega$, is equal to

$$\left(\frac{d\bar{V}}{dJ}\right)_{V=V_n} = R \left\{ 1 - \frac{S\omega_1^2}{8\pi\xi^2\gamma^2} \left[1 + \left(1 + \frac{\gamma}{2\Gamma}\right)^{-1} - \frac{4\Gamma}{\gamma} \ln\left(1 + \frac{\gamma}{2\Gamma}\right) \right] J_n^2\left(\frac{\omega \sim}{\Omega}\right) \right\}. \quad (35)$$

It oscillates as a function of the intensity of the HF radiation (in particular, the slope of the current-voltage characteristic at zero changes in oscillatory fashion, compare (35) at $n = 0$ with (32)).

Thus, summarizing our results, we can state that the effects of the fluctuation conductivity ("paraconductivity" in accordance with the terminology used in

the American literature) in tunnel junctions have a highly distinct behavior, and reveal quantum interference effects in a high-frequency field, namely, generation of radiation at the Josephson frequency $\omega_0 = 2eV/\hbar$ (Sec. 1), the appearance of singularities of the current-voltage curve when exposed to microwave r , an oscillatory dependence of the slope of the dynamic conductivity $d\bar{V}/dJ$ as a function of the amplitude of the HF field. A study of these effects can be of definite interest as part of the investigation of the fluctuation superconductivity at temperatures above T_C .

In conclusion we note that we have calculated only a part of the change of the current of the tunnel junction at a temperature above T_C , that connected with the fluctuation pairing. Besides the fluctuation component of the Josephson current obtained above, there should exist also a correction due to the difference between the normal current and the quantity V/R , this being connected with disturbance of the density of states above T_C ^[8]. Both corrections, if small, make an additive contribution to the total current and can be calculated independently. A similar situation obtains also for the fluctuation conductivity of films. The corrections to the expression of Aslamazov and Larkin^[2], obtained recently by Maki^[9] and by Thompson^[10], can be interpreted as being due to the contribution of the normal component of the current, the magnitude of which changes as a result of the fluctuation pairs⁴⁾.

¹I. O. Kulik, ZhETF Pis. Red. 10, 488 (1969) [JETP Lett. 10, 313 (1969)]; in: Fizika kondensirovannogo sostoyaniya (Condensed-state Physics), Press of Low-temp. Physico-tech. Inst., Ukr. Acad. Sci., Khar'kov, 1970 (in press).

²L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela 10, 1104 (1968) [Sov. Phys.-Solid State 10, 000 (1968)]; Phys. Lett. 26A, 238 (1968).

³I. O. Kulik, Zh. Eksp. Teor. Fiz. 59, 584 (1970) [Sov. Phys.-JETP 32, 318 (1971)].

⁴E. Abrahams and T. Tsuneto, Phys. Rev. 152, 416 (1966).

⁵A. I. Larkin and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 53, 2159 (1967) [Sov. Phys.-JETP 26, 1219 (1968)].

⁶S. Shapiro, A. Janus, and S. Holly, Rev. Mod. Phys. 36, 223 (1964).

⁷I. K. Yanson, Candidate's Dissertation, Low-temp. Physico-tech. Inst. Ukr. Acad. Sci., Khar'kov, 1965.

⁸R. W. Cohen, B. Abeles, and C. R. Fuselier, Phys. Rev. Lett. 23, 377 (1969).

⁹K. Maki, Progr. Theor. Phys. 39, 897 (1968).

¹⁰R. S. Thompson (in press).

Translated by J. G. Adashko
107

⁴⁾This remark is due to A. I. Larkin.

³⁾We note that these singularities should have the same form also in Josephson tunnel junctions at a temperature below T_C , provided the microwave field amplitude satisfies the inequality $J_0 V \ll T \Omega$, where J_0 is the value of the Josephson critical current. This was observed experimentally by Yanson [7].