IONIZATION-DIFFUSION OSCILLATIONS IN A PLASMA

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Low-frequency natural oscillations of a new type are found in a partially ionized bounded plasma. The oscillations are due to ionization, ambipolar diffusion, and recombination of charged particles on the walls.

WE show in this paper that ionization and diffusion can lead to natural oscillations of the charged-particle density in a weakly-ionized bounded plasma. It is assumed that the electrons and ions produced within the volume recombine mainly on the walls, on which they fall as a result of ambipolar diffusion. The oscillations are possible under conditions when the coefficient of ambipolar diffusion greatly exceeds the coefficient of diffusion of the neutral particles. To this end, the electron temperature must be much higher than the ion temperature.

1. We note that the oscillations in question are similar in their nature to the oscillations arising in the well-known Volterra problem (cf., e.g., [1]) of the size of the populations of two species of fish.

In [2,3], the change of the total number of neutral particles and ions, in the case of a partly-ionized plasma, is described by equations analogous to those of the Volterra problem. It is assumed in [2,3] that neutral particles enter the plasma volume from the outside, and that electrons and ions leak out. These equations are

$$dN/dt = \alpha N - \beta N N_i, \tag{1}$$

$$dN_i/dt = -\alpha_i N_i + \beta N N_i, \tag{2}$$

where N and N_i are the total numbers of neutral particles and ions. In (1) and (2), the terms αN and $-\alpha_i N_i$ characterize respectively the influx of neutral particles and the outflow of the ions, while the term $\beta N N_i$ describes the change in the number of particles, due to the ionization process. The plasma is assumed to be quasineutral ($N_i = N_e$).

If the deviations δN and δN_i from the stationary values of the total number of the particles $N_0 = \alpha_i/\beta$ and of the ions $N_{0i} = \alpha/\beta$ are small, then the problem can be solved in the linear approximation. We obtain

$$\delta N \sim \cos\omega t, \quad \delta N_i \sim \sin\omega t, \quad \omega = \sqrt{\alpha\alpha_i}.$$

Thus, in a partly-ionized plasma oscillations of the total number of neutral particles and ions are possible. However, such a formulation of the problem, when there is an influx of neutral particle and an outflow of ions, should lead in principle to a significant spatial inhomogeneity, which is not taken into account in [2,3].

2. Let us consider the self-consistent problem of the natural oscillations of a plane-parallel layer of plasma, when the motion of the electrons and the ions towards the walls is due to ambipolar diffusion and there is an opposing diffusion flow of neutral particles resulting from the recombination of the electrons and ions on the walls.

Let n and n_i be the densities of the neutral particles and the ions. We assume that the plasma is weakly ionized ($n_i \ll n$) and is quasineutral, i.e the densities of the ions and of the electrons are equal. In addition, we shall assume that the electron mean free path is much shorter than the characteristic dimensions of the plasma, and that the effective collision time is small compared with the characteristic times of the processes under consideration. Under these conditions, the equations describing the change of the density of neutral particles and ions are

$$\partial n/\partial t = D(n)\Delta n - a(n)n_i, \tag{3}$$

$$\partial n_i/\partial t = D_i(n)\Delta n + a(n)n_i, \tag{4}$$

where D(n) and $D_i(n)$ are the diffusion coefficients of the neutral particles and of the ambipolar diffusion, and a(n) is the frequency of ionization by one electron. We assume that these coefficients depend only on the density of the neutral particles, since the plasma is weakly ionized and only collisions with neutral particles are of significance.

Assuming that the charged particles recombine completely on the wall and that the density of the diffusion flux of the ions near the wall is equal to the density of the opposing diffusion flux of the neutral particles resulting from the ion recombination on the wall, we get the following boundary conditions:

$$n_i|_\Sigma = 0, \tag{5}$$

$$D(n)\frac{\partial n}{\partial s}\Big|_\Sigma = -D_i(n)\frac{\partial n_i}{\partial s}\Big|_\Sigma, \tag{6}$$

where s is the vector normal to the surface Σ.

3. Let us consider a plasma layer bounded by two parallel walls $x = \pm b$, assuming that the particle densities depend only on one spatial coordinate x. We seek the stationary distribution of the density of the neutral particles in the form $n_0 + \tilde{n}_0$, where n_0 is the coordinate-independent part of the neutral-particle density, with $n_0 \gg \tilde{n}_0$ and $n_0 \gg n_{0i}$. Taking the boundary conditions (5) and (6) into account, we obtain

$$n_{0i} = A\cos\varkappa x, \qquad \tilde{n}_0 = -A\frac{D_i}{D}\cos\varkappa x, \tag{7}$$

where $\kappa^2 = a/D_i = (\pi/2b)^2$ and A is the ion density at x = 0. Here and throughout D_i, D, and a are assumed to be functions of n_0. We note that (7) is the known stationary distribution of the particle density, obtained from the Schottky theory[4] for the positive column of a gas discharge.

4. Let us consider now small perturbation of the system (3) and (4) in the form $\exp i\omega t$. We obtain

$$i\omega\delta n = D\frac{d^2}{dx^2}\delta n - a\delta n_i - \alpha n_{0i}\delta n, \qquad (8)$$

$$i\omega\delta n_i = D_i\frac{d^2}{dx^2}\delta n_i + a\delta n_i + \alpha_i n_{0i}\delta n, \qquad (9)$$

where

$$\alpha = \frac{da}{dn_0} - \frac{dD}{dn_0}\frac{a}{D}, \qquad (10)$$

and α_i is obtained from (10) by replacing D with D_i. We assume that $T_e \gg T$, where T is the temperature of the neutral particles and of the ions. Under this condition, the coefficient of ambipolar diffusion D_i is much larger than the coefficient D of neutral-particle diffusion. We can then assume that those terms of (8) which contain D affect significantly only the density distribution of the neutral particles near the boundary, in a region of thickness $\sim\lambda = \frac{1}{2}(D/\omega)^{1/2}$. We shall consider later separately the distribution of the neutral particles in this region, with allowance for the boundary condition (6), and now we assume that $\lambda \ll b$ and that the perturbation of the neutral particles near the boundary has a negligible effect on the ion-density distribution, and solve the system (8) and (9) assuming the term containing D to be small.

From (8) we obtain δn as a function of δn_i and substitute it in (9). We get

$$D_i\frac{d^2}{dx^2}\delta n_i + (a - i\omega)\delta n_i - \alpha_i n_{0i}\frac{a}{i\omega + \alpha n_{0i}}\left[\delta n_i + D\frac{d^2}{dx^2}\left(\frac{\delta n}{i\omega + \alpha n_{0i}}\right)\right] = 0. \qquad (11)$$

Equation (11) and the boundary condition (5) determine the natural frequency ω of the oscillations. It is easily seen that when $|\omega| \gtrsim a$ Eqs. (11) and (5) have only aperiodically-damped solutions. Let us consider in detail the region $a \gg |\omega| \gg \alpha n_{0i}$.

In this case, Eq. (11) admits of the following solution, which is even in x:

$$\delta n_i = C\left[\cos\varkappa x + \frac{i\omega}{2\varkappa D_i}x\sin x + \frac{\alpha_i A}{2i\omega}\left(1 - D\frac{\varkappa^2}{i\omega}\right) - \frac{1}{6}\frac{\alpha_i A}{i\omega}\left(1 - D\frac{\varkappa^2}{i\omega}\right)\cos 2\varkappa x\right]. \qquad (12)$$

Using the boundary condition (5), which in this case takes the form $\delta n_i(\pm b) = 0$, we obtain for $\omega = \omega' + \omega''$

$$\omega' = \left(\frac{2\pi}{3}\frac{\alpha_i A D_i}{b^2}\right)^{1/2}, \qquad (13)$$

$$\omega'' = \frac{\pi^2}{8}\frac{D}{b^2}. \qquad (14)$$

Under conditions when $a(n) \sim n$ and $D_i \approx \text{const}$, we obtain

$$\omega' \approx \frac{D_i}{b^2}\left(\frac{A}{n_0}\right)^{1/2}. \qquad (15)$$

Thus, the frequency of the ionization-diffusion oscillations in a plane-parallel plasma layer is inversely proportional to the characteristic ion diffusion time, and the damping is inversely proportional to the neutral-particle diffusion time. Formulas (14) and (15), as well as all our other formulas, are valid when

$$A/n_0 \ll D/D_i \ll (A/n_0)^{1/2}. \qquad (16)$$

We note that under condition (16), as was already assumed, the thickness of the boundary region λ is much smaller than the layer width 2b.

Let us find the neutral-particle density distribution. From (8) we obtain for δn the expression

$$\delta n = -\frac{aC}{\varkappa^2 - i\omega}\cos\varkappa x + C_1\left(e^{\varkappa(1+i)/\lambda} + e^{-\varkappa(1+i)/\lambda}\right). \qquad (17)$$

We determine the constant C_1 with the aid of the boundary condition (6), which in our case takes the form

$$D\frac{\partial\delta n}{\partial x}\bigg|_{x=\pm b} = -D_i\frac{\partial\delta n_i}{\partial x}\bigg|_{x=\pm b}.$$

We obtain

$$C_1 = C\frac{D_i}{D}e^{-b(1+i)/\lambda}\varkappa\lambda(1-i). \qquad (18)$$

We note that the ion-number perturbation due to ionization in the boundary layer of thickness λ is small. It can be estimated by using (9) and (18).

In conclusion, we note that the ionization-diffusion oscillations of the particle density considered by us for the case of a plane-parallel plasma layer, can exist also in a volume having a different geometry, particularly cylindrical. These oscillations will occur, for example, in the positive column of a gas discharge.

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