# *POSSIBILITY OF PARAMETRIC RESONANCE IN AN ELECTRON PLASMA*

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An electron plasma placed in a strong high-frequency electric field was investigated. It is shown that the oscillation of the electron mass in an external high-frequency field leads to parametric buildup of both potential and non-potential high-frequency oscillations.

1. Much attention has been paid recently to the question of plasma stability in a strong HF field. It was shown in[1-3] that a plasma in a strong HF field is unstable against the buildup in it of both potential and non-potential oscillations. In the foregoing investigations of the parametric buildup of the potential oscillations, the decisive role was played by ion oscillations.

In the present paper we consider the stability of a purely electronic plasma against natural oscillations in a strong HF field. It is assumed here that the ions do not take part in the oscillations, but only cancel out the equilibrium space charge of the electrons. Allowance for the relativistic effect of motion of the electrons in the external field causes the material parameter that oscillates in the external HF field to be the electron mass. On the other hand, oscillations of the electron mass lead to parametric buildup of both potential and non-potential HF oscillations. It is shown that at certain plasma parameters the growth increment of the potential oscillations can exceed the maximal increment obtained in[2].

The field applied to the plasma is represented in the form $\mathbf{E}_0(t) = \mathbf{E}_0(0)\sin\omega_0 t$, assuming that the wavelength of the natural oscillations is much smaller than the characteristic length of plasma inhomogeneity, and also the length of the external HF wave. Since we are interested in the parametric excitation of the natural HF oscillations in a purely electronic plasma, we use the relativistic equation of single-fluid electron hydrodynamics of a cold plasma and Maxwell's equations

$$\operatorname{rot}\mathbf{H} = \frac{1}{c}\frac{\partial\mathbf{E}}{\partial t} - 4\pi e n\mathbf{v}, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{H}}{\partial t},$$
$$\operatorname{div}\mathbf{E} = 4\pi e(n_{0i} - n), \quad \partial n/\partial t + \operatorname{div} n\mathbf{v} = 0,$$
$$\left[\frac{\partial}{\partial t} + \mathbf{v}\nabla\right]\frac{\mathbf{v}}{\sqrt{1 - v^2/c^2}} = -\frac{e}{m_0}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}\mathbf{H}]\right). \qquad (1)^*$$

The velocity of the plasma electrons in the equilibrium state is determined by

$$\mathbf{v}_0(t) = \frac{\mathbf{v}_E\cos\omega_0 t}{[1 + (v_E^2/c^2)\cos^2\omega_0 t]^{1/2}}, \quad \mathbf{v}_E = \frac{e\mathbf{E}_0(0)}{m_0\omega_0}.$$

Linearizing the system (1) with respect to small deviations from the equilibrium state ($n_0 \gg n_1$, $\nu_0 \gg \nu_1$ etc.), and assuming for the non-equilibrium values a coordinate dependence in the form $\exp(i\mathbf{k}\mathbf{r})$, we obtain a system of equations describing the parametric buildup of the natural oscillations of the plasma:

$$i[\mathbf{k}\mathbf{H}_1] = \frac{1}{c}\frac{\partial\mathbf{E}_1}{\partial t} - \frac{4\pi e}{c}(n_0\mathbf{v}_1 + n_1\mathbf{v}_0), \quad i[\mathbf{k}\mathbf{E}_1] = -\frac{1}{c}\frac{\partial\mathbf{H}_1}{\partial t},$$
$$i(\mathbf{k}\mathbf{E}_1) = -4\pi e n_1, \quad \frac{\partial n_1}{\partial t} + i(\mathbf{k}\mathbf{v}_0)n_1 + i n_0(\mathbf{k}\mathbf{v}_1) = 0,$$
$$\left(\frac{\partial}{\partial t} + i(\mathbf{k}\mathbf{v}_0)\right)\left(\frac{\mathbf{v}_1}{(1 - v_0^2/c^2)^{1/2}} + \frac{\mathbf{v}_0(\mathbf{v}_0\mathbf{v}_1)}{c^2(1 - v_0^2/c^2)^{3/2}}\right) \qquad (2)$$
$$= -\frac{e}{m_0}\left(\mathbf{E}_1 + \frac{1}{c}[\mathbf{v}_0\mathbf{H}_1]\right).$$

We confine ourselves below to an analysis of the system (2) in two limiting cases: when the wave vector $\mathbf{k}$ is parallel to the external field $\mathbf{E}_0$ and when it is perpendicular to $\mathbf{E}_0$.

2. Let us consider the case $\mathbf{k} \parallel \mathbf{E}_0$. The system (2) can be reduced in this case to two equations, the first of which describes longitudinal oscillations, namely

$$\frac{\partial^2 y}{\partial t^2} + \omega_{Le}^2\left(1 - \frac{v_0^2}{c^2}\right)^{3/2} y = 0, \qquad (3)$$

where $\omega_{Le} = \sqrt{4\pi n_0 e^2/m_0}$ is the Langmuir frequency of the electrons in the laboratory coordinate system, and

$$y = \left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}\frac{\partial}{\partial t}\left(n_1\exp\left\{i\int\mathbf{k}\mathbf{v}_0\,dt'\right\}\right). \qquad (4)$$

The second equation represents the oscillations of the magnetic field (purely transverse waves) and is written in the form

$$\frac{\partial^2\mathbf{H}_1}{\partial t^2} + \left[k^2c^2 + \omega_{Le}^2\left(1 - \frac{v_0^2}{c^2}\right)^{1/2}\right]\mathbf{H}_1 = 0. \qquad (5)$$

In view of the complexity of the analytic investigation of Eqs. (3) and (5), we consider the case $\nu_E^2/c^2 \ll 1$, which leads to Mathieu equations[4]. Equation (3) then takes the form

$$\frac{\partial^2 y}{\partial t^2} + \omega_{Le}^2\left(1 - \frac{3}{4}\frac{v_E^2}{c^2}\cos 2\omega_0 t\right)y = 0. \qquad (6)$$

From (6) there follows parametric excitation of the electronic Langmuir oscillations, with the fundamental resonance occurring at an external-field frequency $\omega_0$ on the order of the electron Langmuir frequency $\omega_{Le}$, while the buildup increment of these oscillations is given by[1)]

$$\gamma = {}^3/_{16}\,\omega_{Le}v_E^2/c^2. \qquad (7)$$

On comparing the increment (7) with the maximum increment obtained in[2]

$$\gamma_{max} = \omega_{Le}(m/M)^{1/3} \qquad (8)$$

*$[\mathbf{v}\mathbf{H}] \equiv \mathbf{v}\times\mathbf{H}$.

1)The result (7) was also obtained in a recent paper [5] in the limit as $k \to 0$.

it turns out that the increment (7) can become larger than the increment (8) if the equilibrium density of the electrons is of the order of $10^8$—$10^9$ cm$^{-3}$, the external-field amplitude is of the order of several kV/cm, and the ion mass is of the order of several dozen hydrogen masses.

In Eq. (5) with $v_E^2/c^2 \ll 1$, it follows that the maximum transverse-wave growth increment is

$$\gamma = {}^1/_{16}\omega_{Le} v_E^2 / c^2. \tag{9}$$

3. Let us consider now the case $\mathbf{k} \perp \mathbf{E}_0$. System (2) reduces in this case to the following equations: that of a purely transverse wave

$$\partial^2 u / \partial t^2 + [k^2c^2 + \omega_{Le}^2 (1 - v_0^2/c^2)^{1/2}] u = 0,$$
$$u = v_{1\perp}\left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}, \quad v_\perp = v_1 \frac{[\mathbf{E}_0 \mathbf{k}]}{E_0 k} \tag{10}$$

and of a longitudinally-transverse wave of fourth order

$$k^2c^2 \frac{c}{v_0}\left\{\frac{\partial}{\partial t}\left[\frac{\partial E_x}{\partial t}\left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}\right] + \omega_{Le}^2 \left(1 - \frac{v_0^2}{c^2}\right) E_x\right\}$$
$$+\left[\frac{\partial^2}{\partial t^2} + \omega_{Le}^2 \left(1 - \frac{v_0^2}{c^2}\right)^{3/2}\right] \frac{c}{v_0}\left\{\frac{\partial}{\partial t}\left[\frac{\partial E_x}{\partial t}\left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}\right] + \omega_{Le}^2 E_x\right\} = 0, \tag{11}$$

When $kc \ll \omega_0, \omega_{Le}$, Eq. (11) becomes simpler and goes over into the equation for longitudinal oscillations, $E_x \parallel k_x$, and has the same form as Eq. (3):

$$\frac{\partial^2 X}{\partial t^2} + \omega_{Le}^2 \left(1 - \frac{v_0^2}{c^2}\right)^{3/2} X = 0. \tag{12}$$

$$X = \frac{c}{v_0}\left[\frac{\partial}{\partial t}\left(\frac{\partial E_x}{\partial t}\left(1 - \frac{v_0^2}{c^2}\right)^{-1/2}\right) + \omega_{Le}^2 E_x\right].$$

We note once more that the effects described above are due to periodic variation of the electron mass in an external HF field

$$m_e(t) = m_0\left(1 + \frac{v_E^2}{c^2} \cos \omega_0 t\right)^{1/2}.$$

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