TRANSVERSE MODE LOCKING IN A SOLID-STATE LASER

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The features of simultaneous generation of several transverse modes in a neodymium-glass laser are investigated. The transverse-mode locking regime is studied. Locking and capture of transverse modes due to nonlinear properties of the active medium are considered theoretically.

1. INTRODUCTION

MODE locking in a laser, i.e., the establishment of definite time-independent phase relations between the generating modes, is attained as a rule in a forced manner (for example, by introducing time-modulated losses into the resonator[1,2] or as a result of the nonlinear properties of a passive cell introduced into the resonator[3,4]). In some cases, however, mode synchronization can occur spontaneously, owing to nonlinear properties of the active laser medium.[5,6] We note that such axial-mode locking in a neodymium-glass laser is apparently an event of low probability.[7]

Either axial or transverse modes can become locked. In [8] it was indicated for the first time that the synchronization of transverse modes should lead to the appearance of a wave "packet" executing oscillations in the transverse plane of the resonator. The locking of two or three transverse modes was observed experimentally in He-Ne and CO_2 lasers following introduction of a nonlinear absorber into the resonator.[9–11] Locking of transverse modes has been considered until recently as having low probability.[8] In our paper[12] we reported that under certain conditions locked generation of transverse modes is observed in a neodymium-glass laser.

The necessary condition for the locking of the generating modes is that their frequencies be equidistant. The frequency intervals between the modes in the presence of an active medium are determined, apart from the resonant properties of the passive resonator of the laser, by two principal mechanisms: the mode frequency pulling because of the anomalous dispersion in the medium,[13] and by the interaction between the equidistant combination polarization frequencies induced in the nonlinear active medium with the resonant frequencies of the transverse modes, which in the general case are not equidistant. Under certain conditions this interaction can ensure equidistance of the frequencies of all the generated modes, especially in solid-state lasers, in which the width of the resonance contours of the transverse modes is appreciable ($\sim 10^7$ sec^{-1}).

For self-locking of transverse modes it is necessary to generate many transverse modes simultaneously. A large number of transverse modes can be excited if the aperture is sufficiently large. In addition, it is necessary to eliminate from the laser all the causes of discrimination of transverse modes. Such causes may be the non-uniformity in the distribution of the inversion over the cross section of the resonator, aberrations in the resonator, sphericity of the mirrors, etc.

We present here the results of an experimental investigation of the regime of simultaneous generation of many transverse modes and their locking in a neodymium-glass laser having a resonator of large cross section. We analyze theoretically the conditions necessary for the synchronization and capture of frequencies of several transverse modes as a result of the nonlinear properties of the active medium.

2. THEORETICAL ANALYSIS

We confine ourselves in the analysis to the case when three modes with identical axial but different transverse indices generate in a resonator with flat mirrors. In the case of simultaneous generation of three transverse modes with closed frequencies, three cases can be realized:

1) Each mode generates in the center of its resonant contour and the mode frequencies are not equidistant.

2) Synchronized mode generation. In this case the combination tone of the polarization resulting from the nonlinear properties of the medium under the influence of the frequencies ω_1 and ω_2 acts as a driving force within the limits of the resonant contour of the mode with frequency ω_3, and causes the latter to generate at a frequency $\omega_2 + \Delta\omega \neq \omega_3$, where $\Delta\omega = \omega_2 - \omega_1$.

3) Regime in which all mode frequencies are captured, and generation occurs at a single frequency.

Let us estimate the width of the frequency region within which it is possible to shift the frequency of the mode ω_3 by an amount that is equidistant relative to the modes with frequencies ω_1 and ω_2 as a result of the nonlinear properties of the laser medium, and let us also consider the conditions that lead to the capture and synchronization of these modes. We carry out the analysis using Lamb's method[14] without taking into account the motion of the atoms, but taking into account the transverse structure of the mode fields in the resonator.

The resonator mirrors will be represented in the form of two infinite bands of width D, the distance between which is L. The origin of the Cartesian coordinates is located on the surface of one of the mirrors, and the z axis is directed along the resonator axis. We assume for simplicity that a two-level active medium fills the entire resonator, and that the modes are linearly polarized.

We represent the field and the polarization of the medium in the form of a series in the natural modes of a planar resonator

$$E(x,z,t) = \sum_l E_l(t) U_{m_l q_l}(x,z) \cos[\omega_l t + \varphi_l(t)],$$

$$P(x,z,t)=\sum_l P_l(t)\,U_{m_l q_l}(x,z),$$

$$P_l(t)=\frac{4}{LD}\int_0^L\int_0^D P(x,z,t)\,U_{m_l q_l}(x,z)\,dz\,dx,$$

$$P_l(t)=C_l(t)\cos[\omega_l t+\varphi_l(t)]+S_l(t)\sin[\omega_l t+\varphi_l(t)],$$

where

$$U_{m_l q_l}(x,z)=\sin\frac{m_l\pi}{D}x\sin\frac{q_l\pi}{L}z$$

are the eigenfunctions (not normalized) of the modes of the planar resonator with a transverse index m_l and an axial index $q_l = q$.

The equations connecting the field and the polarization are Maxwell's equations, which in the approximation of slowly-varying amplitudes and phases can be reduced to the form[14]

$$(\omega_l+\dot\varphi_l-\Omega_l)E_l=-\frac{1}{2}\frac{\omega}{\varepsilon}C_l,$$
$$\dot E_l+\frac{1}{2}\frac{\omega}{Q_l}E_l=-\frac{1}{2}\frac{\omega}{\varepsilon}S_l. \tag{1}$$

Here E_l, ω_l, and φ_l are the amplitude, frequency, and phase of the l-th mode, Ω_l and Q_l are the natural frequency and the Q factor of the resonator for the l-th mode, ω is the central radiation frequency, and ϵ is the dielectric constant of the medium. The coefficients S_l and C_l can be determined with the aid of perturbation theory, by representing the polarization of the medium in the form of a series in powers of the mode amplitudes E_μ, a procedure valid for low pump levels. In first order in perturbation theory we obtain for the projection of the polarization on the l-th mode

$$P_l^{(1)}(t)=-\frac{id_{12}^2(n_2-n_1)}{2\hbar}\sum_\mu E_\mu(t)\frac{1}{i(\omega-\omega_\mu)+T_{21}^{-1}}N_{l\mu}$$
$$\times\exp[-i(\omega_\mu t+\varphi_\mu)]+\text{c.c.} \tag{2}$$

$$N_{l\mu}=\frac{4}{LD}\int_0^L\int_0^D U_{m_l q_l}(x,z)\,U_{m_\mu q_\mu}(x,z)\,dz\,dx=\delta_{l\mu},$$

where T_{21} is the relaxation time of the non-diagonal elements of the density matrix, $T_{21}^{-1}=\frac{1}{2}(T_{22}^{-1}+T_{11}^{-1})$, T_{22} and T_{11} are the relaxation times of the upper and lower working levels; d_{12} is the matrix element of the dipole moment of the active center, (n_2-n_1) is the initial excess of atom density at the upper working level (assumed to be homogeneous over the length and cross section of the laser), and $\hbar$ is Planck's constant.

We note that in the analysis of the interaction of modes that are close in frequency, the inhomogeneous character of the luminescence-line broadening of such active media as neodymium glass can be disregarded in practice. In the third order of perturbation theory we obtain analogously

$$P_l^{(3)}(t)=\frac{id_{12}^4(n_2-n_1)}{8\hbar^3}\sum_{\sigma,\mu,n}E_\sigma E_\mu E_n \tag{3}$$
$$\times\left\{\frac{1}{i(\omega_\mu-\omega_n)+T_{22}^{-1}}+\frac{1}{i(\omega_\mu-\omega_n)+T_{11}^{-1}}\right\}$$
$$\times\left\{\frac{1}{i(\omega-\omega_n)+T_{21}^{-1}}+\frac{1}{i(\omega_\mu-\omega)+T_{21}^{-1}}\right\}\frac{N_{\sigma\mu nl}}{i(\omega-\omega_\sigma-\omega_\mu+\omega_n)+T_{21}^{-1}}$$
$$\times\exp[-i(\omega_\sigma-\omega_\mu+\omega_n)t-i(\varphi_\sigma-\varphi_\mu+\varphi_n)]+\text{к. с.},$$

$$N_{\sigma\mu nl}=\frac{4}{LD}\int_0^L\int_0^D U_{m_\sigma q_\sigma}U_{m_\mu q_\mu}U_{m_n q_n}U_{m_l q_l}\,dz\,dx$$
$$=\frac{3}{16}(\delta_{\sigma+\mu,\,n+l}+\delta_{\sigma+n,\,\mu+l}+\delta_{\sigma+l,\,\mu+n}-\delta_{\sigma+\mu+n,\,l}-\delta_{\sigma+l+n,\,\mu}-\delta_{\mu+n+l,\,\sigma}).$$

Here $\sigma, \mu, n, l = 1, 2, 3$, $q_\sigma = q_\mu = q_n = q_l = q$.

Using (2) and (3), we can determine the coefficients C_l and S_l in the first and third orders of perturbation theory. In the first order of perturbation theory we can estimate from (1), for the stationary generation regime ($\varphi_l = E_l = 0$), the frequency pulling of the l-th transverse mode towards the center of the luminescence line:

$$\omega_l-\Omega_l=\frac{1}{2}\frac{\omega}{Q_l}\frac{\omega-\omega_l}{T_{21}^{-1}}.$$

For a neodymium-glass laser we have $T_{21}\sim 10^{-12}$ sec, and therefore for transverse modes whose frequencies are close to the line center, $\omega-\omega_l\sim 10^7\ \text{sec}^{-1}$ and $\omega/2Q_l\sim 10^7\ \text{sec}^{-1}$, the amount of pulling is small ($\sim 10^2\ \text{sec}^{-1}$) and this effect can be neglected in the subsequent analysis.

In third-order perturbation theory we can estimate the width of the region of "capture" and "synchronization" of the mode frequencies. Confining ourselves for simplicity to the "capture" of two transverse modes with different transverse indices (1 and 2) and equal values of Q, we can rewrite the system (1) for such a single-frequency regime in the form

$$(\dot\varphi_1+\delta\omega_1-\theta)E_1=-\varkappa NE_1E_2^2\sin2\Delta\varphi,$$
$$(\dot\varphi_1+\delta\omega_1-2\Delta\Omega-\theta)E_2=\varkappa NE_1^2E_2\sin2\Delta\varphi,$$
$$\dot E_1=\alpha E_1-\varkappa N_{1111}E_1^3-\varkappa NE_1^2E_2(2+\cos2\Delta\varphi),$$
$$\dot E_2=\alpha E_2-\varkappa N_{2222}E_2^3-\varkappa NE_1E_2^2(2+\cos2\Delta\varphi), \tag{4}$$

where $\Delta\varphi=\varphi_2-\varphi_1$, $\omega_1=\omega_2$, $\delta\omega=\omega_1-\Omega$, $\Delta\Omega=(\Omega_1-\Omega_2)/2$,

$$\varkappa=\frac{1}{4}\frac{\omega}{Q}\frac{T_{22}+T_{11}}{T_{21}}\frac{d_{12}^2}{\hbar^2}[(\omega-\omega_1)^2+T_{21}^{-2}]^{-1}R,$$
$$N_{2121}=N_{1221}=N_{2211}=N_{1122}=N_{1212}=N_{2112}=N={}^3/_8,$$
$$N_{1111}=N_{2222}={}^9/_{16},\qquad \theta=\frac{1}{2}\frac{\omega}{Q}\frac{(\omega-\omega_1)}{T_{21}^{-2}}R,$$
$$\alpha=\frac{1}{2}\frac{\omega}{Q}\left[\frac{T_{21}^{-2}}{(\omega-\omega_1)^2+T_{21}^{-2}}R-1\right],$$

R is the excess of pump over threshold.

The system (4) contains in the stationary case four equations for three unknown quantities (E_1, E_2, $\Delta\varphi$). Let us determine at which parameters this system has a simultaneous solution, which is in fact the mode-frequency "capture" regime. To this end, we can easily obtain from (4)

$$\frac{\alpha}{\Delta\Omega}\sin2\Delta\varphi+\cos2\Delta\varphi=-\left(\frac{N_{1111}}{N}+2\right). \tag{5}$$

It is seen from (5) that the width of the "capture" region $2\Delta\Omega$ depends on the phase difference between the captured modes $\Delta\varphi$. Its largest width is reached at

$$\Delta\varphi=\frac{1}{2}\arcsin\left[-\frac{\{(N_{1111}/N+2)^2-1\}^{1/2}}{(N_{1111}/N+2)}\right]=-36.5^\circ.$$

It follows therefore that the "capture" is possible under the condition

$$\Delta\Omega<\alpha\left\{\left(\frac{N_{1111}}{N}+2\right)^2-1\right\}^{-1/2}=$$
$$=\frac{1}{2}\frac{\omega}{Q}\left[\frac{T_{21}^{-2}}{(\omega-\omega_1)^2+T_{21}^{-2}}R-1\right]\left\{\left(\frac{N_{1111}}{N}+2\right)^2-1\right\}^{-1/2}.$$

A numerical estimate in accordance with formula (6) at $R = 2$ for a neodymium-glass laser ($\tau_{21} \sim 10^{-2}$ sec, $(\omega - \omega_1)^2 \ll T_{21}^{-2}$) yields for $\Delta\nu_{\text{capt}}$ a value on the order of 10^6 sec^{-1}. However, "capture" in such a wide band is possible only at an optimal relation between phases of the generated modes, i.e., when $\Delta\varphi \sim 36°$. Such a phase relation is not always realized. In the case of synchronization of three modes as a result of the nonlinear properties of the laser medium, the phase relations established between the modes are such that their phase differences are multiples of π.[15] As seen from (6), in this case (i.e., when $\Delta\varphi \approx k\pi$, k = 0, 1, 2) the width of the "capture" region is very small, on the order of 10^2 sec^{-1} at $\Delta\varphi \sim 10^{-4}$.

A rigorous analysis of the interaction of three transverse modes the frequency intervals between which are much smaller than the widths of their resonant contours entails great difficulties. It is clear, however, that the width of the frequency region within which the frequency of one of the modes can be shifted to a position that is equidistant relative to the other modes (i.e., the width of the synchronization regime) should have the same order of magnitude as the maximum width of the "capture" region.

An estimate of the synchronization band of three modes and of the most favorable conditions for this purpose can be obtained in the following manner. Formula (3) can be used to determine the combination tone of the polarization $P^{(3)}[(\omega_2 + \Delta\omega)t]$ produced under the influence of the frequencies ω_1 and ω_2 in a nonlinear laser medium near the frequency ω_3, which corresponds to the third transverse mode of the resonator:

$$P^{(3)}[(\omega_2+\Delta\omega)t] = \frac{id_{12}^4(n_2-n_1)}{8\hbar^3}E_2^2E_1 \times \exp\{-i[(\omega_2+\Delta\omega)t+(2\varphi_2-\varphi_1)]\}$$
$$\times\left(\frac{1}{i(\omega_2-\omega)+T_{11}^{-1}}+\frac{1}{i(\omega_1-\omega_2)+T_{22}^{-1}}\right)$$
$$\times\left(\frac{1}{i(\omega-\omega_2)+T_{21}^{-1}}+\frac{1}{i(\omega-\omega_1)+T_{21}^{-1}}\right)\frac{1}{i(\omega_1+\omega)+T_{21}^{-1}}N_{2123}+\text{c.c.}$$

If we regard $P^{(3)}[(\omega_2 + \Delta\omega)t]$ as a driving force acting on a tank circuit with central frequency ω_3, then the permissible frequency difference $\delta\omega$ between ω_3 and $\omega_2 + \Delta\omega$, within which oscillations in the circuit will occur at the driving-force frequency, can be determined from the formula (see [16])

$$\delta\omega = \frac{\omega}{2\varepsilon_0}\frac{|P^{(3)}(\omega_2+\Delta\omega)|}{|E_3(\omega_3)|}, \tag{7}$$

where $E_3(\omega_3)$ is the amplitude of the tank-circuit oscillations in the absence of the driving force. Substituting in (7) the value of $P^{(3)}(\omega_2 + \Delta\omega)$, we obtain

$$\delta\omega = \frac{\omega}{4\varepsilon}\frac{d_{12}^4(n_2-n_1)T_{21}^2N_{2123}}{\hbar^3\Delta\omega}\frac{E_1E_2^2}{E_3}. \tag{8}$$

To estimate $\delta\omega$ it is necessary to know E_1, E_2, and E_3. Usually on changing over to transverse modes of higher order, the resonator losses for them increase rapidly. Let us assume that for the mode with the transverse index 3 the threshold pump power has been exceeded by 1.5–2 times. From (1)–(3) we can then obtain for the amplitude of this mode, assuming that it generates alone,

$$E_3 = \left(\frac{3{,}55\hbar^2}{d_{12}^2T_{22}T_{21}}\frac{R_3-1}{R_3}\right)^{1/2}, \tag{9}$$

where R_3 is the excess of the threshold pump power for the mode with the transverse index $m_l = 3$. In the derivation it was assumed that $\omega - \omega_3 \ll T_{21}^{-1}$ and $T_{11} \ll T_{22}$. Substituting in (9) parameters typical of neodymium glass ($d_{12}^2 \approx 2\times10^{-38}$ cgs esu, $T_{22} = 5\times10^{-4}$ sec, $T_{21} \sim 10^{-12}$ sec), we obtain $E_3 \sim 0.4$ cgs esu at R = 2. If we assume that the amplitudes of the modes with the transverse indices m = 1 and m = 2, when generated independently, have a value $E_1 \sim E_2 \sim 3$–5 cgs esu (since the resonator losses for them are much smaller), then we obtain from (9) for the width of the synchronization region at $\Delta\omega \sim 6\times10^6$ sec^{-1} and $n_2 - n_1 \sim 10^{16}$ cm^{-3} the value $\delta\omega/2\pi \sim 10^6$ sec^{-1}.

Thus, an estimate shows that synchronization of a small number (3–5) of transverse modes in a solid-state laser is possible even when the maxima of the resonant curves of these modes are far from equidistant. Most favorable for synchronization are conditions in which the driving combination tone of the polarization is strong (this is caused, in particular, by an increase in the excess over the threshold) and the synchronized modes are weaker than the remaining ones, as is usually the case for transverse modes with increasing order of the mode.

3. EXPERIMENTAL PROCEDURE

The experiments were performed with a laser using active rods of rectangular cross section with 2% Nd_2O_3. The transverse dimensions of the active samples were 120 × 15 mm, and the length was 150 mm (Fig. 1). The pump system ensured uniform distribution of the inverted population over the cross section in the direction of the x axis. The distribution of the inversion along the y axis was uneven, being somewhat larger on the edges of the active element than at the center. The aperture of the resonator was varied with an inserted diaphragm whose dimension along the x axis ranged from 5 to 120 mm, and amounted to 2–2.5 mm along the y axis. With such diaphragm dimensions, the distribution of the inversion over the y axis remained practically uniform within the limits of the investigated apertures.

The pump energy was varied from 3 to 6 kJ (the maximum pump to threshold energy ratio was n ≈ 4). The pump pulse duration was ~700 μsec. The resonator consisted of two flat mirrors with transmissions $\tau_1 = 0.5\%$ and $\tau_2 = 10$–40%; the precision with which the laser elements were constructed was not worse than ~0.1λ. The distance between mirrors was L = 25–200 cm. To investigate the influence of the weak sphericity of the mirrors of the resonator, a lens was introduced inside the resonator, with a focal length F = 1153 cm. The curvature radii of the mirrors of the

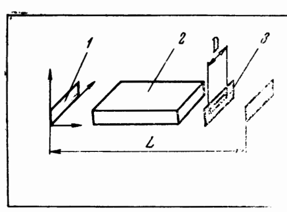

FIG. 1. Diagram of laser: 1—mirror, 2—active element, 3—diaphragm.

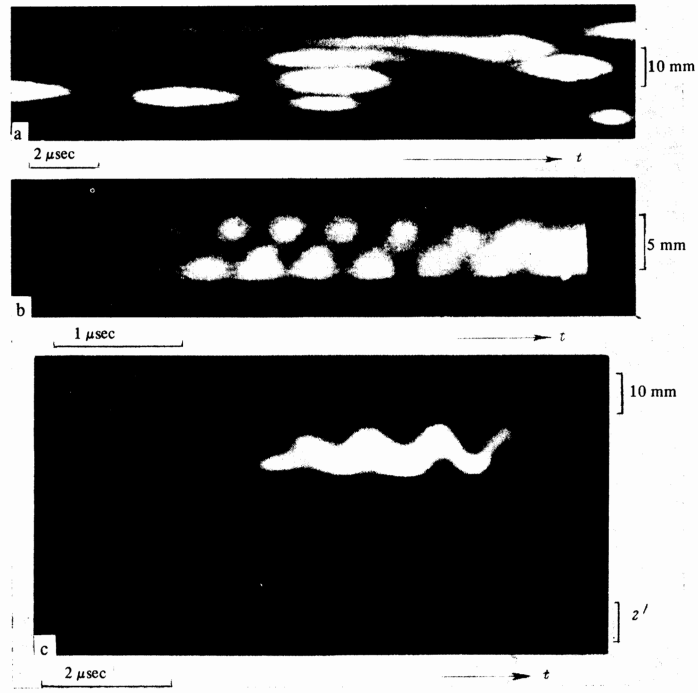

FIG. 2. Fragments of time scans at a resonator length L = 200 cm: a—pictures of near field of the radiation of a laser with flat mirrors and uniform distribution of the inversion over the resonator cross section (D = 30 × 2.5 mm); b—pictures of near field in the case of synchronized generation of four transverse modes (D = 8 × 2.5 mm); c—pictures of near and far fields of laser radiation in the case of synchronized generation of three transverse modes (D = 30 × 25 cm).

equivalent spherical resonator were R_e = 21.6 m, and the resonator length was L_e = 91 cm.

The influence of the uneven distribution of the inversion on the character of the generation was investigated in the same lasers but the diaphragm introduced into the resonator measured ~ 2.5 mm along the x axis, and its y dimension was varied in the range up to 15 mm. The same investigations were carried out on a laser with cylindrical neodymium-glass rods 20 mm in diameter and 240 mm long, in which the pump was a straight flash lamp; in this case the uneven distribution of the inversion over the cross section was much larger. The pump level was chosen in such a way that the average radiation density in the resonator was approximately equal to the radiation density in a generator with a uniform inversion distribution.

A high-speed SFR-2M camera was used to obtain a simultaneous time scan (frame by frame and with a slit) of the near and far fields of laser radiation. The scanning was in a direction in which the dimension of the aperture was ~ 2.5 mm. The time resolution in the slit scanning regime was 0.5–0.05 μsec; in the frame regime, the exposure of each frame was ~0.5 μsec, and the interval between frames was ~ 2 μsec. A diffraction image of the slit was printed on the film for photometry purposes.

4. EXPERIMENTAL RESULTS

An investigation of the general character of free generation in a laser with flat mirrors and a large resonator cross section shows that if the inverted population is uniformly distributed over the cross section, the generation in each spike occurs not over the entire resonator cross section, but in individual small regions (Fig. 2a). The dimensions of these generation regions are D ~ 3–10 mm at L = 26 cm (average dimension ~ 6 mm) and D ~ 6–10 mm at L = 200 cm (average dimension ~7 mm), and depend little on the pump level. The observed local regions of generation within the limits of the large laser cross section are as a rule randomly located relative to each other. However, in addition to generation of this character, one sometimes observes successive displacements of the generation regions from one edge of the generator cross section to another, and the times of these displacements can amount to 40–120 μsec.

If the mirrors are well adjusted, one frequently observes within the limits of individual generating regions (or in the entire aperture, if it is of the same order as the average dimension of the observed generation regions) periodic oscillations executed by a small generating zone (generation spot) over the resonator mirror. These oscillations are accompanied by a similar scanning of the laser beam in the far zone within the limits of the angular divergence of several transverse modes of the local planar resonator.

Such interference phenomena always take place when more than one transverse mode is excited in the spike. The observed character of the oscillations of the generating zone is different within the limits of the radiation region. In some cases the oscillations are not uniform, slow at the edges of the generation region and rapid at the center (Fig. 2b). In other cases the oscillations of the radiating spot over the generation region are more uniform (Fig. 2c). More complicated interference phenomena within the limits of the radiating regions are sometimes observed. The frequency of the oscillations of the generation zone over the mirror coincides in all these cases (within the limits of the measurement accuracy, ~ 20%) with the beat frequency $\Delta\nu_{12}$ of the next-lower transverse modes of the "empty" resonator. According to [17], $\Delta\nu_{12} = 1.2/D^2$ [MHz], where D is the dimension (in cm) of that section of the mirror within which the oscillations of the generation zone take place. The dimension of the scanning zone is (0.3–0.5) D.

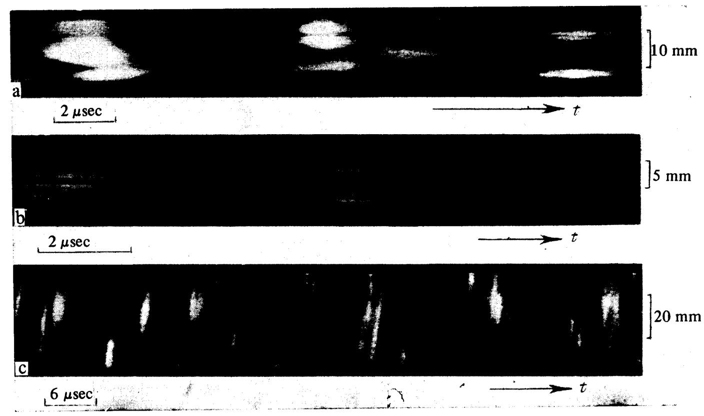

FIG. 3. Time scan of the picture of the near field of laser radiation: a—with flat mirrors and non-uniform distribution of inverted population over the resonator cross section (L = 200 cm, D = 18 × 2.5 mm); b—with weakly-spherical mirrors (mirror curvature radius R_e = 21.6 m, L_e = 0.91 m, D = 8 × 2.5 mm); c—with flat mirrors with a tilt angle α between them (L = 40 cm, D = 50 × 5 mm, α = 4–5″).

Measurement of the angular divergence of the radiation has shown that in the case when the generation spot moves in non-uniform fashion over the mirror, the laser beam executes a similar scanning in the far zone in a range (3.3–3.5)λ/D, corresponding to synchronized generation of approximately four transverse modes of the local planar resonator, and in the case of a more uniform displacement (see, for example, Fig. 2c), the laser beam scans in a range (2.5–2.6)λ/D, corresponding to synchronization of the order of three transverse modes (the values of the angular divergence of the radiation are indicated for a level 0.5 of the maximum intensity). In certain regions the oscillations apparently correspond to beats of two modes.

We note that since the average dimensions of the observed generation regions in the neodymium-glass laser with a planar resonator are small even in the case of a uniform distribution of the inversion, it follows that it is impossible to have a large number of simultaneously excited transverse modes (their maximum number, in accordance with the experimental data, is 4–5).

In a laser with an uneven distribution of the inverted population over the cross section of the resonator, no effects characteristic of synchronization of transverse modes in the spike were observed. In such lasers with flat mirrors, generation also occurs in small regions randomly distributed over the cross section (Fig. 3a). The dimensions of the generation regions in a laser with an uneven distribution of the inversion over the cross section, at a resonator length L = 50 cm, amount to ~1.5–4 mm (the most frequently encountered generation regions have a dimension ~2.5 mm), and at L = 200 cm their dimensions amount to 2–8 mm (the most frequently encountered value is 3–3.4 mm). Thus, their dimensions are on the average smaller by a factor of 2.5 than for a laser with a uniform inversion distribution. Generation with an uneven inversion distribution occurs predominantly in those sections of the laser cross section where the pump density is large. In individual spikes (10–15% of the total number of spikes per pulse) one observes transverse modes of the second–fourth orders, in a form typical of spherical resonators. The measurement of the radiation divergence shows that for a laser with a non-uniform inversion distribution it is close to the diffraction divergence in each spike (~λ/D) for the observed dimensions of the generation region, and for those spikes in which transverse modes of the second–fourth order are observed, it corresponds to the angular divergence of the radiation for these modes. Thus, in the case of nonuniform distribution of the inversion, the generation in each spike has as a rule a single-mode character, explaining the absence of locking.

An investigation of the possibility of locking of transverse modes in a laser with weakly-spherical mirrors has shown that although generation in such lasers occurs over the entire cross section, only one transverse mode of higher order is excited in each spike with a noticeable intensity (Fig. 3b), and no synchronization is observed. Pulsations of the intensity of the generating mode, occurring without a change in the order of the mode, are observed in some spikes. These intensity pulsations apparently are the result of a strong perturbation acting on this mode, for example as a consequence of spatial competition and suppression of other modes by it (see Fig. 3b).

Phenomena characteristic of self-locking of transverse modes were likewise not observed if the flat mirrors of the laser resonator with uniform inversion distribution over the cross section had a slight inclination relative to each other, or else if the end faces of the active elements were not parallel to the planes of the resonator mirrors or to each other. In the presence of linear phase aberration, the average dimension of the generation regions within the limits of the cross section increased (for example, when one of the mirrors is tilted by an angle $\varphi \approx 2''$–$3''$, the average dimension of the generation region increased to 20–25 mm), and the near and far radiation fields within the limits of each generating region acquired a characteristic structure in the form of alternating light and dark bands (Fig. 3c), the intensity of which decreased in the direction of the closer edges of the mirrors. Thus, the presence of linear phase aberration in the laser resonator leads to an appreciable alteration in the distribution of the field in the resonator (to the appearance of complex modes of "misaligned" resonators), as a result of which the lower transverse modes are apparently discriminated.

5. DISCUSSION OF RESULTS

The cause of the excitation of generation in small regions of the cross section of a laser with flat mirrors is the presence of aberrations, which in this case exert a strong influence on the mode formation. These aberrations are due primarily to inaccurate construction of the laser elements (local deviations from ideal flat surface on the order of $\lambda/10-\lambda/20$), and also to thermal deformation during pumping (this is particularly important in the case of non-uniform distribution of the pump density over the resonator cross section). A certain role in the change of the character of the aberrations during the pulse time may apparently be played by nonuniform "burning out" of the inverted population from spike to spike within the limits of the large cross section. It is of interest to compare the experimentally observed dimensions of the generation regions with theoretical estimates [18] that take into account the influence of aberrations. It follows from the results of this reference that if the difference between the resonator lengths of two neighboring "sublasers" (causing small discrepancies between the resonant frequencies of these "sublasers") is of the order of $\lambda/15$, simultaneous generation of these lasers at the same transverse mode is possible if the dimension (overall) of the aperture does not exceed $D \sim 1.2$ mm at $L = 200$ cm and $D \sim 0.6$ mm at $L = 30$ cm. The experimentally observed dimensions of the generation regions are larger than these calculated values by one order of magnitude (at the uniform inversion distribution). Such a disparity between the calculated and experimental data is due to the fact that in the calculations of [18] it is assumed that the coupling between the lasers is due only to the diffraction of the light by the edges of the mirrors, whereas under real conditions this coupling is due both to diffraction and to scattering of light by the resonator elements, and the coupling due to the scattering of the light may apparently greatly exceed the coupling between the "subgenerators" as a result of diffraction. This indeed leads to an increase of the maximum possible dimension of the generation region at a given value of the aberration.

It also follows from the experiments that an analysis of the formation of the transverse-mode spectrum in a laser with flat mirrors, proposed by Tang and Statz,[19] is valid only for lasers with small aperture dimensions (on the order of the average dimensions of the observed generation regions). The beam divergence of a laser with a large resonator cross section, integrated over the entire generation pulse, is the result of averaging of the scatter of the angular divergence of each small radiating region of the cross section. Therefore it has a bell-shaped (Gaussian) distribution without any structure whatever.

It follows from the above experimental results that simultaneous generation of thermal transverse modes within the confines of one radiating generation region becomes possible only if discrimination of certain transverse modes relative to the others is eliminated. Sphericity of the mirrors, non-uniform distribution of the inversion, and linear phase aberrations in the resonator lead to the appearance of generation at only one transverse mode of one type or another, i.e., they serve as such discriminating factors for the transverse modes in the resonator. The most important observed feature of simultaneous generation of several transverse modes is the fact that this generation is always synchronized. The character of the variation of the near radiation field of a laser with flat mirrors in the case of synchronized generation of three and four low transverse modes is shown in Fig. 4 (the initial phases of the modes are

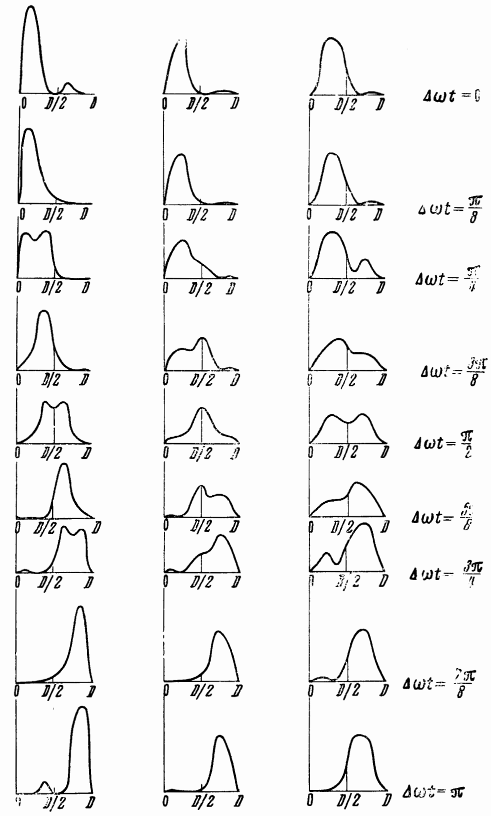

FIG. 4. Diagram of distribution of radiation intensity over mirror of planar resonator for synchronization of four (a), three (b), and the beats of two lowest transverse modes (c); $\Delta\omega$ is the frequency interval between modes.

assumed to be identical). The figure also shows the time variation of the near field upon generation of the two lowest transverse modes (beats). We see that in the case of synchronized generation of four modes the generation zone shifts gradually during the period of oscillations near the edges of the aperture, but passage through the central part of the aperture is rapid. This agrees with the experimentally observed character of the motion of the generating zone (see, for example, Fig. 2b).

The case of synchronization of three transverse modes leads to a smoother displacement of the generation zone over the mirror. The character of the variation of the near field in the case of beats between two transverse modes differs from the case of synchronization of three modes, as can be seen from Fig. 4b, in that the field intensity at the center of the aperture does not change during the beats, whereas in the case of three-mode synchronization the field intensity in the central part of the aperture changes twice during each period of the oscillation. The latter is clearly seen, for example, in Fig. 2c.

Upon synchronization, the laser beam should scan within the limits of the angular divergence of the synchronized modes. Measurement of the limits of beam scanning in the experiment gives a value that agrees well with that expected from the calculations.

It was shown above that the synchronization and capture of frequencies of two transverse modes are approximately equally probable. However, if the phase relations of the generating modes is favorable for synchronization, capture is practically impossible. The phase relations between the modes in the laser do not have a random character. The phase relations established between the modes in the laser are the most favorable from the energy point of view (see, for example, [15, 20]). Synchronized generation is apparently favored from the energy point of view. This evidently explains the fact that locking of transverse modes is always observed in the case of simultaneous generation of several transverse modes.

It is probably possible to obtain synchronization of a larger number of transverse modes in resonators close to concentric, where, as is well known, a rather large number of transverse modes is excited.

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