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MEASUREMENT OF THE WOLFENSTEIN PARAMETER IN pn *SCATTERING AT* 605 MeV

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The Wolfenstein parameter A and the polarization P in pn scattering at 605 MeV were measured at angles 55, 78, 94, and 125° (c.m.s.). The results (Table I) were used to refine two existing sets of phase shifts from the phase shift analysis of NN scattering at 630 MeV[6]. Using the τ test[14,15], the probability of an error of the first kind is estimated if the solution with the large χ^2 (solution II from[6]) is discarded; this error turned out to be $\alpha < 0.01\%$.

THE amplitude for elastic scattering of nucleons by nucleons at energies 50—400 MeV was uniquely determined within the framework of the phase-shift analysis[1]. Above the threshold of meson production, when this process begins to play an important role in NN interaction, the energy interval closest to the non-ambiguity region and most thoroughly investigated experimentally is the one from 600 to 650 MeV. A phase-shift analysis referred to the energy 630 MeV and valid, accurate to within the method used to take inelastic processes into account, was carried out in this case many times[2,3]. As data accumulated in accordance with the previously mentioned program[4] and as statistical selection criteria[5] more stringent than the χ^2 criterion were introduced, the number of solutions decreased continuously. At the present there are two known sets of phase shifts that describe the results of the experiments equally well[6]. These sets have made it possible to plan further experiments for the purpose of eliminating the existing ambiguities. Prevalent among these is the measurement of the Wolfenstein parameter A in pn scattering, since the angular dependences of this parameter, calculated on the basis of the indicated sets of phase shifts, differ noticeably from each other (Fig. 1). This circumstance, and also the fact that this parameter was not measured earlier in the investigated energy interval, stimulated the performance of the experiment described below.

EXPERIMENT

The scattering cross section I_2 and the polarization $\langle\sigma\rangle_f$ of a beam of particles with spin 1/2, after scattering by an unpolarized target with spin 1/2 are given by

Table I

	ϑ(c.m.s.)			
	55°	78°	94°	125°
A_{pn}	0.88±0.18	0.64±0.20	0.75±0.14	0.70±0.21
P_{pn}	0.11±0.06	−0.15±0.07	−0.23±0.04	−0.30±0.08
Number of events	12532	5262	11386	10950

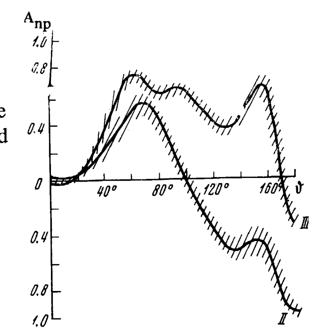

FIG. 1. Angular dependence of the Wolfenstein parameters A_{np} calculated from the phase-shift sets II and III of [7]. The hatched regions indicate the error corridors. ϑ is the scattering angle in the c.m.s.

the well known Wolfenstein formula[7]*

$$I_2\langle\bar{\sigma}\rangle_f = I_{02}\{(P_2 + D\mathbf{P}_1\mathbf{n}_2)\mathbf{n}_2 + (A\mathbf{P}_1\mathbf{k}_2 + R\mathbf{P}_1[\mathbf{n}_2\mathbf{k}_2])\mathbf{S}_2 + (A'\mathbf{P}_1\mathbf{k}_2 + R'\mathbf{P}_1[\mathbf{n}_2\mathbf{k}_2])\mathbf{k}_2'\},$$

where $\mathbf{P}_1$ is the initial polarization of the beam; $\mathbf{k}_2$ and $\mathbf{k}_2'$ are unit vectors in the direction of the momentum before and after the scattering; $\mathbf{n}_2$ is the normal to the scattering plane; $\mathbf{P}_2$ is the polarization occurring when an unpolarized beam is scattering; $\mathbf{S}_2 = \mathbf{n}_2 \times \mathbf{k}_2'$; A, D, R, A′ and R′ are the Wolfenstein parameters; $\mathbf{I}_{02}$ is the cross section for the scattering of the unpolarized beam.

*$[\mathbf{n}_2\mathbf{k}_2] \equiv \mathbf{n}_2 \times \mathbf{k}_2$.

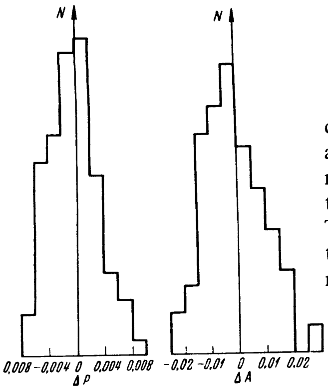

FIG. 3. Errors introduced in the determination of the parameters P and A by the inaccuracy of measurement of the angle of scattering by the analyzer and the spark chambers. The histograms were obtained by the Monte Carlo method. N is the number of events.

tering were calculated from the formulas

$$A_{pn} = (A_{CD_2} - KA_{CH_2}) / (1 - K),$$
$$P_{pn} = (P_{CD_2} - KP_{CH_2}) / (1 - K),$$

where K is the ratio of the number of events from a CH_2 target to the number of events from a CD_2 target, normalized to an equal number of monitor counts. The value of K turned out to be 0.26 ± 0.01, 0.23 ± 0.01, 0.25 ± 0.01, and 0.26 ± 0.01 for the angles 55, 78, 94, and 125°, respectively. Table I gives the values of A_{pn} and P_{pn} obtained as a result of the reduction of 40,130 events. Approximately one-quarter of all the events pertains to the measurement of the background (CH_2 or C).

In addition to the statistical errors, there are also errors in the determination of P_1 and P_3; the latter were taken into account during the reduction process by the maximum-likelihood method. The contribution due to the inaccuracies in the measurement of the projections of the scattering angle ϑ_3, which has a normal distribution, as established by two independent scannings, was estimated by the Monte Carlo method. To this end, the distribution of $I_3(\vartheta_3, \varphi_3)$ was generated at A = 0 and $P_2 = 0$ (10,000 events) with the same dependence on ϑ_3 as obtained in the experiment. This distribution was then deformed in each test in accordance with the algorithm

$$\left.\begin{matrix}\vartheta_3^i\\ \varphi_3^i\end{matrix}\right\} \to \left.\begin{matrix}\theta_1^i\\ \theta_2^i\end{matrix}\right\} \to \left.\begin{matrix}\theta_1^i + \eta_j\Delta\theta\\ \theta_2^i + \eta_{j+1}\Delta\theta\end{matrix}\right\} \to \begin{matrix}\vartheta_3^{i\prime}\\ \varphi_3^{i\prime}\end{matrix}$$

where $\Delta\theta = 0.7°$ is the variance in the distribution of the measurement error of the projections of θ_1 and θ_2; η_j are random numbers with a normal distribution and unity variance; i assumed values from 1 to 10,000 and j from 1 to 20,000 n, where n is the number of tests. For a distribution deformed in this manner we determined each time the parameters A and P_2. The distributions of A and P_2, obtained after 100 tests, are shown in Fig. 3. We see that their variances do not exceed 0.015 and 0.004, respectively.

CONCLUSION

The obtained values of A_{pn} and P_{pn} were used to refine the phase-shift sets obtained in[6]. We included additionally in the analysis the data on the total cross section of the elastic interaction σ_{tot}^{in}, which is equal to

$$\sigma_{tot}^{in} = \frac{\pi}{2k^2}\sum_L\sum_J (2J+1)(1 - \exp\{-4\,\mathrm{Im}\,\delta_{LJ}\}).$$

This expression is valid with the same accuracy that the factor $\cos^2 2\epsilon_2$ ($\epsilon_2 \sim 3°$) can be replaced by unity in front of the terms of the sum corresponding to the mixing states $^3P_2$ and $^3F_2$. The two sets were refined by independently varying the imaginary parts of the phase shifts for the states $^3P_{0,1,2}$, $^1D_2$, and $^3F_{2,3,4}$. It turned out that $\mathrm{Im}\,\delta_{^3P_0}$ and $\mathrm{Im}\,\delta_{^3P_1}$ go off during the course of minimization of the functional χ^2 into the unphysical region (they become negative). In this case they were assumed equal to zero and their values fixed.

During the course of the refinement we found that the aggregate of the experimental information on NN scattering in the energy interval 600–650 MeV is described by the phase-shift sets III and II from[6], with χ^2 values 198.5 and 248.3, respectively, and with $\bar{\chi}^2 = 191$. The contribution made to $\Delta\chi^2 = \chi^2_{II} - \chi^2_{III} = 49.8$ from the values of A_{pn} measured in the present experiment is 31.2.

Using the τ-test[14] as applied to the phase-shift analysis[15], we estimated the probability of an error of the first kind with the solution with large χ^2 (set II) discarded. It turned out to be $\alpha < 0.001\%$. Thus, assuming that the elastic part of the NN-scattering amplitude for the states with L > 5 is described satisfactorily by a one-meson Feynman diagram and that the meson production proceeds mainly from the states $^3P_{0,1,2}$, $^1D_2$, and $^3F_{2,3,4}$, the solution II can be discarded. However, as already mentioned earlier[16], the conclusion that the phase-shift analysis is single-valued in the investigated energy region depends to a considerable degree on

Table II. Phase-shift sets II and III after their refinement (Stapp parametrization)

Phase shift	Set II, χ^2 = 248.3	Set III, χ^2 = 198.5
f^2	0,058±0,007	0,076±0.005
$^1S_0$	—29.20±3.49	—19.81±3.38
$^3S_1$	—6,01±7.43	—17,40±5.44
$^3P_0$	—50.34±7.59	—20.69±2,74
$^1P_1$	—39.66±10.49	—27,30±5.39
$^3P_1$	—39.80±4.49	—29,95±2.19
$^3P_2$	16.75±1.83	34.82±1.77
ε_1	20.67±4.91	9.48±5.15
$^3D_1$	32.82±10.11	—27.30±2.41
$^1D_2$	5.48±2.60	9.26±1.58
$^3D_2$	21,00±4.36	22.53±3.16
$^3D_3$	0.15±2.77	—8.87±1.82
ε_2	—3.44±1.43	3.00±0.89
$^3F_2$	—6.61±1.36	—4.20±0.60
$^1F_3$	2,19±2.69	—6.36±2.06
$^3F_3$	—1.20±1.82	0.69±0.77
$^3F_4$	3.40±0.64	3.64±0.81
ε_3	8.68±3.14	9.79±1.70
$^3G_3$	1.41±2.15	—6.13±2.17
$^1G_4$	4.84±0.74	5.52±0,66
$^3G_4$	1.10±2.24	6.02±1.36
$^3G_5$	—1.90±1.12	—7,02±1.17
ε_4	—1,23±0.78	0.76±0.76
$^3H_4$	1.49±0.69	—2.15±0.60
$^1H_5$	—2,54±0.94	—6.24±1.44
$^3H_5$	—3.22±0.72	—3.21±0.79
$^3H_6$	1.48±0.28	—2,70±0.45
Im $^3P_0$	0 fixed	0 fixed
Im $^3P_1$	1.50±4.6	0 fixed
Im $^3P_2$	3,04±1,87	5.37±1.52
Im $^1D_2$	6,48±3.71	10.96±2.56
Im $^3F_2$	1.29±2.28	0.57±1.07
Im $^3F_3$	8.69±2.91	2.32±1.52
Im $^3F_4$	1.24±1.01	4.14±0,73

If we choose the experimental conditions such that $\mathbf{P}_1 \parallel \mathbf{k}_2$, then, recognizing that $\mathbf{P}_1 \perp \mathbf{P}_2$ and that $I_2 = I_{02}(1 + \mathbf{P}_1\mathbf{P}_2)$ implies $I_2 = I_{02}$, we obtain

$$\langle\bar{\sigma}\rangle_f = \mathbf{P}_2 \pm P_1 A \mathbf{S}_2 \pm P_1 A' \mathbf{k}_2'.$$

After the succeeding, analyzing scattering, the angular distribution of the scattered particles is

$$I_3 = I_{03}(1 + \langle\bar{\sigma}\rangle_f \mathbf{P}_3)$$

or

$$I_3(\vartheta_3, \varphi_3) = I_{03}(\vartheta_3)\{1 + P_3(\vartheta_3) \times [P_2(\vartheta_2)\cos\varphi_3 \pm A(\vartheta_2) P_1 \sin\varphi_3]\}.$$

Here ϑ_3, φ_3, $P_3(\vartheta_3)$, and $I_{03}(\vartheta_3)$ are respectively the polar and azimuthal angles, the analyzing ability, and the cross section for scattering of the unpolarized beam by the analyzer, φ_3 is the angle between $\mathbf{S}_2$ and the plane of the analyzing scattering, reckoned from $\mathbf{S}_2$ to $\mathbf{P}_3$. The signs plus and minus correspond to antiparallel or parallel mutual orientation of the vectors $\mathbf{P}_1$ and $\mathbf{k}_2$.

By measuring the cross section $I_3(\vartheta_3, \vartheta_3)$ we can determine the parameter of interest to us either from the "up-down" asymmetry or by using the maximum-likelihood method[8].

The experiment was performed with a longitudinally polarized proton beam from the JINR synchrocyclotron[9]. The beam polarization was 0.34 ± 0.02, the average energy was 605 ± 9 MeV. At scattering angles 55, 78, 94, and 125° (c.m.s.), corresponding to the elastic kinematics of scattering of free nucleons by nucleons, we measured the difference of the effects from targets made of CD_2 and CH_2 with equal numbers of carbon nuclei. The spin state of the scattered proton beam was analyzed by scattering by carbon plates placed inside optical spark chambers. The analyzer thicknesses were chosen such that the angle of multiple scattering in them did not exceed 2°, and amounted to (9.5 ± 9.5) g/cm² for scattering angles 55, 78, and 94° and (5.7 + 4.8) g/cm² for the angle 125°. The photography was in two mutually perpendicular planes. The construction of the setup (Fig. 2) has made it possible to perform measurements simultaneously at two angles.

The separation of the acts of elastic pn scattering and control of the operation of the spark chambers was with the aid of scintillation counters connected in accordance with the schemes $C_1 + C_2 + H_1 - A_1$ and $C_3 + C_4 + H_2 - A_2$, respectively, for one and the other scattering angle, where C_1, C_2, C_3, and C_4 are counters registering the scattered protons, H_1 and H_2 are the neutron counters installed at conjugate angles, and A_1 and A_2 are anticoincidence counters which eliminate registration of pp scattering. The angular resolution of the proton branch was ±1° for the angles 55, 78, and 94° and ±1.5° for 125°. The efficiency of the neutron counters was approximately 10%. Anticoincidence-channel operation was monitored by additional spark chambers placed ahead of the neutron counters.

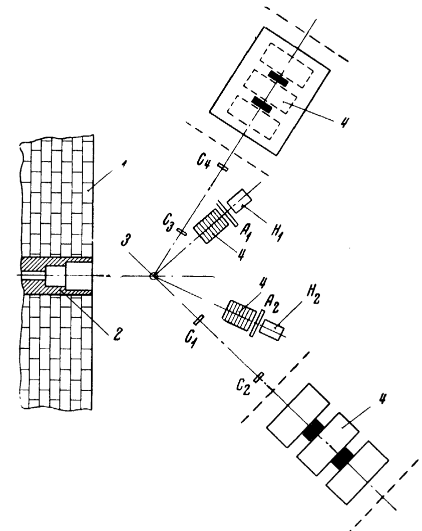

FIG. 2. Experimental setup: 1) Shield, 2) collimator, 3) scatterer, 4) spark chambers, C_1-C_4) proton counters, H_1, H_2) neutron counters, A_1, A_2) anticoincidence counters.

DATA REDUCTION

In reducing the spark-chamber photographs we used the following selection criteria:

1) The presence of a single track in the main chamber and its absence in the control chamber;

2) a kink in the track corresponds to the position of the analyzer (and not of the chamber electrodes);

3) the events to be measured are those in which the sum of the projections of the scattering angle is larger than 3° ($\theta_1 + \theta_2 > 3°$).

During the stage of the preliminary computer reduction, initial requirements were imposed:

a) The deviation of the direction of entry of the particle into the chamber from the direction averaged over the nearest 200 events should not exceed the angle $\Delta\vartheta = 4.5°$ determined by the geometry of the experiment.

b) The analyzing-scattering angle $\vartheta_3 \geq 6°$.

The first of these requirements excludes registration of charged particles not arriving from the target. The second requirement establishes the lower limit with respect to ϑ_3, at which the reduction yields results that are stable within the limits of errors.

The final reduction of the events satisfying the indicated criteria was carried out by the maximum-likelihood method. The likelihood function was written in the form

$$L = \prod_i \{1 + P_3(\vartheta_3^i)[P_2(\vartheta_2)\cos\varphi_3^i \pm P_1 A(\vartheta_2)\sin\varphi_3^i]\}.$$

Measurements with different mutual orientations of the vectors $\mathbf{P}_1$ and $\mathbf{k}_2$ (the signs +, −) make it possible to exclude the possible systematic errors connected with the adjustment of the apparatus and with the systematic errors in the measurement of the projections of the scattering angles on the photographs.

The analyzing ability $P_3(\vartheta_3)$ in the reduction of the information for the angle 125° was taken to be the polarization produced in scattering of protons by carbon, averaged over the energy[10,11,3]. The basis for this was the good agreement between the angular distributions of the pC scattering, measured in our work and in the cited papers. For the angles 55, 78, and 94° the data on $P_3(\vartheta_3)$ were taken from[12,13], in which the experimental conditions were close to ours.

We determined simultaneously the two parameters A and P_2. The values of these parameters for pn scat-

the assumptions made concerning the meson-production mechanism. In this connection, the uniqueness problem possibly requires additional investigations.

Table II lists the phase-shift sets II and III after their refinement using the results of the present paper.

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