KINETIC DIA- AND PARAMAGNETISM UNDER THE INFLUENCE OF THERMOMAGNETIC AND GALVANOMAGNETIC CURRENTS

A. E. GUREVICH and O. A. MEZRIN

A. F. Ioffe Physico-technical Institute, U.S.S.R. Academy of Sciences

Submitted April 21, 1970

Zh. Eksp. Teor. Fiz. 59, 1005–1008 (September, 1970)

It is shown that the Hall and Nernst currents produced in a nonequilibrium conducting medium situated in an external magnetic field can either intensify or weaken this field in the interior of the medium. This phenomenon, called kinetic dia- and paramagnetism, is of importance both in the plasma of stars, which have strong kinetic diamagnetism, and in solids, which can be either paramagnetic or diamagnetic, depending on the mechanism governing the carrier scattering, the sign of their charge, and the direction of the radial temperature gradient in the solid or of the electric field.

1. IT was shown earlier that if a conducting medium in which there is either a temperature gradient ∇T or an electric field $\mathbf{E}$ is placed in a magnetic field $\mathbf{H}$, then the Hall or Nernst currents produced in the medium induce a magnetic field that intensifies or weakens the original magnetic field in the interior of the medium. These phenomena were called kinetic dia- and paramagnetism. We report here a detailed investigation of these phenomena.

In a medium placed in a magnetic field $\mathbf{H}$, there are produced Hall and Nernst currents

$$\mathbf{j} = [(\sigma_1\mathbf{E} + \Lambda\nabla T) \times \mathbf{H}].$$

The sign of the coefficient Λ in most scattering mechanisms is opposite to the sign of the exponent k in the expression $\tau \sim v^k$ for the relaxation time on the velocity. In order of magnitude we have $\Lambda \approx \pm\sigma\tau/mc$.

The fundamental equation of our problem is

$$\frac{c}{4\pi}\operatorname{rot}\mathbf{H} = [(\sigma_1\mathbf{E} + \Lambda\nabla T)\times \mathbf{H}]. \tag{1}$$

2. Kinetic diamagnetism connected with the presence of a temperature gradient can be very strong in a plasma, particularly in stars. Since the temperature gradient in stars is radially directed, Eq. (1) relates the curl of the toroidal field with the toroidal field itself, and analogously for the polhode field. In the case of Coulomb scattering, $\Lambda < 0$ and therefore $\Lambda\nabla T > 0$. This means that the magnetic field becomes weaker inside the star. The characteristic length of this attenuation is

$$L \approx \frac{\lambda^2 Na^3}{4\pi}\frac{m}{m_a}\frac{c^2}{g},$$

In the outer layers of the star we have

$$|\partial T/\partial r| \approx m_a g,$$

where m_a is the mass of the atom and g the acceleration due to gravity. We have further

$$\sigma\tau \approx 1/Na^3\lambda^2,$$

where N is the electron density, $a = e^2/T$ the Coulomb length, and λ the Coulomb logarithm. Therefore the characteristic length for the screening of the magnetic field by the currents is

$$L \approx mc^2/4\pi\sigma\tau|\nabla T|.$$

where m is the electron mass.

Since in a politrope with exponent 3 we have

$$Na^3 \approx \text{const},$$

it is immaterial at which depth N and T are taken. Putting then $N \approx 10^{17}$, $T \approx 10^{-11}$, $g \approx 3\times 10^4$, and $\lambda \approx 20$ (typical values for the sum), we obtain the estimate

$$L \approx 6\cdot 10^8 \approx 10^{-2}R,$$

where R is the sun's radius.

The characteristic length of stars hotter than the sun is even smaller.

3. The kinetic dia- and paramagnetism is important also for solids. Let us consider a conductor in the form of a hollow cylinder with inside radius r_1 and outside radius r_2, the wall thickness $\Delta = r_2 - r_1$ being much smaller than its length L, so that all the physical parameters can be regarded as invariant along the cylinder axis (the z axis).

Let the external magnetic field $\mathbf{H}_0$ be directed along the z axis of the cylinder. A current $\mathbf{J} = 2\pi r\sigma\mathbf{E}$ per unit length flows radially through the wall, the field E being equal to

$$E = E_0\frac{r_2}{r} = V\Big/ r\ln\frac{r_1}{r_2}.$$

In addition, there is a radial temperature drop

$$\Delta T = T(r_2) - T(r_1) = T_0 - T(r_1)$$

(the temperature is expressed in energy units), so that

$$T(r) = T_0 - \Delta T\ln\frac{r_2}{r}\Big/\ln\frac{r_2}{r_1}. \tag{2}$$

Here $\mathbf{H}_0$, $\mathbf{E}_0$ and T_0 are the magnetic and electric field intensities and the temperature on the outside surface of the cylinder, and V is the potential difference between the outside and inside surfaces.

4. The radial temperature gradient ∇T produces a thermoelectric field determined from the condition

$$j_r = \sigma E_T + \beta\nabla T = 0,$$

whence

$$E_T = -(\beta/\sigma)\nabla T,$$

where j_r is the current-density component in the radial direction.

When $L \gg \Delta$, the resultant Hall and Nernst currents produce no noticeable radial and azimuthal field components H_r and H_φ, while the variation of the component $H_z = H$ is given by

$$\frac{\partial H}{\partial r} = \left[\frac{2\sigma_1}{c\sigma} J + \frac{4\pi\Lambda\Delta T}{c\ln(r_2/r_1)}\right]\frac{H}{r}. \quad (3)$$

If the magnetic field is weak in the sense that $\Omega\tau = H/H_c = eH\tau/mc \ll 1$ (Ω is the cyclotron frequency), then

$$H(r) = H_0(r_2/r)^{-a}. \quad (4)$$

If only an electric current or only a temperature drop is present, we have, respectively,

$$H(r) = H_0(r_2/r)^{-a_J}, \quad H = H_0(r_2/r)^{-a_T},$$

where

$$a = \frac{2\sigma_1}{c\sigma} J + \frac{4\pi\Lambda\Delta T}{c\ln(r_2/r_1)} = a_J + a_T. \quad (5)$$

In [1] there was obtained, in the presence of ∇T, an exponential dependence of the magnetic field on the radial coordinate, owing to the fact that in place of (2) it was assumed that $\partial T/\partial r = \text{const}$, an assumption valid if $\Delta \ll r_2$.

The sign of the first term in (5) is determined by the sign of the charge of the predominant carriers and by the current direction. Its magnitude depends on the exponent k in the dependence of the relaxation time on the carrier velocity. For nondegenerate electrons with $k = 2n$ and $k = 2n + 1$ (n is an integer), a_J is given respectively by

$$a_J = -\frac{2(4n+3)!!}{3^n(2n+3)!!}\frac{J}{cH_c}, \quad a_J = -\frac{\sqrt{\pi}(4n+5)!!}{2\cdot 6^{(2n+1)/2}(n+2)!}\frac{J}{cH_c},$$

and for degenerate electrons by

$$a_J = -2J/cH_c.$$

An estimate yields for the last case

$$a_J \approx \frac{2J}{cH_c} = \pm\left|\frac{2\mu J}{c^2}\right|.$$

For the alloy InSb (mobility $\mu \approx 3\times10^7$ at room temperature) we have $a_J \approx 1$ at $J \approx 5\times10^3$ A/cm, whereas for bismuth ($\mu \approx 10^{10}$ at helium temperature) we have $a_J \approx 1$ at $J \approx 20$ A/cm.

the exponent of the dependence of the relaxation time on the carrier velocity and by the direction of the temperature drop. For nondegenerate electrons with $k - 2n$ and $k = 2n + 1$ we have, respectively,

$$a_T = -\frac{4\pi(4n+3)!!}{3^n(2n+3)!!\ln(r_2/r_1)}\frac{\sigma\tau\Delta T}{mc^2},$$

$$a_T = -\frac{\pi^{3/2}(2n+1)(4n+5)!!}{2\cdot 6^{(2n+1)/2}(n+2)!\ln(r_2/r_1)}\frac{\sigma\tau\Delta T}{mc^2},$$

and for degenerate ones

$$a_T = -\frac{2\pi k}{3\ln(r_2/r_1)}\frac{T}{\zeta}\frac{\sigma\tau\Delta T}{mc^2},$$

where σ stands throughout for the conductivity in the absence of a magnetic field and ζ is the chemical potential.

If the magnetic field is strong, i.e., $H \gg H_c$, then we have $\Lambda \sim 1/H^2$ in (3), and σ_1/σ does not depend on H ($N^+ \neq N^-$). Therefore a solution of (3) is

$$H^2(r) = (H_0^2 + K)(r_2/r)^{-2a_J^*} - K. \quad (6)$$

In the presence of an electric current only, we thus have

$$H(r) = H_0(r_2/r)^{-a_J^*},$$

and in the case of a temperature drop only

$$H^2(r) = H_0^2 + 2a_J^* K\ln(r_2/r).$$

In the second case the dependence obtained in [1] is not logarithmic, for the reason indicated above.

For nondegenerate electrons at $k = 2n$ and $k = 2n + 1$ we have for a_J^* and K

$$a_J^* = \frac{3(2n+3)!!}{(3-2n)!!(4n+3)!!}a_J = -\frac{2}{3^{n-1}}\frac{1}{(3-2n)!!}\frac{J}{cH_c},$$

$$a_J^* = \frac{3\cdot 2^{2n}(n+2)!}{(1-n)!(4n+5)!!}a_J = -\left(\frac{2}{3}\right)^{(2n-1)/2}\frac{\sqrt{\pi}}{(1-n)!}\frac{J}{cH_c},$$

$$K = -\frac{3^{3n}(3-2n)!!}{2(4n+3)!!}H_c^3\frac{c}{J}a_T = \frac{2\cdot 3^{2n}\pi n(3-2n)!!}{(3+2n)!!\ln(r_2/r_1)}H_c^2\frac{\sigma\Delta T}{eJ},$$

$$K = -\frac{3^{3n+3/2}(1-n)!}{2^{n-1/2}\sqrt{\pi}(4n+5)!!}H_c^3\frac{c}{J}a_T = \left(\frac{3}{2}\right)^{2n+1}\frac{\pi(2n+1)(1-n)!}{(n+2)!\ln(r_2/r_1)}H_c^2\frac{\sigma\Delta T}{eJ},$$

and for degenerate electrons

$$a_J^* = a_J = 2J/cH_c,$$

$$K = -\frac{\pi}{2}H_c^3\frac{c}{J}a_T = \frac{\pi^2 k}{3\ln(r_2/r_1)}\frac{T}{\zeta}\frac{\sigma\Delta T}{eJ}.$$

The signs of a_J^* and K are determined by the same parameters as above.

The solution (6) is valid when r satisfies the inequality

$$(r/r_2)^{2a_J^*} > (H_c^2 + K)/(H_0^2 + K).$$

At smaller values of r, the magnetic field is already weak and solution (3) is applicable.

Finally, if the magnetic field inside the cylinder is quantizing then, as shown by calculation, the effect is vanishingly small and will therefore not be discussed here.

[1] L. E. Gurevich, ZhETF Pis. Red. 11, 269 (1970) [JETP Lett. 11, 175 (1970)].

Translated by J. G. Adashko
115