DOMAIN STRUCTURE PRODUCED IN AN ANTIFERROMAGNET WHEN THE CHARACTER OF THE MAGNETIC ANISOTROPY IS ALTERED

V. G. BAR'YAKHTAR, A. E. BOROVIK, V. A. POPOV, and E. P. STEFANOVSKIĬ

Physicotechnical Institute of the Ukrainian Academy of Sciences

Submitted April 14, 1970

Zh. Eksp. Teor. Fiz. 59, 1299–1306 (October, 1970)

We consider the temperature first-order phase transition, at which the magnetic anisotropy of the "easy axis" type goes over into magnetic anisotropy of the "easy plane" type. We investigate the distribution of the magnetic moments of the sublattices in the boundary between the two different phases, and calculate the surface energy of the separation boundary. It is shown that the phase transition can give rise to a thermodynamically-stable domain structure due to the weak ferromagnetism of the phase with the "easy-plane" magnetic anisotropy. The dimensions of the domains are calculated. We investigate also the distribution of the magnetic moments in 180° domain boundaries and calculate the surface energies of these boundaries. It is shown that the surface energy of an 180° boundary is greatly reduced when the phase-transition temperature is approached.

1. INTRODUCTION

As is well known, in a number of antiferromagnets (AFM) (for example, in α-Fe_2O_3), the character of the magnetic anisotropy changes appreciably with changing temperature, namely, above a certain temperature T_M (T_M is the Morin temperature) the magnetic moments are oriented perpendicular (parallel) to the chosen axis, and below this temperature the magnetic moments of the sublattices are parallel (perpendicular) to the chosen axis.

Such a change in the orientation of the magnetic moments, in the case of uniaxial AFM, i.e., AFM in which the anisotropy in the basal plane is small, indicates either a first-order phase transition, or the presence of at least two second-order phase transitions. Indeed, from symmetry considerations it is clear that in a uniaxial AFM there can be realized three phases in sequence: a phase $\Phi_{\parallel}$, in which the antiferromagnetism vector $\mathbf{l}$ is oriented parallel to the symmetry axis, a phase $\Phi_{\perp}$, in which the vector $\mathbf{l}$ is oriented perpendicular to the symmetry axis, and a phase $\Phi_{<}$, in which the vector $\mathbf{l}$ is oriented at a certain angle to the anisotropy axis. The transition from the phase $\Phi_{\parallel}$ to the phase $\Phi_{\perp}$ is of first order if these phases can coexist; the transition from the phase $\Phi_{\parallel}(\Phi_{\perp})$ to the phase $\Phi_{<}$ can be regarded as a second-order phase transition, for in such a transition one of the symmetry elements is lost and one of the frequencies of the homogeneous oscillations vanishes on the transition line.

We consider in the present paper, in the main, the case of a first-order phase transition $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$. As is well known, in a first order phase transition there is coexistence of the phases, and consequently separation boundaries are produced between these phases.

In the phase $\Phi_{\perp}$, as a rule, there occurs a small magnetic moment due to the Dzyaloshinskiĭ interaction. The presence of the magnetic moment in the phase $\Phi_{\perp}$ makes it possible for the AFM sample to break up at temperatures close to T_M into domains that are thermodynamically stable. These domains are analogous to those investigated in[1], which arise when the antiferromagnetism vector $\mathbf{l}$ in an AFM with magnetic anisotropy of the "easy axis" (EA) type turns over in an external magnetic field.

We obtain in this paper the distributions of the magnetic moments of the AFM sublattices in the boundaries between the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ (90° boundaries), and also in the 180° introduction boundaries in AFM with weak ferromagnetism; we calculate the surface energies of the 90 and 180° interdomain walls, determine the domain structures (DS) for a plane-parallel plate, and estimate the domain dimensions.

2. THE PHASE TRANSITION $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$ IN AN UNBOUNDED AFM

Before we investigate the DS, let us consider the question of the phase transition $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$ in an infinite AFM. As already noted in the introduction, this transition can occur either via a first-order transition, or via two second-order phase transitions. The simplest density of the non-equilibrium thermodynamic potential, which makes it possible to describe these two possibilities, is

$$w = \frac{1}{2}\delta(\mathbf{M}_1 + \mathbf{M}_2)^2 - \frac{1}{8}\beta'(\mathbf{M}_1 - \mathbf{M}_2, \mathbf{n})^2 - \frac{1}{8}\beta(\mathbf{M}_1 + \mathbf{M}_2, \mathbf{n})^2 - \frac{1}{2}d(\mathbf{n}[\mathbf{M}_1\mathbf{M}_2]) - \frac{1}{4\cdot 16}\beta_3(\mathbf{M}_1 - \mathbf{M}_2, \mathbf{n})^4 + f\left(\frac{1}{2}M_1^2, \frac{1}{2}M_2^2\right), \tag{2.1}$$

where δ is the exchange-interaction constant, β, β', and β_3 are the magnetic-anisotropy constants, d is the Dzyaloshinskiĭ constant, $f(\frac{1}{2}M_1^2, \frac{1}{2}M_2^2)$ is the exchange energy, which determines the values of the magnetic moments at a given temperature, $\mathbf{M}_1$ and $\mathbf{M}_2$ are the sublattice magnetizations, and $\mathbf{n}$ is a unit vector along the symmetry axis; we shall henceforth need also the quantities

$$\mathbf{l} = \frac{1}{2}\left(\frac{\mathbf{M}_1}{M_1} - \frac{\mathbf{M}_2}{M_2}\right) \qquad \mathbf{m} = \frac{1}{2}\left(\frac{\mathbf{M}_1}{M_1} + \frac{\mathbf{M}_2}{M_2}\right).$$

The dimensionless quantities δ, β′, and d are functions of the temperature, and the exchange energy f depends not only on the values of the magnetic moments of the sublattices, but also on the temperature.

Formula (2.1) pertains to a homogeneous quasi-equilibrium state such that the magnetic moments of the sublattices have reached neither their equilibrium values nor their equilibrium directions.

We note, finally, that allowance for only the second-order terms in the magnetic-anisotropy energy is insufficient for the description of the character of the phase transition $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$, and it is necessary to take into account in the magnetic-anisotropy energy also the fourth-order terms. An analysis of the fourth-order terms in the magnetic-anisotropy energy of AFM is given in[2], where it is shown that the most important role is played by the term $(\mathbf{M}_1 - \mathbf{M}_2, \mathbf{n})^4$, which is given in formula (2.1).

To determine the equilibrium directions of $\mathbf{M}_1$ and $\mathbf{M}_2$ and their equilibrium values, it is convenient to change over from $\mathbf{M}_1$ and $\mathbf{M}_2$ to their components in spherical coordinates:

$$M_{1z} = (\mathbf{M}_1\mathbf{n}) = M_1 \cos\theta_1, \quad M_{1y} = M_1 \sin\theta_1 \sin\varphi_1, \quad M_{1x} = M_1 \sin\theta_1 \cos\varphi_1,$$
$$M_{2z} = (\mathbf{M}_2\mathbf{n}) = M_2 \cos\theta_2, \quad M_{2y} = M_2 \sin\theta_2 \sin\varphi_2,$$
$$M_{2x} = M_2 \sin\theta_2 \cos\varphi_2, \tag{2.2}$$

where θ_i and φ_i are the polar and azimuthal angles of the vector $\mathbf{M}_i$.

Substituting these formulas in (2.1) and equating to zero the derivatives of w with respect to θ_1, θ_2, $\varphi = \varphi_1 - \varphi_2$, M_1, and M_2, we can easily verify that the following solutions exist for θ_1 and θ_2:

1) $$\theta_1 = \theta_2 = \pi/2, \tag{2.3}$$

2) $$\theta_1 = 0, \quad \theta_2 = \pi. \tag{2.4}$$

It can also be shown that in either of the two cases the magnetizations of the sublattices are the same, $M_1 = M_2$.

The angle φ and the value of the magnetization of the sublattice are determined in the former case from the equations

$$\operatorname{tg}\varphi = \frac{d}{2\delta}, \quad f_1 + \delta(1+\cos\varphi) + d\sin\varphi = 0, \tag{2.5}$$

where

$$f_1 \equiv 2\frac{\partial}{\partial M_1} f\left(\frac{1}{2}M_1^2, \frac{1}{2}M_2^2\right)\Big|_{M_1=M_2}.$$

In the second case, the determination of the angle φ has no meaning, and to determine M we have the equation

$$f_1 - {}^1\!/_2(\beta' + \beta_3 M^2) = 0. \tag{2.6}$$

The solution of (2.3) together with Eq. (2.5) determines the phase in which the AFM vector **l** is perpendicular to the chosen axis, and the solution (2.4) together with Eq. (2.6) determines the phase in which the AFM vector **l** is parallel to the chosen axis. It follows from (2.5) and (2.6) that simultaneously with the change of the direction of **l** a change takes place in the magnitude of the sublattice magnetization M. To be sure, this change is relatively small and is proportional to the ratio of the relativistic interactions to the exchange interaction (compare (2.5) and (2.6))[1)].

Let us determine now the conditions that must be satisfied in order for the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ to be thermodynamically stable. To this end it is necessary to investigate the second derivatives of the potential w. Examining the second derivatives of w with respect to θ_1, θ_2, and φ, we arrive at the conclusion that the phase $\Phi_{\perp}$ corresponding to the solution of (2.3) and (2.5) is stable if

$$(2\delta + \beta') \leqslant \sqrt{(2\delta)^2 + d^2}, \quad {}^1\!/_2(\beta + \beta') \leqslant \sqrt{(2\delta)^2 + d^2} \tag{2.7}$$

and $\cos\varphi < 0$ (in the derivation of the conditions (2.7) it was assumed that the constant d is negative). Recognizing that δ is due to exchange, and that β, β′ and d are of relativistic origin, we can readily see that the second condition of (2.7) is no limitation at all[2)], and the first can be represented approximately in the form

$$\beta' - \frac{1}{4}\frac{d^2}{\delta} \leqslant 0. \tag{2.8}$$

The conditions for the stability of the phase $\Phi_{\parallel}$ are

$$2\delta + \beta' + \beta_3 M_{\parallel}^2 \geqslant 0, \quad \beta' + \frac{1}{2}\beta_3 M_{\parallel}^2 \geqslant \frac{d^2}{4\delta + \beta' + \beta_3 M_{\parallel}^2}. \tag{2.9}$$

The first condition of (2.9) is satisfied by virtue of the exchange nature of δ and the relativistic nature of β′ and β_3, and the second, accurate to terms of second order in the ratio of the relativistic-interaction constants to the exchange-interaction constants, can be represented in the form

$$\beta' - \frac{1}{4}\frac{d^2}{\delta} \geqslant \frac{-\beta_3 M_{\parallel}^2}{2}. \tag{2.10}$$

The conditions (2.8) and (2.10) (or more accurately, the conditions (2.7) and (2.9) determine the temperature interval at which the phases $\Phi_{\perp}$ and $\Phi_{\parallel}$ are stable. To determine these temperature intervals, it is necessary to know the temperature dependence of the quantities β′, β_3, d, δ, and $M_{\parallel}$.

Let us consider the simplest case (bearing in mind the application of our theory to hematite), when the difference $\beta' - {}^1\!/_4 d^2/\delta \equiv \psi(T)$ is a monotonically decreasing function of the temperature in the entire region of the existence of the AFM[3)]. Then the condition (2.8) can be rewritten in the form

$$T_1 < T < T_N,$$

where T_1 is determined from the condition $\psi(T_1) = 0$. Expanding ψ(T) in a series about $T = T_1$, $\psi(T) = -A(T - T_1)$, and assuming that $|\beta_3 M_{\parallel}^2| \ll \beta'$, we can readily find the region of stability of the phase $\Phi_{\parallel}$:

1)We note that besides the solutions (2.3) and (2.4) there exists a solution corresponding ot the phase $\Phi_<$. This solution and its analysis are of fundamental importance when the transition $\Phi_{\parallel} \rightleftarrows \Phi_{\perp}$ proceeds via two second-order phase transitions. We shall not discuss this here, however.

2)This statement is valid only if T is not too close to T_N, where T_N is the Neel temperature, since δ becomes small at temperatures close to T_N. We recall that $\delta(T_N) \approx 0$ at $T = T_N$.

3)In the case when the coefficients of the expansion of w are not monotonic functions of the temperature, there can exist several temperature intervals in which the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ are stable.

$$0 < T < T_2, \quad T_2 = T_1 + \beta_3(T_1) M_{\parallel}^2(T_1) / A.$$

If $\beta_3(T_1) > 0$, then the regions of existence of the phases $\Phi_\perp$ and $\Phi_\parallel$ overlap and the transition $\Phi_\parallel \rightleftarrows \Phi_\perp$ is a phase transition of first order.

On the other hand, if $\beta_3(T_1) < 0$, then the regions of existence of the phases $\Phi_\parallel$ and $\Phi_\perp$ are separated by a finite temperature interval $\Delta T = \beta_3(T_1) M_\parallel^2(T_1)/A$. As shown by a more detailed analysis, there exists in this temperature interval a phase $\Phi_<$ in which the AFM vector $\mathbf{l}$ is oriented at an angle to the anisotropy axis, n, neglecting the anisotropy in the basal plane, the transition $\Phi_\perp \rightleftarrows \Phi_\parallel$ via the phase $\Phi_<$ constitutes two second-order phase transitions.

In order to determine the temperature at which the first-order phase transition $\Phi_\perp \rightleftarrows \Phi_\parallel$ takes place, it is necessary to equate the thermodynamic potentials $w_\parallel$ and $w_\perp$ obtained from (2.1) by substituting the equilibrium values of M_1 and M_2 respectively for the phases $\Phi_\parallel$ and $\Phi_\perp$. After simple calculations we find that the condition $w_\parallel = w_\perp$ leads to[2]

$$\beta' - \frac{1}{4}\frac{d^2}{\delta} + \frac{1}{2}\beta_3 M_\parallel^2 = 0,$$

whence

$$T_M = T_1 + \frac{\beta_3 M_\parallel^2}{2A} = \frac{1}{2}(T_1 + T_2) \approx \sqrt{T_1 T_2}. \qquad (2.11)$$

We see from this expression that $T_1 < T_M < T_2$.

Differentiating the expression $w_\parallel - w_\perp$ with respect to the temperature, we find the jump of the entropy in the phase transition; when this jump is multiplied by T_M we obtain the heat of the transition

$$Q = T_M \Delta S = {}^1/_2 A T_M. \qquad (2.12)$$

In concluding this section, we note that the possible AFM phase diagrams in terms of the variables H and T ($H \parallel n$) can be represented schematically in the manner shown in Fig. 1. For simplicity, these diagrams disregard the possible existence of the phase $\Phi_<$. Allowance for this phase leads to splitting of the ends of the line separating the phases $\Phi_\perp$ and $\Phi_\parallel$ (a splitting of the entire line is also possible).

3. INTERDOMAIN BOUNDARIES IN AFM

We shall first consider the distribution of the magnetic moments on the boundary between the phases $\Phi_\perp$ and $\Phi_\parallel$. Since the AFM vector $\mathbf{l}$ rotates through 90° inside this boundary, it is necessary, in order to describe this rotation, to take into account in the expression for w also the terms connected with the inhomogeneity of the magnetic moments. These terms, as is well known, have mainly an exchange nature and can be represented in the form

$$w_{\text{inhom}} = \left[\frac{1}{2}\alpha\left(\frac{\partial \mathbf{m}}{\partial x_i}\right)^2 + \frac{1}{2}\alpha'\left(\frac{\partial \mathbf{l}}{\partial x_i}\right)^2\right] M_0^2, \qquad (3.1)$$

where α and α' are the exchange constants, $\alpha \sim \alpha \sim \delta a^2$.

The total thermodynamic potential can be represented in the form

$$w_{\text{tot}} = w_{\text{inhom}} + w_{\text{hom}}, \qquad (3.2)$$

where w_{hom} is given by (2.1). We shall consider a domain wall (Fig. 2) such that the AFM vector $\mathbf{l}$ is rotated in the xz plane (the x axis, along which $\mathbf{l}$ is directed in the phase $\Phi_\perp$, is fixed by the anisotropy in the basal plane), the domain wall being parallel to the yz plane, i.e., the angle of rotation θ of the vector $\mathbf{l}$ and the magnitude of the magnetic moment m depend only on the coordinate x.

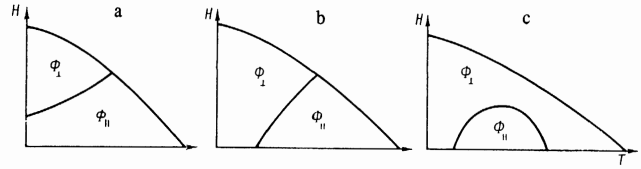

FIG. 1. a) The AFM does not have a Morin point, b) the AFM has one Morin point, c) two Morin points.

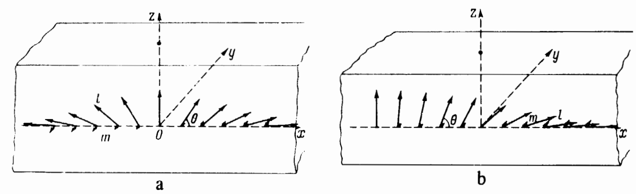

FIG. 2. a) 180°–domain wall in phase $\Phi_\perp$; b) 90°–domain wall separating the phases $\Phi_\parallel$ and $\Phi_\perp$.

The energy W_{tot} can be represented in this case in the form

$$W_{\text{tot}} = \int dr \left\{\frac{1}{2}\alpha'(\theta')^2 + 2\delta m^2 - \frac{1}{2}\beta' \sin^2\theta + \frac{1}{2}\beta' m^2 \sin^2\theta + dm\cos\theta - \frac{1}{2}dm^3\cos\theta - \frac{1}{4}\beta_3 M_0^2(1-m^2)^2\sin^4\theta\right\} M_0^2, \qquad (3.3)$$

where $\theta' \equiv d\theta/dx$. In writing down this expression we have discarded the terms proportional to the derivatives of m, and neglected the small difference between $M_\parallel(T)$ and $M_\perp(T)$.

Varying W_{tot} with respect to θ and m, we obtain the following equations describing the distribution of the magnetic moments in the domain boundary:

$$(4\delta + \beta'\sin^2\theta)m + d\cos\theta - {}^3/_2 dm^2\cos\theta + \beta_3 M_0^2(1-m^2)m\sin^4\theta = 0,$$
$$\alpha'\theta'' + \beta'\sin\theta\cos\theta - \beta' m^2\sin\theta\cos\theta + dm\sin\theta + \beta_3 M_0^2(1-m^2)^2\sin^3\theta\cos\theta = 0. \qquad (3.4)$$

From the first equation of the system (3.4) we readily get

$$m = -\frac{d}{4\delta}\cos\theta\left(1 - \frac{\beta'}{4\delta}\sin^2\theta - \frac{\beta_3 M_0^2}{4\delta}\sin^4\theta\right), \qquad (3.5)$$

recalling that $l \approx 1 - {}^1/_2 m^2$. Substituting this value for the magnetic moment in the second equation of (3.4), we obtain

$$\alpha'\theta'' + a\sin\theta\cos\theta - {}^1/_2 b\sin 2\theta\cos 2\theta = 0, \qquad (3.6)$$

where

$$a = \beta' - \frac{1}{4}\frac{d^2}{\delta} + \frac{1}{2}\beta_3 M_0^2, \quad b = \frac{1}{2}\beta_3 M_0^2 + \beta'\left(\frac{d}{4\delta}\right)^2. \qquad (3.7)$$

It is easy to see that the first integral of this equation is

$${}^1/_2\alpha'(\theta')^2 + {}^1/_2 a\sin^2\theta - {}^1/_8 b\sin^2 2\theta = \text{const}. \qquad (3.8)$$

It follows therefore that when $a \neq 0$ it is possible to have domain boundaries in which the direction of the vector $\mathbf{l}$ is rotated through 180° (as shown in Fig. 2). We recall that far from the domain boundary the distribution is assumed to be homogeneous, i.e., $\theta' \to 0$ as $x \to \pm\infty$.

The case $a > 0$ corresponds to a 180° domain boundary (DB) with rotation of l from $\theta = \pi$ to $\theta = 0$. Comparing the condition $a > 0$ with the condition (2.9), we see that it coincides with the condition for thermodynamic stability of the phase $\Phi_{\parallel}$.

The case $a < 0$ (which coincides with the condition for the existence of the phase $\Phi_{\perp}$, see (2.8)) corresponds to a 180° DB with rotation of the vector l from $\theta = -\pi/2$ to $\theta = \pi/2$.

Finally, in the case $a = 0$ a rotation of l from $\theta = \pi/2$ to $\theta = 0$ is possible[4)].

The dependence of the angle θ on the coordinate x can in all these cases be described with the aid of the formulas

$$\theta(x) = \begin{cases} \operatorname{arcctg}\left[\sqrt{\dfrac{a}{a+b}}\,\operatorname{sh}\dfrac{x}{x_{01}}\right], & a > 0, \quad \Phi_{\parallel}; \\ \operatorname{arctg}\exp(-x/x_{02}), & a = 0; \\ -\operatorname{arctg}\left[\sqrt{\dfrac{|a|}{|a|+b}}\,\operatorname{sh}\dfrac{x}{x_{03}}\right], & a < 0, \quad \Phi_{\perp}, \end{cases} \tag{3.9}$$

where

$$x_{0i} = \begin{cases} (|a| + b)^{-1/2}, & a \gtrless 0, \\ b^{-1/2}, & a = 0. \end{cases} \tag{3.10}$$

Let us stop briefly to analyze the obtained formulas. Since $|b| \ll |a|$ far from $T = T_M$, it follows from (3.10) that the thickness of the 90° DB which separates the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ is much larger than the thickness of the 180° DB in the phases $\Phi_{\parallel}$ or $\Phi_{\perp}$ far from the Morin point. As seen from (3.5) the magnetic moment m reverses direction in a 180° DB of the phase $\Phi_{\perp}$ ($a < 0$) (from $m = -d/\delta$ at $x \to -\infty$ to $m = d/\delta$ at $x \to +\infty$), and in a 180° DB of the phase $\Phi_{\parallel}$ ($a > 0$) the magnetic moment m reaches a maximum absolute magnitude $|m| = d/\delta$ in the middle of the domain wall at $x = 0$.

Finally, when the phases $\Phi_{\parallel}$ and $\Phi_{\perp}$ coexist ($a = 0$), the magnetic moment, as follows from (3.5), has the following values on the different sides of the 90° DB: $m = 0$ for the phase $\Phi_{\parallel}$ and $m = d/\delta$ for the phase $\Phi_{\perp}$.

Knowing the distributions (3.5), (3.9), and (3.10) for l and m in the domain boundaries, we can easily find the surface energy of the DB. Let us determine it from the formula

$$\sigma = \frac{1}{S}[W_{\text{tot}} - W'_{\text{hom}}] = \int_{-\infty}^{\infty} dx\,(w_{\text{tot}} - w'_{\text{hom}}) = \alpha \int_{-\infty}^{\infty} (\theta' M_0)^2\, dx \tag{3.11}$$

where S is the surface of the DB and w'_{hom} is obtained by substituting in (3.2) the homogeneous values of l and m for the corresponding phase.

Using this formula, as well as formulas (3.5), (3.9) and (3.10), (3.2), and (2.5)–(2.8), we get

$$\sigma = \alpha^{1/2} M_0^2 \begin{cases} \sqrt{|a| + b} + \dfrac{|a|}{\sqrt{b}} \ln \dfrac{\sqrt{b} + \sqrt{|a| + b}}{\sqrt{|a|}}, & a \neq 0; \\ \dfrac{1}{2}\sqrt{b}, & a = 0. \end{cases} \tag{3.12}$$

It is seen from the foregoing formulas that, generally speaking, the surface energy of the 90° DB is much smaller than the surface energy of the 180° DB. However, as $T \to T_M$, as seen from (3.12), the surface energy of the 180° DB decreases greatly and becomes of the same order as the surface energy of the 90° DB.

4)We emphasize that the condition $a = 0$ coincides with the condition for the equality of the thermodynamic potentials $w_{\parallel}$ and $w_{\perp}$.

4. DOMAIN STRUCTURE OF AFM

In concluding this paper, let us stop to discuss the domain structure of the AFM. Since an energy σS is consumed in the production of the DB, it is clear that the 180° DB, which can exist in the phase $\Phi_{\parallel}$, are not in equilibrium thermodynamically, since their occurrence cannot lead to a decrease of any other part of the AFM energy.

The 180° DB in the phase $\Phi_{\perp}$ may turn out to be thermodynamically stable in AFM samples of finite dimensions in the case when the magnetization on the opposite sides of the DB has different directions, since the appearance of these boundaries reduces the energy of the magnetic field outside the body[5)]. In analogy, 90° DB will be thermodynamically stable, since the subdivision of the AFM into domains with such DB leads to a closing of the magnetic flux inside the body and to a decrease of the energy of the magnetic field outside the body.

To estimate the domain dimensions, we start from the following expression for the energy of an AFM broken down into domains[1,3] [6)]:

$$E = \sigma l_1 l_2 \frac{l_3}{D} + \sigma n l_2 D \frac{l_3}{D} + m^2 l_2 \frac{D^2}{n} \frac{l_3}{D} M_0^2, \tag{4.1}$$

where D are the domain dimensions. $l_1 < l_2 < l_3$ the plate dimensions, n the number of "wedges" per domain. Minimizing this expression with respect to n and D, we obtain

$$D = (\sigma^{1/2} l_1 / m M_0)^{2/3}, \quad n = (m^2 M_0^2 l_1 / \sigma)^{1/3}. \tag{4.2}$$

Substituting in these formulas the values for the surface energy σ of the 90° DB, we obtain

$$D \approx 2[\alpha\beta_3 M_0^2]^{1/6} l_1^{2/3} (\delta / d)^{2/3}, \quad n \approx {}^1\!/_2\, l_1^{1/3} (d / \delta)^{2/3} [\alpha\beta_3 M_0^2]^{-1/6}.$$

Analogously, for the 180° DB of the phase $\Phi_{\perp}$ we have

$$D \approx 2\left[\alpha\left(\beta' - \frac{d^2}{4\delta} + \frac{1}{2}\beta_3 M_0^2\right)\right]^{1/6} l_1^{2/3} \left(\frac{\delta}{d}\right)^{2/3},$$

$$n \approx \frac{1}{2} l_1^{1/3} \left(\frac{d}{\delta}\right)^{2/3} \left[\alpha\left(\beta' - \frac{d^2}{4\delta} + \frac{1}{2}\beta_3 M_0^2\right)\right]^{-1/6}.$$

Let us estimate the dimensions of the domains near T_M. Assuming, in accordance with[2], $\beta_3 M_0^2 = 0.9$, $\delta \sim 10^4$, $d \sim 10^2$, $2M_0 \approx 9 \cdot 10^2$ G, and $l_1 = 1$ mm, we obtain for domains separated by 90° DB $n \approx 1$ and $D \approx 0.1$ cm.

5)Farztdinov [4] investigated 180° DB located in the basal plane, and the subdivision of AFM with weak ferromagnetism into thermodynamic-equilibrium domains separated by such boundaries. Although the surface energy of such domains, generally speaking, is smaller than the surface energy of the domains considered by us in the phase $\Phi_{\perp}$, the situation may change near $T = T_M$, when $\sigma_{180°}(\Phi_{\perp})$ is greatly decreased. In this case, competition between the DB, considered in [4], and the DB considered in the present paper, is possible.

6)We note that in our earlier paper [1] the work of emergence of the magnetic moments to the surface of the sample was estimated incorrectly. The last term in formula (5) of [1] should be replace by $m^2 l_2 (D^2/n)(l_3/D)$, so that $\chi_{\perp}$ in Eq. (6) of that paper should be replaced by $\chi_{\perp}^2$.

In conclusion, the authors thank A. I. Akhiezer, A. S. Borovik-Romanov, and V. V. Eremenko for a discussion of the results.

[1] V. G. Bar'yakhtar, A. E. Borovik, and V. A. Popov, ZhETF Pis. Red. 9, 634 (1969) [JETP Lett. 9, 391 (1969)].

[2] V. I. Ozhogin and V. G. Shapiro, Zh. Eksp. Teor. Fiz. 55, 1737 (1968) [Sov. Phys.-JETP 28, 915 (1969)].

[3] L. D. Landau, Sobranie trudov (Collected Works) Nauka, 1969, p. 423.

[4] M. M. Farztdinov, Izv. Akad. Nauk SSSR, ser. fiz. 28, 590 (1964).

Translated by J. G. Adashko
149