SOVIET PHYSICS JETP VOLUME 32, NUMBER 4 APRIL, 1971

CONTRIBUTION TO THE THEORY OF WEAK TURBULENCE OF COUPLED WAVES IN A MAGNETOACTIVE SOLID-STATE PLASMA

A. A. BULGAKOV, S. I. KHANKINA, and V. M. YAKOVENKO

Institute of Radiophysics and Electronics, Ukrainian Academy of Sciences

Submitted April 18, 1970

Zh. Eksp. Teor. Fiz. 59, 1327–1335 (October, 1970)

We consider the problem of nonlinear interaction of waves with random phases in a magnetoactive solid-state plasma. To describe this interaction, a system of kinetic equations is used. It is demonstrated that a stationary state can be established as a result of turbulent phenomena in the development of the instability of acoustic waves under the influence of an external constant electric field. In the stationary state, the phonon-energy level is determined and the influence of the turbulent phonons on the electron drift velocity is considered.

1. In a solid-state plasma situated in a constant electric and a constant magnetic field, acoustic waves build up under certain conditions as a result of the inductive coupling between the conduction electrons and the lattice[1,2]. Limitation of the gain and establishment of a stationary amplitude are made possible by the nonlinear interaction of the waves. This interaction varies with the phases of the waves. If the phase of the wave remains constant during the characteristic time of its amplitude variation, then the stationary state is the result of energy transfer from the amplified wave to damped waves of lower frequency (decay instability). It is obvious that a problem of this kind is meaningful in the case of amplification of a wave generated by an external energy source with a narrow spectrum, and such a problem was considered in[3].

In this paper we consider the problem of nonlinear interaction of waves having random phases, i.e., the correlation between the initial and final values of the phase vanishes before the amplitude of the wave changes. The state of a plasma in which waves with random phases are excited is called turbulent. If the energy of the interaction between waves is small compared with the energy of the waves themselves, then the turbulence is weak, and we can use the kinetic equations for its description[4]. We assume that a weakly turbulent plasma is produced as a result of the development of instability of acoustic waves under the influence of the constant electric field. In this case wave generation occurs in the region of small wave vectors determined by the dimensions of the system, and as a result of the nonlinear interaction the energy is transferred to the waves with large wave vectors. In the region of large wave vectors, energy dissipation takes place. The energy level of the acoustic waves (phonons) in the stationary state is determined by the growth increment. In the stationary state, under the influence of the fields of the excited phonons, a change takes place in the constant drift velocity of the electrons, and leads to violation of Ohm's law.

2. To describe processes occurring in a magnetoactive solid-state plasma in the presence of a constant electric field, we use the following system of equations

$$c \operatorname{rot} \mathbf{E} = -\partial \mathbf{H} / \partial t, \tag{1}$$

$$c \operatorname{rot} \mathbf{H} = 4\pi e N (\partial \mathbf{u} / \partial t - \mathbf{v}), \tag{2}$$

$$\mathbf{E}_0 + \mathbf{E} + \frac{1}{c}[\mathbf{v}, \mathbf{H}_0 + \mathbf{H}] + \frac{m\nu}{e}\left(\mathbf{v} - \frac{\partial \mathbf{u}}{\partial t}\right) = 0, \tag{3}^*$$

$$M\left[\frac{\partial^2 \mathbf{u}}{\partial t^2} - s_t^2 \Delta \mathbf{u} - (s_l^2 - s_t^2)\nabla(\nabla \mathbf{u})\right] = e(\mathbf{E} + \mathbf{E}_0)$$

$$+ \frac{e}{c}\left[\frac{\partial \mathbf{u}}{\partial t}, \mathbf{H}_0 + \mathbf{H}\right] + m\nu\left(V - \frac{\partial \mathbf{u}}{\partial t}\right). \tag{4}$$

Here $\mathbf{E}$ and $\mathbf{H}$ are alternating electric and magnetic fields; e, m, N and $\mathbf{v}$ are the charge, mass, concentration, and velocity of the electrons; $\mathbf{u}$ is the lattice displacement vector; s_t and s_l are the velocities of the transverse and longitudinal sounds; M is the "effective ion mass," equal to $M = \rho/N$, where ρ is the crystal density; ν is the effective electron collision frequency. The constant electric field $\mathbf{E}_0$ is parallel to the constant magnetic field $\mathbf{H}_0$. In the absence of alternating fields, the conduction electrons drift with velocity $\mathbf{v}_0$ along the constant electric field, where $v_0 = e|E_0|/m\nu$, $\mathring{u}_Z^0 = 0$.

We have omitted the inertial term from the electron equations of motion, since we are interested in frequencies that are small compared with the electron cyclotron frequency $\omega_H = |e|H_0/mc$. In addition, we disregard the change of the electron concentration and the displacement current in Maxwell's equations, since the carrier concentration is assumed to be sufficiently large.

We represent the variable quantities in Eqs. (1)–(4) in the form

$$\mathbf{v} = \sum_{k=-\infty}^{\infty} C_\mathbf{k}(t)\mathbf{v}_\mathbf{k} e^{i(\mathbf{kr} - \omega_\mathbf{k} t)},$$

$$\mathbf{u} = \sum_{k=-\infty}^{\infty} C_\mathbf{k}(t)\mathbf{u}_\mathbf{k} e^{i(\mathbf{kr} - \omega_\mathbf{k} t)}$$

etc., where $C_\mathbf{k}(t)$ is the slowly varying amplitude of the harmonic with frequency $\omega_\mathbf{k}$ and wave vector $\mathbf{k}$; $\mathbf{v}_\mathbf{k}$ and $\mathbf{u}_\mathbf{k}$ constitute the solution of the linear system of equations (1)–(4),

$$C_\mathbf{k} = C_{\mathbf{k}-}^*, \quad \mathbf{v}_\mathbf{k} = \mathbf{v}_{\mathbf{k}-}^*, \quad \omega(\mathbf{k}) = -\omega^*(-\mathbf{k}).$$

*$[\mathbf{v}, \mathbf{H}_0 + \mathbf{H}] \equiv \mathbf{v} \times (\mathbf{H}_0 + \mathbf{H})$.

The frequency and the wave vector are connected by the dispersion relation

$$(\omega^2 - k^2 s_t^2)^2(\omega^2 - k^2 s_l^2)\left[\left(\omega - k_z v_0 + \frac{ik^2c^2}{\omega_0^2}\nu\right)^2 - \frac{k^2 k_z^2 c^2 H_0^2}{(4\pi e N)^2}\right]$$
$$= -\frac{H_0^2}{4\pi\rho}(\omega^2 - k^2 s_t^2)\left(\omega + i\frac{k^2c^2\nu}{\omega_0^2}\right)[\omega^2(k^2 + k_z^2) - k^2(k_x^2 + k_y^2)s_t^2$$
$$- 2k^2 k_z^2 s_l^2] - \frac{\omega^2 H_0^4 k_z^2}{(4\pi\rho)^2}\left[k^2 - \left(\frac{4\pi e N v_0}{cH_0}\right)^2\right][\omega^2 - (k_x^2 + k_y^2)s_t^2 - k_z^2 s_l^2]. \quad (5)$$

The z axis is parallel here to the magnetic field H_0.

Since the parameter $H_0^2/4\pi\rho s_{t,l}^2$ of the coupling between the electromagnetic and acoustic waves is small, Eq. (5) can be solved by successive approximations with respect to this parameter[1)]. When $\rho \to \infty$, relation (5) breaks up into equations for the transverse ($\omega = ks_t$) and longitudinal ($\omega = ks_l$) acoustic waves and an equation for the helical waves (helicons)

$$\omega = k_z v_0 \pm \frac{kk_z c H_0}{4\pi e N} + i\frac{k^2c^2\nu}{\omega_0^2},$$

where $\omega_0^2 = 4\pi e^2 N/m$. Taking into account the finite value of ρ, we can find the correction to the linear dispersion laws and the electronic damping of the acoustic waves. Acoustic waves with such a dispersion and such a damping have electromagnetic-field components and are therefore called coupled. We shall henceforth be interested in the instability of only the transverse acoustic waves, since it sets in at relatively low electron drift velocities. Depending on the relations between the plasma parameters, different expressions are obtained for the growth increment. It can be shown that if the condition

$$\left(\frac{v_0\cos\theta - s_t}{s_t\cos\theta}\right)^3 \frac{\nu^2}{\omega_H^2} > \frac{H_0^2}{4\pi\rho s_t^2},$$

is satisfied, where θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$ (weak coupling between the helicons and the acoustic waves), then the electronic correction to the dispersion is

$$\delta\omega = \delta\omega' + i\gamma = -\frac{H_0^2}{8\pi\rho}\left(\frac{4\pi e N}{cH_0}v_0 - k\right) \quad (6)$$
$$\times \cos^2\theta\left[s_t - v_0\cos\theta + \frac{kcH_0}{4\pi e N}\cos\theta + i\frac{kc^2\nu}{\omega_0^2}\right]^{-1}.$$

if $v_0 > cH_0 k/4\pi e N$, then the electronic damping of the sound reverses sign.

For acoustic oscillations to build up it is necessary that the electronic growth increment exceed the lattice absorption. Since the occurrence of instability can be expected for long-wave phonons, then the condition $\omega\tau_{ph} \ll 1$ (where τ_{ph} is the relaxation time of the thermal phonons) is well satisfied, and the lattice absorption is described by the Akhiezer mechanism[5] and is proportional to the square of the wave vector. A comparison of the electronic growth increment and of the lattice-absorption coefficient shows that in the wave vector region $k < 4\pi e N s_t/cH_0$ ($v_0 \sim s_t$) and for semiconducting materials of the type PbTe ($N \approx 4 \times 10^{17} - 2 \times 10^{18}$ cm^{-3}, $\rho = 3.5$ g/cm^3, $s_t \sim 10^5$ cm/sec, $\nu \approx 2 \times 10^{10} - 10^{11}$ sec^{-1}, $m \approx 10^{-29}$ g, and $H_0 = 10^3 - 10^4$ Oe) at helium temperatures the lattice absorption is negligibly small[6]. (All the numerical estimates will henceforth be carried out for PbTe, since interaction of helical and acoustic waves was recently observed in it experimentally[7].) As seen from (6), the growth increment reaches a maximum when the frequencies and wave vectors of the sound and of the helicon coincide, i.e.,

$$v_0\cos\theta - s_t = cH_0 k\cos\theta / 4\pi e N.$$

Thus, at $k \approx 2\pi/L = 1.5$ cm^{-1} (L is the length of the sample), $H_0 \sim 10^3$ Oe, and $\theta \approx 0$ we obtain $v_0 - s_t \approx 7.5 \times 10^3$ cm/sec and $\gamma \approx 3 \times 10^2$ sec^{-1}.

In strong magnetic fields ($H_0 \sim 10^4$ Oe) it is possible to satisfy the strong-coupling condition

$$\left(\frac{v_0\cos\theta - s_t}{s_t\cos\theta}\right)^3 \frac{\nu^2}{\omega_H^2} < \frac{H_0^2}{8\pi\rho s_t^2},$$

and the growth increment at the point of resonance of the helical and acoustic waves reaches the value

$$|\gamma_p| = \left[\frac{eH_0}{2Mc}k\cos^2\theta\left(v_0 - \frac{cH_0 k}{4\pi e N}\right)\right]^{1/2} \qquad \gamma_p \approx 3\cdot 10^3\,\text{sec}^{-1}. \quad (7)$$

3. We proceed now to analyze nonlinear effects of wave interaction. The simplest forms of such an interaction are three-wave processes of the type of decay and coalescence of waves. These processes are characterized in the equations of motion (3) and (4) by linear terms (Lorentz force). Taking these terms into account, we can find the nonlinear current $\mathbf{j}^{nl}$ [8]. In the Lagrangian of the system, the wave interaction governed by the nonlinearity of the Lorentz force is described by the term $\mathbf{j}^{nl}\mathbf{A}/c$ ($\mathbf{A}$ is the vector potential of the electromagnetic field, $\mathbf{H} = \text{curl}\,\mathbf{A}$, $\mathbf{E} = -c^{-1}\partial\mathbf{A}/\partial t$, the oscillations are assumed nonpotential, $\varphi = 0$). In addition, the interaction of the waves is connected also with the anharmonicity of the lattice[9].

We are interested primarily in the time evolution of a packet of transverse acoustic waves. Let us assume for simplicity that there is no constant electric field. Since we are interested only in the orders of magnitude of the matrix elements, it is clear that the order of magnitude cannot depend strongly on the constant electric field. This is also seen from the fact that the values of the matrix elements for weakly-damped waves are determined by their frequencies $\omega = \omega'(\mathbf{k})$ and do not depend on the wave damping. The frequencies of the acoustic waves, in turn, are practically independent of the electron drift.

The term $\mathbf{j}^{nl}\mathbf{A}/c$ in the Lagrangian of the system can be transformed into

$$L^{(3)} = \int d^3\mathbf{r}\left\{\frac{eN}{c}\sum_{\mathbf{k}\mathbf{k}'\mathbf{k}''}\frac{i\mathbf{v}_\mathbf{k}^*}{\omega_\mathbf{k}}[\mathbf{v}_{\mathbf{k}'}\mathbf{H}_{\mathbf{k}''}] - i\omega_{\mathbf{k}'}\mathbf{u}_\mathbf{k}^*[\mathbf{u}_{\mathbf{k}'}\mathbf{H}_{\mathbf{k}''}]\right\}\cdot \quad (8)$$
$$\times \exp\{i[(-\mathbf{k} + \mathbf{k}' + \mathbf{k}'')\mathbf{r} - (-\omega_\mathbf{k} + \omega_{\mathbf{k}'} + \omega_{\mathbf{k}''})t]\}C_\mathbf{k}^* C_{\mathbf{k}'} C_{\mathbf{k}''} + \text{c.c.}$$

The physical meaning of $C_\mathbf{k}$ can be explained from the following considerations. Namely, the coupled-wave energy per unit volume can be written, on the basis of the system (1)–(4) in the form ($\nu = 0$, $E_0 = 0$)

$$W = \sum_k |C_\mathbf{k}|^2\left\{\frac{|\mathbf{h}_\mathbf{k}|^2}{8} + \frac{\rho|\dot{\mathbf{u}}_\mathbf{k}|^2|\omega_\mathbf{k}|^2}{2} + \frac{\rho s_t^2}{2}|[\mathbf{k}\mathbf{u}_\mathbf{k}]|^2 + \frac{\rho s_l^2}{2}|(\mathbf{k}\mathbf{u}_\mathbf{k})|^2\right\} \quad (9)$$

($\mathbf{h}_\mathbf{k}$ and $\mathbf{u}_\mathbf{k}$ are connected by the system of linear relations (1)–(4)). On the other hand, the energy of the

---

[1)]The solution of Eq. (5) for the one-dimensional case when the wave vector is directed along the magnetic field H_0 was investigated in [1].

coupled electromagnetic and acoustic waves can be represented in the form $W = \sum_k n_k |\omega_k|$, where $|\omega_k|$ is the energy of the k-th harmonic (particle) and n_k is the number of particles. We then obtain

$$\frac{|\mathbf{h}_k|^2}{8\pi} + \frac{\rho}{2}\left(|\omega_k|^2 |\mathbf{u}_k|^2 + |[\mathbf{k}\mathbf{u}]|^2 s_t^2 + |(\mathbf{k}\mathbf{u})|^2 s_l^2\right) = |\omega_k|, \quad (9a)$$

$n_k = |C_k|^2$, with C_k^* and C_k having the meaning of particle creation and annihilation operators.

Expressing the quantities v_k, h_k, and u_k in terms of ω_k with the aid of the linear equations, we can represent the Lagrangian $L^{(3)}$ describing the interaction of three waves in the form

$$L^{(3)} = \sum_{kk'k''} V^e_{kk'k''} C_k^* C_{k'} C_{k''} \delta_{k,k'+k''} + \text{c.c.} \quad (10)$$

where $V^e_{kk'k''}$ is the matrix element. The explicit form of the matrix element is quite cumbersome and will not be presented here. However, the order of magnitude of $V^e_{kk'k''}$ can be estimated by starting from the expression (18), the linear relations between the components of u, v, and H, and the normalization condition (9a). It must be borne in mind here that in formula (9a) for the helicons we can neglect the terms proportional to u and $|h_k| \approx (8\pi\omega_k)^{1/2}$, whereas for sound $|u_k| \approx (\rho\omega_k)^{-1/2}$. We did not write out terms of the type $C_k C_{k'} C_{k''}$ etc. in the Lagrangian, since, owing to the conservation laws, they make no contribution to the probabilities of the processes.

The matrix element $V^a_{kk'k''}$, due to the lattice anharmonicity, is of the following order of magnitude[9]:

$$V^{\bar{a}}_{kk'k''} \sim Bk^3 |u_k|\,|u_{k'}|\,|u_{k''}|, \quad (11)$$

where B is the effective value of the third-order modulus of elasticity. In order of magnitude, $B \approx \rho s^2$.

Knowing the form of the matrix elements, we can write down a system of kinetic equations[10] describing the variation of the number of particles as a result of decay and coalescence, and estimate the characteristic relaxation times:

$$\frac{\partial n_k^\alpha}{\partial t} = 4\pi \sum_{\beta,\gamma,k',k''} |V^{\alpha\beta\gamma}_{kk'k''}|^2 \delta(\omega_k^\alpha - \omega_{k'}^\beta - \omega_{k''}^\gamma)(n_{k'}^\beta n_{k''}^\gamma - n_k^\alpha n_{k''}^\gamma - n_k^\alpha n_{k'}^\beta)\delta_{k,k'+k''} + 8\pi \sum_{\beta,\gamma,k',k''} |V^{\gamma\alpha\beta}_{k''kk'}|^2 \delta(\omega_{k''}^\gamma - \omega_k^\alpha - \omega_{k'}^\beta)(n_{k'}^\beta n_{k''}^\gamma + n_k^\alpha n_{k''}^\gamma - n_{k'}^\beta n_k^\alpha)\delta_{k'',k+k'}. \quad (12)$$

The summation is carried out here over positive frequencies. The indices α, β, and γ denote the types of interacting waves and each can stand for the following: t—transverse sound, l—longitudinal sound, h—helicon.

It can be shown that wave-interaction processes in which two helicons and sound take part are determined by the matrix element[2)] V_{hhs}. (In estimating the matrix element, we assume that $s_t \sim s_l \sim s$. This condition is equivalent to ku = 0. For all the known semiconductors $s_l/s_t \sim 1.4-3$.) we have

$$V^e_{hhs} \approx \frac{k^{5/2} cH_0}{2\pi e N(\rho s)^{1/2}} \quad \text{for} \quad k < \frac{4\pi eNs}{cH_0}, \qquad V^e_{hhs} \approx \left(\frac{k^3 s}{\rho}\right)^{1/2} \quad \text{for} \quad k > \frac{4\pi eNs}{cH_0}. \quad (13a)$$

In this case

$$R \equiv \left|\frac{V^e_{hhs}}{V^a_{hhs}}\right| \sim \frac{4\pi\rho s^2}{H_0^2} \quad \text{for} \quad k < \frac{4\pi eNs}{cH_0}, \qquad R \sim \frac{4\pi\rho s^2}{H_0^2}\left(\frac{kcH_0}{4\pi eNs}\right)^3 \quad \text{for} \quad k > \frac{4\pi eNs}{cH_0}. \quad (13b)$$

In processes in which one helicon and two acoustic waves take part we have

$$V_{hss} \sim \frac{k}{\rho}\left(\frac{eNH_0}{c}\right)^{1/2}, \quad (14a)$$

$$R \sim \frac{4\pi eNs}{kcH_0} \quad \text{for} \quad k < \frac{4\pi eNs}{cH_0}, \qquad R \sim \frac{kcH_0}{4\pi eNs} \quad \text{for} \quad k > \frac{4\pi eNs}{cH_0}. \quad (14b)$$

Finally, in interactions of three acoustic waves, the matrix element due to the lattice turns out to be the largest and equal to

$$V^a_{sss} \sim \rho s^2 \frac{k^3}{(\rho\omega)^{3/2}} \sim \left(\frac{k^3 s}{\rho}\right)^{1/2}. \quad (15a)$$

Here

$$V^e_{sss} \sim V^a_{sss} \frac{H_0^2}{4\pi\rho s^2}\left(\frac{4\pi eNs}{kcH_0}\right) \quad \text{for} \quad k < \frac{4\pi eNs}{cH_0}, \qquad V^e_{sss} \sim V^a_{sss} \frac{H_0^2}{4\pi\rho s^2}\left(\frac{4\pi eNs}{kcH_0}\right)^2 \quad \text{for} \quad k > \frac{4\pi eNs}{cH_0}, \quad (15b)$$

i.e., we have $V^a_{sss} > V^e_{sss}$ for all possible k.

Assume that at the initial instant of time a packet of transverse acoustic waves with energy W_0 is located in the region $k \sim k_0$ and its width is $\Delta k \sim k_0$. Two waves interact in the interior of the packet and form either a helicon or longitudinal sound. The occurrence of a transverse-sound wave is impossible by virtue of the energy and momentum conservation laws. It should be noted that, in principle, the interaction of three transverse acoustic waves is possible if the wave damping greatly exceeds the correction that must be introduced into the frequency because of the dispersion of the speed of sound[12,13]. However, when the inductive interaction between the lattice and the conduction electrons is taken into account, the frequency increment due to the electronic dispersion of sound is larger than the electronic damping of the sound by a factor ω_H/ν.

From the kinetic equation (12) we find that the nonlinear increments of the damping of sound γ^{nl} with formation of a helicon or of longitudinal sound are respectively equal to

$$\gamma^{tth} \approx \frac{W_0}{\rho s^2}\frac{eH_0}{Mc}, \quad (16)$$

$$\gamma^{ttl} \approx \frac{W_0}{\rho s^2}\omega, \quad (17)$$

i.e., $\gamma^{tth} \ll \gamma^{ttl}$. Thus, the time evolution of the packet is determined by the interaction of the acoustic waves with one another. As a result, the number of transverse phonons in the region k_0 decreases.

4. If the phonon damping is offset by generation, then a stationary state is established. It is clear that in the presence of electron drift in a constant electric field, a situation is possible when the growth increment (6) is equal to the nonlinear damping decrement (17). Under the influence of these two competing factors, the phonon energy turns out to be

$$W_0 = \frac{\gamma}{\omega(k_0)}\rho s^2. \quad (18)$$

---

2)The matrix element describing scattering of helicons by phonons in deformation interaction was obtained by Suramlishvili [11].

Thus, the nonlinear damping of the transverse acoustic waves is effected by transferring their energy to the longitudinal sound, which in turn is damped. If the linear damping of the longitudinal sound in the region $k \sim k_0$ turns out to be very small, then it can be assumed that the damping of the transverse and longitudinal sounds in the interval $k \sim k_0$ is effected via further transformation of the energy towards large $k \gg k_0$, where the damping of the sound is large, since it is proportional to k^2. In this case apparently the energy-containing region is separated from the damping region by a certain intermediate or intertial region. In the latter region, starting from the assumption that the weak turbulence is local[14], we can find the spectral distribution of the phonons. Namely, the amount of energy dissipated per unit time, i.e., the energy flux, is a constant quantity equal to $p_0 = V^2 n_k^2 k^6$. Hence

$$n_k \approx p_0^{1/2} / Vk^3 \sim (p_0 / k^6)^{1/2}, \tag{19}$$

and the energy spectrum is $W_k \sim n_k \omega_k k^2 \sim (p_0/k^3)^{1/2}$.

Let us estimate the influence of the nonlinear processes on the electron drift velocity, assuming that the external electric field remains unchanged. From (2) and (3) it follows that

$$v_z = v_0 - \frac{e}{mc\nu}\left(H_y \frac{\partial u_x}{\partial t} - H_x \frac{\partial u_y}{\partial t}\right). \tag{20a}$$

As seen from this formula, the sign of the increment of the drift velocity depends on the phase relations between H and $\partial u/\partial t$. For growing waves, H and $\partial u/\partial t$ are in phase and therefore the electron drift velocity decreases under the influence of the alternating fields. Let us change over to the Fourier representation and use the linear relations between H and u:

$$H_x = \frac{Mc}{e} \frac{\omega^2 - k^2 s_t^2}{v_0 - kcH_0/4\pi eN} u_y; \quad H_x = iH_y, \quad u_x = iu_y.$$

For simplicity we have put here $k = k_z$. With the aid of the dispersion equation (5) we obtain, after averaging over the volume,

$$\langle v_z \rangle = v_0 - \frac{H_0 c}{2\pi eN} \sum_{k=0}^{\infty} \frac{\omega^2 k^3 |u_k|^2}{(\omega - kv_0 + k^2 cH_0/4\pi eN)^2 + k^4 c^4 \nu^2/\omega_0^4}. \tag{20b}$$

When $v_0 \ll s_t$, the growth increment is $\gamma \sim m\nu v_0^2/Ms^2$, $W_0 \sim \nu Nmv_0^2/\omega(k_0)$, and the change of the drift velocity is equal to

$$\Delta v = \langle v_z \rangle - v_0 \approx -\frac{k_0 cH_0}{2\pi eN} \frac{W_0}{\rho s^2} \approx -\frac{\gamma}{m\nu} \frac{W_0}{v_0 N} \approx -\frac{\gamma(k_0) v_0}{\omega(k_0)}. \tag{21}$$

It follows therefore that in a weakly turbulent plasma the effective electron collision frequency increases, $\nu_{\text{eff}} \approx \nu(1 + \gamma/\omega(k_0))$.

At resonance, obviously, the change of the drift velocity will be more appreciable. In this case a narrow wave packet with the maximum growth increment (6) is separated out of the set of growing waves. The width of the packet is determined, for weak coupling, from the condition

$$\left| v_0 \cos\theta - s_t - \frac{k \cos\theta cH_0}{4\pi eN} \right| \lesssim \frac{kc^2\nu}{\omega_0^2}, \qquad \frac{\Delta k}{k_0} \lesssim \frac{\nu}{\omega_H \cos\theta} \ll 1. \tag{22}$$

In the wave-vector region Δk, the acoustic waves have a linear dispersion and interaction of three transverse phonons is therefore possible. Such a situation is connected with the fact that in the resonance region the correction to the linear dispersion law is much smaller than the growth increment. The growth increment is ω_H^2/ν^2 times larger than outside the resonant region. The matrix element describing the interaction of these waves is determined as before by formula (15a).

Establishment of the stationary state causes, first, the phonons to diffuse by collision out of the resonant region, and second, the electron drift velocity to change under the influence of the alternating fields of the resonant waves. The change of the drift velocity can take the system out of the resonance state (the condition (22) is violated), leading to a decrease of the growth increment.

If the correction to the drift velocity is small, then a stationary state is established in the resonant region, and the phonon energy W_0 reaches the value $W_0 \sim \omega_H^2 Nmv_0^2/v\omega$. Since the range of v_0 corresponding to resonance is determined by formula (22), $\Delta v < kc^2\nu/\omega_0^2$, and (see (20b))

$$|\Delta v| \approx \frac{\omega_H \omega_0^2 W_0}{\omega(k_0) \nu^2 \rho c^2} v_0, \tag{23}$$

it follows that the resonance-conservation condition for a developed turbulence takes the form

$$k_0 > \frac{\omega_H}{\nu} \frac{\omega_0}{c} \left(\frac{m}{M} \frac{v_0}{c} \frac{\omega_0}{\nu}\right)^{1/3}. \tag{24}$$

In this case the change of the drift velocity is

$$\Delta v \approx -\frac{m}{M} \frac{\omega_H^3}{\nu^3} \frac{\omega_0^2}{k_0^2 c^2} v_0. \tag{25}$$

If the inequality $\Delta v > kc^2\nu/\omega_0^2$ is satisfied, i.e., $k_0 < \omega_0^2 c^{-2}(\omega_H W_0/\rho\nu^3)^{1/2}$, then the change of the drift velocity causes an appreciable change in the growth increment:

$$\gamma^t(v - |\Delta v|) \approx \frac{1}{2} \frac{m}{M} \frac{\nu^5 c^6 k_0^2 \omega(k_0) \rho^2}{\omega_H \omega_0^6 W_0^2}. \tag{26}$$

The energy of the phonons in the stationary state is determined from the condition $\gamma^t(v - |\Delta v|\Delta v|) = \omega(k_0)W_0/\rho s^2$ and turns out to be

$$W_0 \approx \rho c^2 \frac{\nu^2}{\omega_0^2} \left(\frac{m}{M} \frac{\omega^2(k_0)}{\omega_H \nu}\right)^{1/3}. \tag{27}$$

The change of the electron drift velocity is given by

$$\Delta v = -v_0 \left(\frac{m}{M} \frac{\omega_H^2}{\omega(k_0)\nu}\right)^{1/3}. \tag{28}$$

Estimates show that to find the stationary state under strong-coupling conditions it is also necessary to take into account the change of the growth increment under the influence of the alternating fields. The phonon energy level and the change in the constant electron velocity can be determined from the same considerations as in weak coupling:

$$W_0 \approx N \frac{mv_0^2}{2} \left(\frac{\nu \omega_H \omega_0^2}{k_0^4 s^2 c^2}\right)^{1/3}, \qquad \Delta v = -\frac{H_0 c k_0^3 W_0}{4\pi eN\rho\gamma_p^2(k_0)} = -v_0 \left(\frac{\nu \omega_H k_0^2 c^4}{\omega_0^4 s^2}\right)^{1/3} \tag{29}$$

(It is assumed in (23)—(29) that $v_0 \approx s$.) For the parameters of PbTe, the change of the electron drift velocity in the resonance region increases appreciably compared with the nonresonant region and can lead to an appreciable deviation from Ohm's law: $\Delta v \approx v_0(10^{-1}-10^{-2})$.

It should be noted that although we have considered an unbounded medium, the results apparently can be valid also for a qualitative description of processes in samples with finite dimensions. Since the interaction has a resonant character, the waves in which the phase-velocity direction coincides with the drift direction have a growth increment larger than the damping decrement of waves traveling in opposite directions. For this reason, multiple reflections from the boundaries lead to an increase of the effective interaction length.

In conclusion, the authors thank É. A. Kaner for interest in the work.

[1]F. G. Bass and V. M. Yakovenko, Fiz. Tverd. Tela, 8, 2793 (1966), [Sov. Phys.-Solid State, 8, 2231 (1966)]. É. A. Kaner, V. M. Yakovenko, Zh. Eksp. Teor. Fiz. 53, 712 (1967) [Sov. Phys.-JETP 26, 446 (1968)].

[2]M. K. Balakirev and S. V. Bogdanov, Fiz. Tekh. Poluprov. 3, 1267 (1969) [Sov. Phys.-Semicond. 3, 1063 (1970)].

[3]A. A. Bulgakov, S. I. Khankina and V. M. Yakovenko, Fiz. Tverd. Tela 11, 2749 (1969) [Sov. Phys.-Solid State 11, 2226 (1970)].

[4]A. A. Galeev and V. I. Karpman, Zh. Eksp. Teor. Fiz. 44, 592 (1963) [Sov. Phys.-JETP 17, 403 (1963)].

[5]A. I. Akhiezer, Zh. Eksp. Teor. Fiz. 1, 277, 1939.

[6]Yu. I. Ravich, B. A. Efimova and I. A. Smirnov, Metody issledovaniya poluprovodnikov v primenenii k khal'kogenidam svintsa PbTe, PbSe, PbS, (Semiconductor Research Methods Applied to the Lead Chalcogenides PbTe, PbSe, and PbS), Nauka, 1968.

[7]W. Schilz, Phys. Rev. Lett., 20, 104 (1968).

[8]V. N. Tsitovich, Nelineinye éffekty v plazme (Nonlinear Effects in Plasma), Chap. 2, Nauka, 1967.

[9]L. D. Landau and E. M. Lifshitz, Teoriya uprugosti (Elasticity Theory), Nauka, 1965, p. 155.

[10]A. A. Vedenov, Voprosi teorii plazmy, (Problems of Plasma Theory) v. 3, Atomizdat, 1963, p. 203.

[11]G. I. Suramlishvili, Fiz. Tverd. Tela 12, 329 (1970) [Sov. Phys.-Solid State 12, 267 (1970)].

[12]S. Simons, Proc. Phys. Soc., 82, 401 (1963).

[13]K. Ozaki and N. Mikoshiba, J. Phys. Soc. Japan, 21, 2486 (1966). M. Tsaji and S. Jnoue, Japan J. Appl. Phys. 8, 704 (1969).

[14]V. E. Zakharov, Zh. Eksp. Teor. Fiz. 51, 688 (1966) [Sov. Phys.-JETP 24, 455 (1967)]; V. E. Zakharov and N. N. Filonenko, Prik. Mat. Tekh. Fiz. 5, 62 (1967); V. M. Yakovenko, Zh. Eksp. Teor. Fiz. 57, 574 (1969) [Sov. Phys.-JETP 30, 315 (1970)].

Translated by J. G. Adashko
152