DEPENDENCE OF THE INTERMEDIATE STATE STRUCTURE PARAMETERS ON THE CURRENT

P. TEKEL[1)]

Institute for Physical Problems, USSR Academy of Sciences

Submitted April 29, 1970

Zh. Eksp. Teor. Fiz. 59, 1043–1046 (September, 1970)

The dependence of the structure parameters of the intermediate state on the value of the electric current flowing through a sample is considered. The calculations were carried out for the case of negligibly small Hall effect in the normal phase. The period of the structure and its velocity of motion are increasing functions of the current.

THE problem of the intermediate state, which arises if the value of the current exceeds a critical value, was first considered by London.[1] The structure obtained by him represented alternating immobile layers of superconducting and normal phases, perpendicular to the direction of the current. A different possible structure was proposed by Gorter,[2] in which the phase separation boundaries are coaxial cylinders moving continuously toward the axis of the sample. In the works of Sharvin[3] and Andreev and Sharvin,[4] the possibility of motion of the structure as a whole with a constant velocity was demonstrated and a set of structures was obtained, the limiting examples of which are the London and Gorter structures. Andreev[5] found a realizable structure. Consideration was given to the case in which the current is close to critical. In the present research we shall not limit ourselves to this case.

1. DETERMINATION OF THE MAGNETIC FIELD

Let us consider a cylindrical conductor of radius R, along which flows the current J, which exceeds the critical value. An intermediate state appears in the region $0 < r < r_0$, and the region $r_0 < r < R$ is occupied by the normal phase. On the basis of macroscopic electrodynamics, equations were obtained in[4] for the shape of the phase separation boundaries and their velocity of motion. This velocity, in the direction of the z axis, which coincides with the axis of the sample, is equal to

$$u = \frac{c^2\alpha}{4\pi\sigma r_0}, \tag{1}$$

where σ is the conductivity of the normal phase, α a constant. The equations of the phase separation boundaries in the region $0 < r < r_0$ can be written, for a periodic structure, in the form

$$z_\pm(r) = z_0(r) \pm rd/2r_0, \tag{2}$$

where d is the period of the structure.

In layers of normal phase, in the region considered, the magnetic field, i.e., only the component $H_\varphi(r, z - ut)$ differs from zero, satisfies the equation

$$\frac{\partial^2 H}{\partial r^2} + \frac{1}{r}\frac{\partial H}{\partial r} - \frac{H}{r^2} + \frac{\partial^2 H}{\partial z^2} = 0. \tag{3}$$

The term with the time derivative is omitted because of the smallness of u. If we seek a solution of (3) in the form of an expansion in the small parameter $(z - z_0)/r$ and take into consideration the fact that $H = H_c$ on the phase boundary, we then obtain

$$H = H_c\left[1 + \frac{(z-z_0)^2 - (rd/2r_0)^2}{2r^2(1+z_0'^2)}\right]. \tag{4}$$

In the region $r_0 < r < R$, the thickness of which is much greater than the period of the structure, we can average Eq. (3) over z and use the averaged field $\bar{H}$. Then we get

$$\frac{d}{dr}\left(r\frac{d\bar{H}}{dr}\right) - \frac{\bar{H}}{r} = 0. \tag{5}$$

The solution of the latter equation is

$$\bar{H} = \frac{A}{r} + Br. \tag{6}$$

From the condition of continuity of $\bar{H}$ for $r = r_0$ and the boundary condition for $r = R$ ($\bar{H} = 2J/cR$), we get

$$A = \frac{H_c r_0}{2}\left[1 - \frac{R^2}{R^2 - r_0^2}\frac{d^2}{6r_0^2(1+\alpha^2)}\right], \quad B = \frac{H_c}{2r_0}\left[1 + \frac{r_0^2}{R^2 - r_0^2}\frac{d^2}{6r_0^2(1+\alpha^2)}\right]. \tag{7}$$

2. MINIMIZATION OF THE MAGNETIC ENERGY

We now construct the thermodynamic potential

$$\tilde{\mathcal{F}} = F - \frac{\mathbf{HB}}{4\pi} - F_s,$$

computer per unit length of the sample. Inasmuch as F is the free energy of the region considered and F_s the energy of the superconducting phase, this potential is equal to zero for the superconducting phase. In the region $0 < r < r_0$, we obtain

$$\tilde{\mathcal{F}}_1 = \frac{1}{d}\int_0^{r_0} 2\pi r\,dr \int_{z_-(r)}^{z_+(r)} dz \frac{H_c^2 - H^2}{8\pi} = \frac{1}{24}H_c^2 d^2 I_1(\alpha), \tag{8}$$

where

$$I_1(\alpha) = \frac{1}{\alpha^2} - \frac{1}{\alpha^3}\operatorname{arctg}\alpha.$$

The energy connected with the surface tension at the phase separation boundary is equal to

$$\tilde{\mathcal{F}}_2 = \frac{H_c^2\Delta}{8\pi}\frac{2}{d}\int_0^{r_0} 2\pi r\,dr[1 + z_0'^2]^{1/2} = \frac{H_c^2\Delta r_0^2}{2d}I_2(\alpha), \tag{9}$$

[1)]Institute of Low Temperatures and Structure Investigations of the Polish Academy of Sciences.

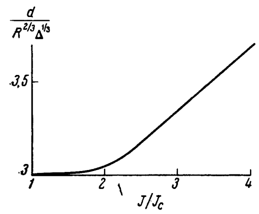

FIG. 1

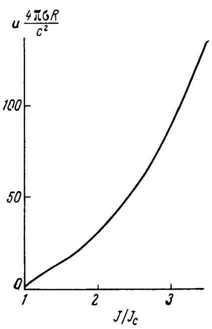

FIG. 2

where

$$I_2(\alpha) = \frac{1}{3\alpha^2}[(1+\alpha^2)^{3/2} - 1].$$

The energy in the region $r_0 < r < R$, which is occupied by the normal phase, is

$$\tilde{\mathcal{F}}_0 = \int_{r_0}^{R} \frac{r\,dr}{4}(H_c^2 - \bar{H}^2). \tag{10}$$

Substituting Eqs. (6) and (7) here, integrating, and then adding the expressions (8) and (9) to the result, we get the magnetic energy. Assuming that $d \ll r_0$, we have

$$\tilde{\mathcal{F}} = \frac{H_c^2 R^2}{8}\left(K + L\frac{1}{x} + Mx^2\right), \tag{11}$$

where

$$x = d/r_0\sqrt{6(1+\alpha^2)}, \tag{12}$$

$$K = \frac{1}{2}\frac{\beta^2}{8} - \frac{3}{8\beta^2} - \frac{\ln\beta}{2\beta^2}, \quad \beta = \frac{R}{r_0},$$

$$L = \frac{4\Delta I_2(\alpha)}{R\beta\sqrt{6(1+\alpha^2)}}, \quad M = \frac{2}{\beta^2}[W + (1+\alpha^2)I_1(\alpha)],$$

$$W = \frac{\beta^2}{8} + \frac{\beta^2\ln\beta}{2(\beta^2-1)} - \frac{3}{8}. \tag{13}$$

Minimizing Eq. (11) in x, we get

$$x_{min} = (L/2M)^{1/3}. \tag{14}$$

The value of the potential (11) at the point of the minimum is

$$\tilde{\mathcal{F}} = \frac{H_c^2R^2}{8}\left(K + \frac{3}{2^{2/3}}L^{2/3}M^{1/3}\right).$$

Minimizing this expression with respect to α for different values of β, we get the curve $\alpha_{min} = \alpha_{min}(\beta)$ or, assuming that

$$\beta = \left\{\left(\frac{J}{J_c}\right) - \left[\left(\frac{J}{J_c}\right)^2 - 1\right]^{1/2}\right\}^{-1},$$

we obtain the curve $\alpha_{min} = \alpha_{min}(J/J_c)$. From this and (14), (12) and (1), with account of (13), we obtain the curves $d = d(J/J_c)$ and $u = u(J/J_c)$, which are plotted in Figs. 1 and 2. For $J/J_c \gg 1$, we can write the formulas

$$\alpha_{min} \approx \frac{4}{\pi}\left(\frac{J}{J_c}\right)^2, \quad d \approx \frac{4}{\pi}\left(\frac{J}{J_c}\right)^{2/3} R^{2/3}\Delta^{1/3},$$

$$u \approx \frac{c^2}{4\pi\sigma R}\frac{8}{\pi}\left(\frac{J}{J_c}\right)^3.$$

I express my gratitude to A. F. Andreev for suggesting the theme and for his interest in the research.

[1] F. London, Superfluids, 1, New York—London, 1950.
[2] C. J. Gorter, Physica 23, 45 (1957).
[3] Yu. V. Sharvin, Works (Trudy) of the 10th International Conference on Low Temperature Physics, 2B, All-union Inst. Sci. Tech. Information, p. 323.
[4] A. F. Andreev and Yu. B. Sharvin, Zh. Eksp. Teor. Fiz. 53, 1499 (1967) [Soviet Phys.-JETP 26, 865 (1968)].
[5] A. F. Andreev, Zh. Eksp. Teor. Fiz. 54, 1510 (1968) [Soviet Phys.-JETP 27, 809 (1968)].

Translated by R. T. Beyer
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ERRATUM

Formula (19) of the translation of the article by R. A. Zhitnikov, P. P. Kuleshov, A. I. Okunevich, and B. N. Sevast'yanov (Sov. Phys.-JETP 31, No. 3, 1970, p. 448), should read

$$\Delta(\Phi) = \begin{cases} 1 \text{ for } \Phi = \Phi_{max} \text{ and a right-polarized RF field,} \\ 1 \text{ for } \Phi = \Phi_{min} \text{ and a left-polarized RF field,} \\ 0 \text{ for all remaining cases.} \end{cases}$$

The translation editor regrets this error.