# THE POSSIBILITY OF THE EXCITATION OF SPIN WAVES IN MAGNETICALLY ORDERED FERROELECTRICS

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Excitation of spin waves in ferroelectric magnetically ordered crystals is investigated. It is shown that if the crystal is simultaneously a semiconductor, excitation of spin waves by an external stationary electric field should be possible. The mechanism of such an excitation differs in principle from the well-known mechanism of excitation of spin waves by an electric field in ordinary (nonferroelectric) magnetically ordered semiconductors and should lead to considerably greater growth increments.

1. A whole series of ferroelectric magnetically ordered crystals is known at present (see the review[1]). We show that if such a crystal is simultaneously a semiconductor it is possible for spin waves to be excited in it by a constant external electric field. The mechanism of this excitation differs in principle from the known excitation mechanism of spin waves Gurevich and Korenblit[2,3]) by an electric field in ordinary (nonferroelectric) magnetically ordered semiconductors and should lead to considerably greater growth increments.

In an ordinary magnetically ordered semiconductor the coupling between the current and the spin system is, as is well known, accomplished through the transverse electromagnetic field; the growth increments are, therefore, relativistically small (proportional to $u^2/c^2$; u is the average directed velocity of the current carriers). On the other hand, in the case of magnetically ordered ferroelectrics the coupling between the electric and spin system should be accomplished directly (because of the presence of mixed magnetoelectric terms in the expression for the internal energy of the crystal; the small parameter characterizing such a magnetoelectric coupling is obviously proportional to $v_0^2/c^2$ where $v_0$ is the velocity of the atomic electrons). In this sense the excitation of spin waves in magnetically ordered ferroelectrics is analogous to the mechanism of the instability of sound waves in piezoelectric semiconductors considered by White.[4]

2. In describing the spin system of a crystal we shall start from the equation of motion of the magnetic moments of the sublattices and from the equations of magnetostatics (see, for example,[5])

$$\frac{\partial \mathbf{M}_i}{\partial t} = g[\mathbf{M}_i\mathbf{H}_i], \quad \operatorname{div}\Big(\mathbf{H} + 4\pi\sum_i \mathbf{M}_i\Big) = 0, \quad \operatorname{rot}\mathbf{H} = 0, \quad (1)^*$$

where $\mathbf{M}_i$ is the magnetic moment density connected with the i-th magnetic sublattice, g is the gyromagnetic ratio, H, is the magnetic field and $\mathbf{H}_i$ is the effective field related to the internal energy of the crystal U by the relation

$$\mathbf{H}_i = -\delta U / \delta \mathbf{M}_i. \quad (2)$$

*$[\mathbf{M}_i\mathbf{H}_i] \equiv \mathbf{M}_i \times \mathbf{H}_i$.

We shall describe the motion of the current carriers with the aid of the equations of hydrodynamics and hydrostatics[1)]

$$\mathbf{v}_\pm = \pm \varkappa_\pm \mathbf{E}, \quad \frac{\partial \rho_\pm}{\partial t} + \operatorname{div}(\rho_\pm \mathbf{v}_\pm) = 0,$$
$$\operatorname{div}(\mathbf{E} + 4\pi\mathbf{P}) = 4\pi(\rho_+ + \rho_-), \quad \operatorname{rot}\mathbf{E} = 0, \quad (3)$$

where $\rho_\pm$ and $\mathbf{v}_\pm$ are the charge densities and directed velocities of the carriers (the subscripts + and − refer to holes and electrons), $\kappa_\pm$ are the carrier mobilities, E is the electric field and P is the polarization vector related to the internal energy of the crystal by the relation

$$\mathbf{P} = -\left(\frac{\delta U}{\delta \mathbf{E}} - \frac{\mathbf{E}}{4\pi}\right). \quad (4)$$

The internal energy U can be represented in the form

$$U = U_M + U_E + U_c, \quad (5)$$

where $U_M$ is the energy of the spin system, $U_E = (8\pi)^{-1}\int\epsilon_{ij}E_iE_j dV$ is the energy of the electric field, and $U_c$ is the coupling energy between the spin system of the crystal and the electric field ($\epsilon_{ij}$ is the dielectric permittivity tensor of the crystal).

Substituting (5) in (4) and solving Eq. (3), we find the oscillating component of the electric field:

$$\mathbf{e} = -\frac{4\pi}{k^2}\frac{\mathbf{k}(\mathbf{kp})}{\varepsilon^*}, \quad (6)$$

where p is the oscillating component of the vector $\mathbf{P}_c = -\delta U_c/\delta \mathbf{E}$,

$$\varepsilon^* = \varepsilon + 4\pi i \rho_0\left(\frac{\varkappa_+}{\omega - \mathbf{ku}_+} + \frac{\varkappa_-}{\omega - \mathbf{ku}_-}\right), \quad (7)$$

$\mathbf{u}_\pm$ is the average value of the directed velocities of the carriers, $\rho_0$ is the absolute value of the charge density of each type of carrier, ω and k are the frequency and wave vector of the oscillations (for simplicity we assume that the crystal is isotropic with respect to its dielectric properties $\epsilon_{ij} = \delta_{ij}\epsilon$). We note that if mainly one type of carrier only makes a contribution to the current (for example, if $\kappa_- \gg \kappa_+$), then the function $\epsilon^*$

1)We assume for simplicity that $H_0 \ll c\kappa^{-1}$; then one need not add in the first of Eqs. (3) the term taking into account the Lorentz force.

is expressed in terms of the conductivity $\sigma = \rho_0 \kappa_-$:

$$\varepsilon^* = \varepsilon + \frac{4\pi i\sigma}{\omega - \mathbf{ku}_-}.$$

3. Returning to relations (1) and (2), we represent the equations of motion of the magnetic moments in the form

$$\frac{\partial \mathbf{M}_i}{\partial t} = g[\mathbf{M}_i, (\mathbf{H}_i^M + \mathbf{H}_i')] + g[\mathbf{M}_i \mathbf{h}_i], \tag{8}$$

where $\mathbf{h}_i$ is the portion of the vector $\mathbf{H}_i^C = -\delta U_C/\delta \mathbf{M}_i$ proportional to the oscillating electric field $\mathbf{e}$ and $\mathbf{H}_i' = \mathbf{H}_i^C - \mathbf{h}_i$. It is readily seen that the term with $\mathbf{H}_i'$ leads only to a small overestimate of the frequencies of the spin waves and can therefore be omitted; as regards the component with $\mathbf{h}_i$, it can lead to an appreciable change in the damping constant of the spin waves and even to their growth.

We shall restrict ourselves below to the case of a crystal with two equivalent magnetic sublattices (an antiferromagnet or a weak ferromagnet). For such crystals

$$U_M = \int \Big\{ \frac{H^2}{8\pi} - \mathbf{H}_0(\mathbf{M}_1 + \mathbf{M}_2) + \frac{\alpha}{2}\left(\frac{\partial \mathbf{M}_1}{\partial x_j}\frac{\partial \mathbf{M}_1}{\partial x_j} + \frac{\partial \mathbf{M}_2}{\partial x_j}\frac{\partial \mathbf{M}_2}{\partial x_j}\right) + \alpha'\frac{\partial \mathbf{M}_1}{\partial x_j}\frac{\partial \mathbf{M}_2}{\partial x_j} - \frac{\beta}{2}[(\mathbf{M}_1\mathbf{n})^2 + (\mathbf{M}_2\mathbf{n})^2] - \beta'(\mathbf{M}_1\mathbf{n})(\mathbf{M}_2\mathbf{n}) + \eta\mathbf{M}_1\mathbf{M}_2 - d\mathbf{n}[\mathbf{M}_1\mathbf{M}_2] \Big\} dV, \tag{9}$$

where $\mathbf{H}_0$ is an external magnetic field; $\alpha$, $\alpha'$, and $\eta$ are exchange interaction constants; $\beta$ and $\beta'$ are magnetic anisotropy constants; d is the weak ferromagnetism constant (the Dzyaloshinskiĭ constant) and $\mathbf{n}$ is the unit vector along the anisotropy axis (the z axis). Introducing the vectors $\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$ and $\mathbf{L} = \mathbf{M}_1 - \mathbf{M}_2$ and taking into account the fact that $\eta \gg 1$, we bring the equations of motion of the magnetic moments into the form

$$\frac{\partial \mathbf{M}}{\partial t} = \frac{g}{2}[\mathbf{L}, (\alpha - \alpha')\Delta\mathbf{L} + (\beta - \beta')\mathbf{n}(\mathbf{Ln}) + \mathbf{h}^-] + \frac{g}{2}d[\mathbf{n}[\mathbf{ML}]] + \frac{g}{2}[\mathbf{M}, (\alpha + \alpha')\Delta\mathbf{M} + (\beta + \beta')\mathbf{n}(\mathbf{Mn}) + \mathbf{h}^+ + 2\mathbf{H}],$$

$$\frac{\partial \mathbf{L}}{\partial t} = g[\mathbf{L}, \mathbf{H} - \eta\mathbf{M}] + \frac{g}{2}([\mathbf{Lh}^+] + [\mathbf{Mh}^-]), \tag{10}$$

where $\mathbf{h}^{\pm} = \mathbf{h}_1 \pm \mathbf{h}_2$.

In the case of an antiferromagnet with an easy-plane type anisotropy (as well as in the case of a ferromagnet) in the absence of an external magnetic field, solving Eqs. (10) and taking into account the fact that $d \ll \eta$, we obtain

$$\left(1 - \frac{\omega^2}{\omega_1^2}\right) m_z = \frac{1}{2\eta} h_z^+ - \frac{i\omega g M_0}{\omega_1^2} h_y^-,$$

$$\left(1 - \frac{\omega^2}{\omega_2^2}\right) m_y = \frac{1}{2\eta} h_y^+ + \frac{i\omega g M_0}{\omega_2^2} h_z^-, \tag{11}$$

where $\omega_1$ and $\omega_2$ are the frequencies of the two branches of spin waves

$$\omega_1 = V_s k, \quad \omega_2 = [(gM_0)^2 2\eta|\beta - \beta'| + V_s^2 k^2]^{1/2},$$

$$V_s = gM_0\sqrt{2\eta(\alpha - \alpha')}, \tag{12}$$

$\mathbf{m}$ is the oscillating component of the vector $\mathbf{M}$ and $M_0$ is the magnetic moment density associated with each sublattice (the x axis is chosen in the direction of the vector $\mathbf{L}_0$—the equilibrium value of the vector $\mathbf{L}$).

In the case of an antiferromagnet with an easy-axis type anisotropy we have according to (10)

$$\left(1 - \frac{\omega^2}{\omega_2^2}\right)\mathbf{m} = \frac{1}{2\eta}\{\mathbf{h}^+ - \mathbf{n}(\mathbf{h}^+\mathbf{n})\} - \frac{i\omega g M_0}{\omega_2^2}[\mathbf{nh}^-]. \tag{13}$$

The excitation of spin waves in an antiferromagnet in a strong external magnetic field equal in order of magnitude to the exchange field $H_0 \sim \eta M_0$ can be of special interest. The point is that, as shown in the following section, in this case the growth increment of the spin waves apparently turns out to be considerably larger than in the absence of a magnetic field. Solving Eqs. (10) and being only interested in the low-frequency branch of oscillations, we obtain

$$\left(1 - \frac{\omega^2}{\omega_3^2}\right) m_z = \frac{1}{2\eta} h_z^+ - \frac{i\omega g L_0}{2\omega_3^2} h_y^- \tag{14}$$

where $\omega_3$ is the frequency of the low-frequency spin wave

$$\omega_3 = V_h k, \quad V_h = 2^{-1/2} g L_0 \eta^{1/2} (\alpha - \alpha')^{1/2} \tag{15}$$

and the quantity $L_0$ is connected with the external magnetic field $H_0$ by the relation

$$L_0 = (4M_0^2 - H_0^2/\eta^2)^{1/2} \tag{16}$$

(the field $H_0$ is directed along the anisotropy axis).

4. The explicit form of the growth increment of the spin waves depends appreciably on the structure of the energy of the interaction between the spin system of the crystal and the electric field $U_C$. Apparently (see[1]) in the known magnetically ordered ferroelectrics the spontaneous polarization vector $\mathbf{P}_0$ is perpendicular to the spontaneous magnetic moment $\mathbf{M}$; we shall therefore choose the coupling energy $U_C$ in the simplest form leading to the perpendicularity of the indicated vectors:

$$U_c = -\lambda \int \mathbf{E}[\mathbf{M}_1\mathbf{M}_2] dV, \tag{17}$$

where $\lambda$ is a constant (if one interprets the results of[1] starting from the coupling energy (17), then $\lambda \sim 10^{-3}$).

According to (17), (2), and (4), in ferromagnets with an easy-axis type of anisotropy

$$\mathbf{h}^+ = -2\lambda M_0[\mathbf{ne}], \quad \mathbf{h}^- = 0, \quad \mathbf{p} = \lambda M_0[\mathbf{nm}]. \tag{18}$$

Substituting these relations in (6) and (13), we obtain

$$1 - \frac{\omega^2}{\omega_2^2} = -\frac{4\pi}{\eta\varepsilon^*}(\lambda M_0)^2 \sin^2\theta_z, \tag{19}$$

where $\theta_z$ is the angle between the wave vector of the oscillations and the anisotropy axis. Solving Eq. (19), we find the growth increment of the spin waves due to their interaction with the current:

$$\gamma = -\frac{2(4\pi)^2(\lambda M_0)^2 \sin^2\theta_z \omega_2 \rho_0}{\eta|\varepsilon^*|^2}\left(\frac{\varkappa_+}{\omega_2 - \mathbf{ku}_+} + \frac{\varkappa_-}{\omega_2 - \mathbf{ku}_-}\right). \tag{20}$$

Taking into account (3), we see that for a not very strong electric field $E_0 < E_2(k)$ where $E_2(k) = \omega_2/\kappa k$ ($\kappa$ is the larger of the quantities $\kappa_+$ and $\kappa_-$) the increment $\gamma$ is negative so that the oscillations with the wave vector $\mathbf{k}$ are damped. For $E_0 > E_2$ the value of $\gamma$ becomes positive; in this case the interaction of the current with spin waves leads to a growth of the latter. Of course, for the spin waves to start growing the increment $\gamma$ must be greater than the damping constant due to the interaction of the spin waves with one another, with phonons and with the defects of the crystal.

Obviously, the condition $E_0 > E_2(k)$ represents simply the condition for the Cerenkov excitation of spin waves by a flux of charged particles; the point is that this condition is equivalent to the inequality $u > \omega_2/k$ where u is the average directed velocity of those current carriers which are characterized by the largest mobility.

In antiferromagnets with an easy-plane type of anisotropy

$$\mathbf{h}^+ = -\lambda[\mathbf{L}_0\mathbf{e}], \quad \mathbf{h}^- = 0, \quad \mathbf{p} = {}^1/_2\lambda[\mathbf{L}_0\mathbf{m}]. \tag{21}$$

One can also use the same relations for weak ferromagnets since, although in this case $\mathbf{h}^- \neq 0$, the contribution of the field $\mathbf{h}^-$ to the growth increment turns out to be proportional (compared with the contribution of the field $\mathbf{h}^+$) to the small parameter $\eta^{-1/2}$.

Substituting (21) in (6) and (11), we obtain

$$1-\frac{\omega^2}{\omega_1^2} = -\frac{4\pi(\lambda M_0)^2}{\eta\varepsilon^*}\cos^2\theta_y, \quad 1-\frac{\omega^2}{\omega_2^2} = -\frac{4\pi(\lambda M_0)^2}{\eta\varepsilon^*}\cos^2\theta_z \tag{22}$$

where $\theta_y$ is the angle between the vector **k** and the y axis. Solving these equations, we find the growth increments of the two branches of spin waves:

$$\gamma_1 = -\frac{2(4\pi)^2(\lambda M_0)^2\cos^2\theta_y kV_s\rho_0}{\eta|\varepsilon^*|^2}\left(\frac{\varkappa_+}{kV_s-\mathbf{ku}_+}+\frac{\varkappa_-}{kV_s-\mathbf{ku}_-}\right),$$

$$\gamma_2 = -\frac{2(4\pi)^2(\lambda M_0)^2\cos^2\theta_z\omega_2\rho_0}{\eta|\varepsilon^*|^2}\left(\frac{\varkappa_+}{\omega_2-\mathbf{ku}_+}+\frac{\varkappa_-}{\omega_2-\mathbf{ku}_-}\right). \tag{23}$$

According to (23), on increasing the external field the low-frequency spin wave will first of all commence to grow. For this the electric field must exceed the critical value $E_1 = \kappa^{-1}V_S$, and the carrier velocity must exceed the phase velocity $V_S \sim 3\times10^5$ cm/sec. For the growth increment to be a maximum, the external electric field should be directed perpendicular to the anisotropy axis and to the equilibrium direction of the vector **L**. As regards the excitation of a high-frequency spin wave, for this, as before, the condition $E > E_2(k)$ must be fulfilled.

The relative growth increment of the spin waves $\gamma/\omega$ in antiferromagnets (as well as in weak ferromagnets) turns out according to (20) and (23) as in the absence of an external magnetic field to be proportional to the small parameter $\eta^{-1}$. In antiferromagnets placed in a strong magnetic field, $H_0 \sim \eta M_0$, the relative growth increment will not contain this small parameter. In fact, it follows from (17) that

$$h_z^+ = -\lambda L_0 e_y, \quad h_y^- = \lambda\frac{H_0}{\eta}e_x, \quad p_x = i\frac{gL_0}{2\omega}\lambda H_0 m_z. \tag{24}$$

Substituting these relations in (6) and (14), we obtain

$$1-\frac{\omega^2}{\omega_3^2} = -\frac{2\pi}{\eta\varepsilon^*}(\lambda L_0)^2\frac{(gH_0)^2}{\omega_3^2}\cos^2\theta_x, \tag{25}$$

where $\theta_x$ is the angle between the vectors **k** and $\mathbf{L}_0$. Hence we find for the growth increment

$$\gamma_3 = -\frac{(4\pi)^2(\lambda L_0)^2\cos^2\theta_x(gH_0)^2kV_H\rho_0}{\eta|\varepsilon^*|^2\omega_3^2}\left(\frac{\varkappa_+}{kV_H-\mathbf{ku}_+}+\frac{\varkappa_-}{kV_H-\mathbf{ku}_-}\right). \tag{26}$$

We draw attention to the fact that in antiferromagnets located in a strong magnetic field (as in antiferromagnets with the easy-plane type of anisotropy) the excitation of spin waves does not require very large carrier velocities, $u = V_C \sim 10^5$ cm/sec. The critical value of the electric field is in this case $E_3 = V_C/\kappa$. It is important that the required carrier velocity and the critical electric field $E_3$ decrease with increasing external magnetic field, vanishing in fields close to $H_C = 2\eta M_0$; the growth increment itself, on the other hand, increases with increasing field like $H_0^2$. We emphasize that formula (26) [as well as (14)–(16) and (24) and (25)] are valid for $H_0 < H_C$; if $H_0 > H_C$, then $L_0 = 0$ and the crystal behaves like a ferromagnet.


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