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THE THEORY OF CYCLOTRON AND SPIN WAVES IN AN ANISOTROPIC ELECTRON FLUID

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The cyclotron and spin wave spectra are studied for an electron fluid with an ellipsoidal Fermi surface. In contrast with previous investigations,[17,18] no simplifications are made regarding the form of the correlation functions describing the interaction between the fluid quasiparticles. The dependence of possible spin and cyclotron wave frequencies on the orientation of the stationary magnetic field relative to the crystal axis is determined. Effects connected with the Cerenkov damping of cyclotron waves are considered; in particular, the dependence of the maximum possible wave vector on the orientation of the stationary magnetic field relative to chosen directions in the crystal is established.

1. The Fermi surfaces of real metals are anisotropic and can differ radically from the spherical in a number of cases.[1-3] It is therefore of interest to study the effect of the anisotropic character of the Fermi surface on the propagation of spin and cyclotron waves in an electron fluid. The existence of such waves was first predicted by Silin[4] on the basis of the theory of an electron fluid.[5] At the present moment, the spectra of spin and cyclotron waves in the electron fluid of metals with an isotropic Fermi surface have been considered in the researches of a whole series of authors.[6-16,4] Spin waves in an anisotropic electron fluid were studied in[17,18]. However, it was assumed in[17,18] that the function $\psi(\mathbf{p}, \mathbf{p}')$ characterizing the correlation of the electrons does not depend on the momenta, i.e., $\psi(\mathbf{p}, \mathbf{p}') \equiv$ const. The assumption that ψ is independent of the momenta is a significant simplification. Nevertheless, it can be thought that such an approximation always takes validly into account the qualitative effect of the appearance of spin waves. However, it is known[11,12] that even qualitatively correct results cannot be obtained when cyclotron waves are studied under the assumption that the correlation function is independent of the momenta.

In the present research we determine the spectra of spin and cyclotron waves in an electron fluid with an ellipsoidal Fermi surface. There are no metals with such a Fermi surface, but analysis shows that, in a number of cases, for a definite arrangement of a stationary magnetic field relative to the crystallographic axes, the situation can in fact be close to such a model.

Let us consider a metal whose constant energy surfaces in momentum space $\mathbf{p}$ (p-space) are arbitrary ellipsoids, described by the equation

$$2\varepsilon = \mathbf{p}\hat{\delta}\mathbf{p}, \tag{1.1}$$

where ϵ is the energy and $\hat{\delta}$ the reciprocal effective mass tensor.

We transform from real p-space to a space $\mathbf{w}$ (w-space) such that the surfaces of constant energy in it are spheres.[19,20] The transformation

$$\mathbf{w} = \delta_0^{-1/2}\hat{\delta}^{1/2}\mathbf{p} \equiv A\mathbf{p} \tag{1.2}$$

reduces Eq. (1.1) to the form

$$2\varepsilon = \delta_0 \mathbf{w}\mathbf{w}, \tag{1.3}$$

where δ_0 is an arbitrary constant having the dimensionality of $\hat{\delta}$. The presence of the arbitrary constant δ_0 in the intermediate results is unimportant, since it does not enter into the final result. It is seen from Eq. (1.3) that the constant-energy surfaces are spheres in w-space. In a set of coordinates with axes directed along the principal axes of the ellipsoid, the matrix $\hat{A}$ is diagonal and has, according to (1.1) and (1.2), the following form:

$$\hat{A} = \begin{pmatrix} (\delta_0 m_1)^{-1/2} & 0 & 0 \\ 0 & (\delta_0 m_2)^{-1/2} & 0 \\ 0 & 0 & (\delta_0 m_3)^{-1/2} \end{pmatrix}, \tag{1.4}$$

where $1/m_1$, $1/m_2$, and $1/m_3$ are nonvanishing elements of the tensor $\hat{\delta}$, which is also diagonal in the chosen set of coordinates.

2. Before proceeding to study the spin waves connected with the oscillations of the spin-density vector function $\boldsymbol{\sigma}$, let us consider the equilibrium state of an electron fluid with ellipsoidal Fermi surface, located in a stationary homogeneous magnetic field $\mathbf{B}$. As is known,[4] in the approximation linear in the field $\mathbf{B}$, the distribution function f does not differ from the Fermi distribution and the vector function of the spin density in phase space has the form

$$\boldsymbol{\sigma}_0(\mathbf{p}) = \frac{\partial f_0}{\partial \varepsilon}\Delta\boldsymbol{\varepsilon}_2(\mathbf{p}), \tag{2.1}$$

where $\Delta\epsilon_2$ is determined from the equation

$$\Delta\boldsymbol{\varepsilon}_2(\mathbf{p}) = -\mu_0\mathbf{B} + \int d^3p'\psi(\mathbf{p}, \mathbf{p}')\frac{\partial f_0}{\partial \varepsilon}\Delta\boldsymbol{\varepsilon}_2(\mathbf{p}'). \tag{2.2}$$

Here μ_0 is the magnetic moment of the electron, and $\psi(\mathbf{p}, \mathbf{p}')$ is the correlation function describing the spin-dependent part of the interaction of the quasiparticles of the electron fluid. Using Eq. (1.2), we transform from p-space to w-space. In w-space, Eq. (2.2) takes the form

$$\Delta\boldsymbol{\varepsilon}_2^*(\mathbf{w}) = -\mu_0\mathbf{B} + \int d^3w'\, \psi^*(\mathbf{w}, \mathbf{w}')\frac{\partial f_0}{\partial \varepsilon}\Delta\boldsymbol{\varepsilon}_2^*(\mathbf{w}'), \tag{2.3}$$

Here

$$\Delta\boldsymbol{\varepsilon}_2^*(\mathbf{w}) = \Delta\boldsymbol{\varepsilon}_2(\hat{A}^{-1}\mathbf{w}), \tag{2.4}$$

$$\psi^*(\mathbf{w}, \mathbf{w}') = |\hat{A}^{-1}|\psi\hat{A}^{-1}\mathbf{w}, \hat{A}^{-1}\mathbf{w}'). \tag{2.5}$$

According to Eq. (1.3), preferred directions are absent in w-space. One can then assume the function $\psi^*(\mathbf{w}, \mathbf{w}')$ to be a function only of the angle Θ between the vectors $\mathbf{w}, \mathbf{w}'$, which lie on the Fermi sphere in w space. Therefore, it is apparently possible to make the expansion of the function $\psi^*(\mathbf{w}, \mathbf{w}')$ in Legendre polynomials

$$\frac{1}{\pi^2\hbar^3}\left(\frac{2\varepsilon_F}{\delta_0^3}\right)^{1/2}\psi^*(\mathbf{w},\mathbf{w}')=\frac{(2\varepsilon_F m_1m_2m_3)^{1/2}}{\pi^2\hbar^3}\psi(\hat{A}^{-1}\mathbf{w},\hat{A}^{-1}\mathbf{w}')$$
$$=\sum_{l=0}(2l+1)\beta_l P_l(\cos\Theta), \tag{2.6}$$

which is widely used in the theory of a Fermi liquid[4,21] Here ϵ_F is the Fermi energy, β_l are constant coefficients, and

$$P_l(z)=\frac{1}{2^l l!}\frac{d^l}{dz^l}(z^2-1)^l.$$

Using the expansion (2.6), we obtain a solution of Eq. (2.3)[4,6]

$$\Delta\varepsilon_2^*=-\gamma\mathbf{B},\ \text{where}\ \gamma=\mu_0/(1+\beta_0).$$

Equations (2.1) and (2.4) then permit us to write for the equilibrium state.

$$\sigma_0=-\gamma\frac{\partial f_0}{\partial\varepsilon}\mathbf{B}.$$

Let us now consider small departures $\delta\sigma$ of the spin density from its equilibrium value σ_0, i.e., we assume that the vector function of the spin density in phase space has the form

$$\sigma=-\gamma\frac{\partial f_0}{\partial\varepsilon}\mathbf{B}+\delta\sigma.$$

The small nonequilibrium contribution $\delta\sigma\alpha\exp(-i\omega t+i\mathbf{k}\cdot\mathbf{r})$ obeys the following kinetic equation:[4,6]

$$-i\omega\delta\sigma+i(\mathbf{kv})\left(\delta\sigma-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right)+\frac{e}{c}\left([\mathbf{vB}]\frac{\partial}{\partial\mathbf{p}}\right)\left(\delta\sigma-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right)$$
$$-\frac{2\gamma}{\hbar}\left[\mathbf{B},\ \delta\sigma-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right]=\delta\mathbf{I}_2, \tag{2.7*}$$

where $\mathbf{v}=\partial\epsilon/\partial\mathbf{p}$ is the velocity of the quasiparticle, $\delta\mathbf{B}$ the nonequilibrium magnetic induction and

$$\delta\varepsilon_2=-\mu_0\delta\mathbf{B}+\int d^3p'\,\psi(\mathbf{p},\mathbf{p}')\,\delta\sigma(\mathbf{p}',\mathbf{r}).$$

As the collision integral, we use the following model expression:[23]

$$\delta\mathbf{I}_2=-\left(\frac{1}{\tau}+\frac{1}{\tau_2}\right)\left(\delta\sigma-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right)$$
$$+\frac{1}{\tau}\frac{\partial f_0}{\partial\varepsilon}\left[\int d^3p\frac{\partial f_0}{\partial\varepsilon}\right]^{-1}\int d^3p\left(\delta\sigma-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2\right).$$

We transform from p-space to w-space. We make use of Eq. (1.2) and also the evident fact that the matrix $\hat{A}$ is symmetric in our case, i.e., that $\mathbf{p}_1(\hat{A}\mathbf{p}_2)=(\hat{A}\mathbf{p}_1)\mathbf{p}_2$ for any two vectors $\mathbf{p}_1$ and $\mathbf{p}_2$. We then have the following relations:[19,20]

$$\frac{\partial\delta\sigma_i^*}{\partial\mathbf{w}}=\hat{A}^{-1}\frac{\partial\delta\sigma_i}{\partial\mathbf{p}},\quad i=1,2,3, \tag{2.8a}$$

$$\left([\mathbf{vB}]\frac{\partial}{\partial\mathbf{p}}\right)\delta\sigma=\left([(\hat{A}^{-1}\mathbf{v})(|\hat{A}|\hat{A}^{-1}\mathbf{B})]\frac{\partial}{\partial\mathbf{w}}\right)\delta\sigma^*. \tag{2.8b}$$

Here $\delta\sigma^*(\mathbf{w})=\delta\sigma(\hat{A}^{-1}\mathbf{w})$.

*$[\mathbf{vB}]\equiv\mathbf{v}\times\mathbf{B}$.

The velocity of the quasiparticle $\mathbf{v}=\partial\epsilon/\partial\mathbf{p}$ in p-space is connected with the velocity $\mathbf{u}=\partial\epsilon/\partial\mathbf{w}$ in w-space by the relation

$$\mathbf{u}=\hat{A}^{-1}\mathbf{v}. \tag{2.9}$$

Using Eqs. (2.8) and (2.9), we transform Eq. (2.7) in w-space:

$$-i\omega\delta\sigma^*+i(\mathbf{k}^*\mathbf{u})\left(\delta\sigma^*-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^*\right)+\frac{e}{c}\left([\mathbf{uB}^*]\frac{\partial}{\partial\mathbf{w}}\right)\left(\delta\sigma^*-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^*\right)$$
$$-\frac{2\gamma}{\hbar}\left[\mathbf{B},\ \delta\sigma^*-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^*\right]=\delta\mathbf{I}_2^*, \tag{2.10}$$

where

$$\delta\varepsilon_2^*=-\mu_0\delta\mathbf{B}+\int d^3w'\,\psi^*(\mathbf{w},\mathbf{w}')\,\delta\sigma^*(\mathbf{w}',\mathbf{r}),$$

$\psi^*(\mathbf{w}, \mathbf{w}')$ is determined by Eq. (2.5)

$$\delta\mathbf{I}_2^*=-\left(\frac{1}{\tau}+\frac{1}{\tau_2}\right)\left(\delta\sigma^*-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^*\right)$$
$$+\frac{1}{\tau}\frac{\partial f_0}{\partial\varepsilon}\left[\int d^3w\frac{\partial f_0}{\partial\varepsilon}\right]^{-1}\int d^3w\left(\delta\sigma^*-\frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_2^*\right)$$

and the following notation is introduced:

$$\mathbf{k}^*=\hat{A}\mathbf{k}, \tag{2.11}$$

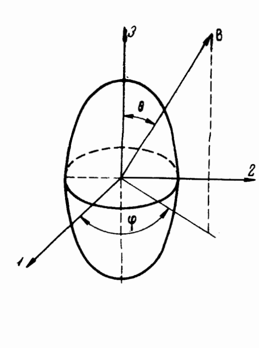
$$\mathbf{B}^*=|\hat{A}|\hat{A}^{-1}\mathbf{B}. \tag{2.12}$$

The Fermi surface in w-space is a sphere and Eq. (2.10) differs from the initial kinetic equation (2.7) only by the replacement of $\mathbf{k}, \mathbf{v}$ in the latter by $\mathbf{k}^*$ and $\mathbf{u}$, respectively, and also of $\mathbf{B}$ in the third term of the left side by $\mathbf{B}^*$. Equation (2.10) can be solved in a manner similar to the solution of the kinetic equation in the case of an isotropic electron fluid. After solution of the kinetic equation (2.10), the nonequilibrium density of magnetization $\delta\mathbf{M}(\mathbf{r}, t)$, by means of which the high-frequency magnetic susceptibility is determined, can be found from the relation

$$\delta\mathbf{M}(\mathbf{r},t)=|\hat{A}^{-1}|\int d^3w\,\delta\sigma^*(\mathbf{w},\mathbf{r},t)=|\hat{A}^{-1}|\,\delta\mathbf{M}^*(\mathbf{r},t). \tag{2.13}$$

The spectrum of spin waves in an electron fluid is actually determined by the pole of the high-frequency magnetic susceptibility[6,21] and, by using the relation (2.13) it is not difficult to note that the transformation (1.2) does not change the dispersion relation. Therefore, one can indeed use the expression for the frequencies previously obtained[4,6-9,21] in the case of an isotropic fluid for the spectra of spin waves in an electron fluid with an ellipsoidal Fermi surface, if only the wave vector $\mathbf{k}$, the velocity $\mathbf{v}$, and the stationary magnetic field $\mathbf{B}$ are replaced, where required, by $\mathbf{k}^*$, $\mathbf{u}$ and $\mathbf{B}^*$.

The following equations are obtained for the eigen-



frequencies of the electron system with ellipsoidal Fermi surface:

$$\omega^{z}_{l,m} = -m\Omega^*(1+\beta_l), \quad \text{if} \quad \delta\sigma \parallel \mathbf{B},$$
$$\omega^{\pm}_{l,m} = -(m\Omega^* \pm \Omega_0)(1+\beta_l), \quad \text{if} \quad \delta\sigma \perp \mathbf{B}, \tag{2.14}$$

where $\Omega_0 = 2\gamma B/\hbar = 2\mu_0 B/\hbar(1+\beta_0)$, and the coefficients β_l are given by the expansion (2.6). The cyclotron frequency Ω^*, according to Eqs. (1.4) and (2.12), is determined by the formula

$$\Omega^*(\theta,\varphi) = \frac{eB}{c}\left(\frac{\sin^2\theta\cos^2\varphi}{m_1 m_2} + \frac{\sin^2\theta\sin^2\varphi}{m_1 m_3} + \frac{\cos^2\theta}{m_1 m_2}\right)^{1/2}. \tag{2.15}$$

Here θ and φ are the angles which determine the orientation of the vector $\mathbf{B}$ relative to the principal axes of the ellipsoid (see the drawing). We note that Eq. (2.15) for the cyclotron frequency naturally agrees with what was obtained from the general formula

$$\Omega^* = 2\pi\frac{eB}{c}\left(\frac{\partial S(\varepsilon, p_\parallel)}{\partial\varepsilon}\right)^{-1},$$

where $S = S(\epsilon, p_0)$ is the area of the intersection of the surface $\epsilon(\mathbf{p}) = \epsilon_F$ with the plane $\mathbf{p}\cdot\mathbf{B} = p_\parallel B$ = const. According to Eq. (2.15), the cyclotron frequency depends on the angles θ and φ, and the relations (2.14) and (2.15) determine the dependence of the eigenfrequencies on the orientation of the magnetic field relative to the axes of the crystal.

Let us consider transversely polarized spin waves ($\delta\sigma \perp \mathbf{B}$) with a frequency close to the resonance frequency $\omega^{\pm}_{00} = \mp\omega_S = \mp 2\mu_0 B/\hbar$. Using the results obtained in[6,7], we find the following expression for the spectrum of spin waves in an electron fluid with an ellipsoidal Fermi surface, an expression that is valid in the limit of long-wave oscillations:

$$\omega = \mp\omega_s\left[1 + \frac{v^2_{\text{eff}}(\theta,\varphi)}{3\omega_s^2}\frac{(1+\beta_0)(1+\beta_1)}{\beta_0-\beta_1}\left(k_\parallel^2 - \frac{k^2_{\text{eff}}(\theta,\varphi)}{A(\theta,\varphi)-1}\right)\right]. \tag{2.16}$$

Here

$$A(\theta,\varphi) = \frac{(1+\beta_0)^2(1+\beta_1)^2}{(\beta_0-\beta_1)^2}\frac{\Omega^{*2}(\theta,\varphi)}{\omega_s^2}, \tag{2.17}$$

$$v^2_{\text{eff}} = 2\varepsilon_F m^{-1}_{\text{eff}}(\theta,\varphi),$$
$$m_{\text{eff}}(\theta,\varphi) = (m_1\cos^2\varphi + m_2\sin^2\varphi)\sin^2\theta + m_3\cos^2\theta, \tag{2.18}$$
$$k^2_{\text{eff}} = m_{\text{eff}}(\theta,\varphi)\left(\frac{k_1^2}{m_1} + \frac{k_2^2}{m_2} + \frac{k_3^2}{m_3}\right) - k_\parallel^2,$$

where k_1, k_2, and k_3 are the components of the vector $\mathbf{k}$ in the set of coordinates with axes directed along the principal axes of the ellipsoid, and $k_\parallel$ is the component of the vector $\mathbf{k}$ along the direction of the magnetic field $\mathbf{B}$. We introduce the effective Larmor radius:

$$R_{\text{eff}}(\theta,\varphi) = \frac{v_{\text{eff}}}{\Omega^*} = \frac{c}{eB}\frac{(2\varepsilon_F m_1 m_2 m_3)^{1/2}}{m_{\text{eff}}(\theta,\varphi)}. \tag{2.19}$$

The formula (2.16) is applicable for long waves that satisfy the condition

$$[k^2_{\text{eff}}(\theta,\varphi) + k_\parallel^2]^{1/2} R_{\text{eff}} \ll 1.$$

The characteristic mark of Eq. (2.16) is that it determines the dependence of the frequency of the spin wave in an electron fluid with an ellipsoidal Fermi surface both on the wave vector $\mathbf{k}$ and on the angles θ, φ, i.e., on the orientation of the stationary magnetic field relative to the crystallographic axes. If, among the coefficients of expansion (2.6) only $\beta_0 \neq 0$, and also if $m_1 = m_2 = m_\perp$, and $m_3 = m_\parallel$, then Eqs. (2.15)–(2.19) reduce to an expression for the spectrum obtained previously[18] in the approximation $\psi(\mathbf{p},\mathbf{p}') \equiv$ const for the case of a Fermi surface in the form of a biaxial ellipsoid of revolution.

Let us now consider spin waves whose limiting frequencies are determined by Eqs. (2.14), with $l \neq 0$. In correspondence with the expressions for spectra obtained in[8,9], we find the following expression for the spin wave frequencies; with accuracy to terms of order $(k^2_{\text{eff}} + k^2_\parallel)R^2_{\text{eff}}$:

$$\omega_{l,m}(k,\theta,\varphi) = (1+\beta_l)[\Omega_0 - m\Omega^*(\theta,\varphi)] + v^2_{\text{эфф}}[D'(l,m)k_\parallel^2 + D''(l,m)k^2_{\text{eff}}]$$

$$D'(2,m) = \frac{9-m^2}{35\Omega^*(\lambda-m)}\frac{1+\beta_3}{\beta_2-\beta_3}, \quad m \neq 0,$$

$$D'(l,m) = \frac{1}{\Omega^*(\lambda-m)}\left(\frac{l^2-m^2}{4l^2-1}\frac{1+\beta_{l-1}}{\beta_l-\beta_{l-1}} + \frac{(l+1)^2-m^2}{4(l+1)^2-1}\frac{1+\beta_{l+1}}{\beta_l-\beta_{l+1}}\right)$$
$$l \geqslant |m| \neq 0, \quad l > 2, \tag{2.20}$$

$$D''(2,m) = \frac{1}{140\Omega^*}\frac{(\lambda-m)^2(1+\beta_2)}{\beta_2^2(\lambda-m)^2-1}$$
$$\times\left[\frac{\beta_2^2(\lambda-m)(1+\beta_3)+\beta_2-\beta_3}{(\beta_2-\beta_3)(\lambda-m)+(1+\beta_3)}(3+m)(4+m) - \frac{\beta_2^2(\lambda-m)(1+\beta_3)-(\beta_2-\beta_3)}{(\beta_2-\beta_3)(\lambda-m)-(1+\beta_3)}(3-m)(4-m)\right],$$

$$D''(lm) = \frac{1}{4(2l-1)\Omega^*}\frac{(\lambda-m)^2(1+\beta_l)}{\beta_l^2(\lambda-m)^2-1}$$
$$\times\left[\frac{(l-1-m)(l-m)}{2l-1}\frac{\beta_l^2(1+\beta_{l-1})(\lambda-m)+\beta_l-\beta_{l-1}}{(\beta_l-\beta_{l-1})(\lambda-m)+(1+\beta_{l-1})} - \frac{(l-1+m)(l+m)}{2l-1}\frac{\beta_l^2(1+\beta_{l-1})(\lambda-m)-\beta_l+\beta_{l-1}}{(\beta_l-\beta_{l-1})(\lambda-m)-(1+\beta_{l-1})}\right.$$
$$+\frac{(l+1+m)(l+2+m)}{2l+3}\frac{\beta_l^2(1+\beta_{l+1})(\lambda-m)+\beta_l-\beta_{l+1}}{(\beta_l-\beta_{l+1})(\lambda-m)+(1+\beta_{l+1})}$$
$$\left.-\frac{(l+1-m)(l+2-m)}{2l+3}\frac{\beta_l^2(1+\beta_{l+1})(\lambda-m)-\beta_l+\beta_{l+1}}{(\beta_l-\beta_{l+1})(\lambda-m)-(1+\beta_{l+1})}\right], \quad l > 2,$$

where

$$\lambda = \Omega_0[\Omega^*(\theta,\varphi)]^{-1}.$$

3. We now consider the cyclotron waves in an electron fluid with ellipsoidal Fermi surface. For a small deviation $\delta f \propto \exp(-i\omega t + i\mathbf{k}\cdot\mathbf{r})$ of the particle distribution from the equilibrium spatially-homogeneous Fermi distribution f_0, we have the following kinetic equation:[4,21]

$$-i\omega\delta f + i(\mathbf{kv})\left(\delta f - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_1\right) + e\mathbf{Ev}\frac{\partial f_0}{\partial\varepsilon} + \frac{e}{c}\left([\mathbf{vB}]\frac{\partial}{\partial\mathbf{p}}\right)\left(\delta f - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_1\right) = \left[\frac{\partial f}{\partial t}\right]_{\text{st}}, \tag{3.1}$$

where

$$\delta\varepsilon_1 = \int d^3p'\,\varphi(\mathbf{p},\mathbf{p}')\delta f(\mathbf{p}'), \tag{3.2}$$

and $\varphi(\mathbf{p},\mathbf{p}')$ is the correlation function which describes the spin-independent part of the interaction of the fluid quasiparticles. We transform from p-space to w-space. Here we use Eq. (2.9), and also the following relations, which are similar to Eqs. (2.8):

$$\frac{\partial\delta f^*}{\partial\mathbf{w}} = \hat{A}^{-1}\frac{\partial\delta f}{\partial\mathbf{p}},$$
$$\left([\mathbf{vB}]\frac{\partial}{\partial\mathbf{p}}\right)\delta f = \left([(\hat{A}^{-1}\mathbf{v})(|\hat{A}|\hat{A}^{-1}\mathbf{B})]\frac{\partial}{\partial\mathbf{w}}\right)\delta f^*. \tag{3.3}$$

Equation (3.1) then takes the form

$$-i\omega\delta f^* + i(\mathbf{k}^*\mathbf{u})\left(\delta f^* - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_1^*\right) + e\mathbf{E}^*\mathbf{u}\frac{\partial f_0}{\partial\varepsilon} + \frac{e}{c}\left([\mathbf{uB}^*]\frac{\partial}{\partial\mathbf{w}}\right)\left(\delta f^* - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_1^*\right) = \left[\frac{\partial f}{\partial t}\right]^*_{\text{st}}, \tag{3.4}$$

where

$$\delta\varepsilon_1^* = \int d^3w'\,\varphi^*(\mathbf{w},\mathbf{w}')\,\delta f^*(\mathbf{w}'), \quad \delta f^*(\mathbf{w}) = \delta f(\hat{A}^{-1}\mathbf{w}), \tag{3.5}$$

$$\varphi^*(\mathbf{w},\mathbf{w}') = |\hat{A}^{-1}|\,\varphi(\hat{A}^{-1}\mathbf{w},\hat{A}^{-1}\mathbf{w}'), \quad \mathbf{E}^* = \hat{A}\mathbf{E}, \tag{3.6}$$

and $\mathbf{u}$, $\mathbf{k}^*$, and $\mathbf{B}^*$ are determined respectively by Eqs. (2.9), (2.11) and (2.12). Similarly to what was done for the function $\psi^*(\mathrm{w}, \mathrm{w}')$, we can also use the expansion in a series of Legendre polynomials for the function $\varphi^*(\mathrm{w}, \mathrm{w}')$:

$$\frac{1}{\pi^2\hbar^3}\left(\frac{2\varepsilon_F}{\delta_0^3}\right)^{1/2}\varphi^*(\mathbf{w},\mathbf{w}') = \frac{(2\varepsilon_F m_1 m_2 m_3)^{1/2}}{\pi^2\hbar^3}\varphi(\hat{A}^{-1}\mathbf{w},\hat{A}^{-1}\mathbf{w}')$$

$$= \sum_{l=0}^{\infty}(2l+1)\,a_l P_l(\cos\Theta). \tag{3.7}$$

Equation (3.4) differs from the initial kinetic equation (3.1) by substitution of $\mathbf{k}^*$, $\mathbf{u}$, $\mathbf{B}^*$, $\mathbf{E}^*$ for $\mathbf{k}$, $\mathbf{v}$, $\mathbf{B}$, $\mathbf{E}$ in the latter. In addition, since the Fermi surface in w space is a sphere, Eq. (3.4) is solved in similar fashion to the solution of the kinetic equation in the case of an isotropic electron fluid. Solution of Eq. (3.4) permits us to compute the nonequilibrium current density from the formula

$$\mathbf{j} = \hat{\sigma}\mathbf{E} = |\hat{A}^{-1}|\hat{A}(\hat{\sigma}'\mathbf{E}^*) = |\hat{A}^{-1}|\hat{A}\left[\int d^3w\,\mathbf{u}\left(\delta f^* - \frac{\partial f_0}{\partial\varepsilon}\delta\varepsilon_1^*\right)\right]. \tag{3.8}$$

We note that expressions were obtained in[23] for the components of the conductivity tensor $\hat{\sigma}$ in an electron gas with an ellipsoidal Fermi surface. The calculation of the components of the tensor σ for an electron fluid is difficult even in the case of a spherical Fermi surface. However, there is no need to calculate the conductivity tensor $\hat{\sigma}$ in order to find the cyclotron-wave frequencies. Under the experimental conditions of study of cyclotron resonance, $\mathrm{j} = 0$,[11,12] and the dispersion equation of the oscillations has the form $|\sigma_{ij}| = 0$ in this case, in accord with (3.8). Since $|\hat{A}^{-1}| \neq 0$, the cyclotron wave frequencies can be found from the condition that $|\sigma'_{ij}| = 0$. Therefore, to obtain the spectra of the cyclotron waves in an electron fluid with an ellipsoidal Fermi surface, we can directly use the results obtained earlier for the isotropic case[10-16], and in a manner similar to what was done by us in the case of spin waves. Thus, with neglect of collisions, $\omega\tau \gg 1$, we have for the eigenfrequencies of the electron system in the longwave limit $(\mathbf{k} \to 0)$

$$\omega_{lm} = -m\Omega^*(\theta,\varphi)(1+a_l), \tag{3.9}$$

Here and everywhere below, Ω^*, v_{eff}, k_{eff}, and R_{eff} depend on θ and φ and are determined by Eqs. (2.15), (2.18), and (2.19). Equation (3.9), together with (2.15), establishes the dependence of the eigenfrequencies of the system on the orientation of the stationary magnetic field $\mathbf{B}$ relative to the axes of the crystal.

We consider the cyclotron waves propagating at such an angle to the direction of the vector $\mathbf{B}$ that $k_{\mathrm{eff}} = 0$. According to the relations (2.11), (2.12), and (2.18), we have $\mathbf{k}^* \parallel \mathbf{B}^*$ in this case. For an isotropic electron fluid ($\hat{A} \equiv 1$), this case corresponds to the propagation of the wave along the magnetic field. We shall assume that only the components with $l \leq 2$ in the expansion (3.7) differ from zero. We shall further assume that $[\partial f/\partial t]_{\mathrm{st}} = -\delta f/\tau_0$,

We use the result obtained in[10,21] for the spectrum of isotropic waves propagating in an isotropic electron fluid along the direction of the stationary magnetic field $\mathbf{B}$. In the case of an ellipsoidal Fermi surface, we find the following dispersion equation for the dependence of the cyclotron wave frequency both on the wave vector and on the angles θ and φ:

$$(1 + a_2 + {}^5/_6 a_2\omega iN_{22}^{\pm1})\,iN_{11}^{\pm1} - {}^5/_6 a_2\omega[iN_{12}^{\pm1}]^2 = 0, \tag{3.10}$$

where

$$iN_{nl}^{\pm1}(\theta,\varphi) = \frac{1}{2}\int_0^{\pi}\frac{dt\sin t P_n^1(\cos t)P_l^1(\cos t)}{-\omega \pm \Omega^*(\theta,\varphi) + k_\parallel v_{\mathrm{eff}}(\theta,\varphi)\cos t - i/\tau_0}. \tag{3.11}$$

It is seen from Eq. (3.11) that Cerenkov collision-free damping is possible if

$$\omega \mp \Omega^*(\theta,\varphi) = k_\parallel v_{\mathrm{eff}}(\theta,\varphi)\cos t.$$

For long waves, when $k_\parallel R_{\mathrm{eff}} \ll 1$, the following expression is obtained from Eq. (3.10) for the cyclotron wave frequency:

$$\omega = \left[\pm\Omega^*(\theta,\varphi) - \frac{i}{\tau_0}\right](1+a_2)\left(1 + \frac{8}{35}\frac{k_\parallel^2 R_{\mathrm{eff}}^2(\theta,\varphi)}{a_2}\right). \tag{3.12}$$

Upon increase in $k_\parallel$, the frequency ω approaches a value satisfying the relation

$$|\omega \pm \Omega^*(\theta,\varphi)| = k_\parallel v_{\mathrm{eff}}(\theta,\varphi).$$

Cerenkov interaction of the wave with electrons becomes possible for such a value of ω. Therefore, for cyclotron waves propagating at such an angle to the direction of the vector $\mathbf{B}$ that $k_{\mathrm{eff}} = 0$, the value of $k_\parallel$ cannot exceed some maximum. Following the analysis given in[13], we obtain the value

$$k_{\parallel_{max}}(\theta,\varphi) = \left|\frac{\Omega^*(\theta,\varphi)}{v_{\mathrm{eff}}(\theta,\varphi)}\frac{5a_2}{3-2a_2}\right|.$$

for the maximum $k_\parallel$.

Thus the dependence of the maximum value of $k_\parallel$ on the orientation of the vector $\mathbf{B}$ relative to the axes of the crystal is also established. Spectra were obtained in[13] for cyclotron waves propagating in an isotropic electron fluid along the magnetic field, under the assumption of the possibility of the expansion in powers of kR, but without limitation to a finite number of Legendre polynomials in the expansion of the correlation function $\varphi(\mathbf{p}, \mathbf{p}')$. In correspondence with the results of this research, we find the following expression for the frequencies of the cyclotron waves in an electron fluid with an ellipsoidal Fermi surface, propagating at such an angle to the direction of the magnetic field $\mathbf{B}$ that $k_{\mathrm{eff}} = 0$:

$$\omega_{2m} = m\Omega^*(\theta,\varphi)(1+a_2)\left\{1 + \frac{k_\parallel^2 R_{\mathrm{eff}}^2(\theta,\varphi)}{m^2}\left[\frac{9-m^2}{35}\frac{1+a_3}{a_2-a_3}\right]\right\}$$
$$(m \neq 0),$$

$$\omega_{lm} = m\Omega^*(\theta,\varphi)(1+a_l)\left\{1 + \frac{k_\parallel^2 R_{\mathrm{eff}}^2(\theta,\varphi)}{m^2}\left[\frac{l^2-m^2}{4l^2-1}\frac{1+a_{l-1}}{a_l-a_{l-1}}\right.\right.$$
$$\left.\left. + \frac{(l+1)^2-m^2}{4(l+1)-1}\frac{1+a_{l+1}}{a_l-a_{l+1}}\right]\right\}, \quad l \geqslant |m| \neq 0, \quad l > 2. \tag{3.13}$$

We now consider cyclotron waves propagating transversely to the magnetic field $\mathbf{B}$, i.e., $k_\parallel = 0$. According to the relations (2.11), (2.12), and (2.18), the vector $\mathbf{k}^*$ in this case is directed at right angles to the direction of the vector $\mathbf{B}^*$. We choose a set of coordinates in which axis 1 is directed along the vector $\mathbf{k}^*$. Then the tensor σ' will have the form[11]

$$\hat{\sigma}' = \begin{pmatrix} \sigma'_{11} & \sigma'_{12} & 0 \\ \sigma'_{21} & \sigma'_{22} & 0 \\ 0 & 0 & \sigma'_{33} \end{pmatrix}. \quad (3.14)$$

The dispersion relation $|\sigma'_{ij}| = 0$, which corresponds to this and determines the spectrum of the cyclotron waves, is divided into the following two parts:

$$\sigma_{33}' = 0, \quad (3.15)$$

$$\sigma_{11}'\sigma_{22}' - \sigma_{12}'\sigma_{21}' = 0. \quad (3.16)$$

Equation (3.15) describes waves in which $\mathbf{E}^* \times \mathbf{B}^* = 0$. Hence, using the relations (2.12) and (3.6), we obtain the result that the electric field vector $\mathbf{E}$ of these waves is oriented at such an angle to $\mathbf{B}$ that

$$[(\hat{A}^{-1}\mathbf{B})(\hat{A}\mathbf{E})] = 0.$$

In the case of an isotropic electron fluid ($\hat{A} \equiv 1$), this mode corresponds to "normal" waves, with the electric field vector directed along the stationary magnetic field $\mathbf{B}$. Equation (3.16) describes waves in which $\mathbf{E}^* \cdot \mathbf{B}^* = 0$.

By using (2.12), (3.6), and the fact that the matrix $\hat{A}$ is symmetric, it is not difficult to note that the electric field vector $\mathbf{E}$ of such waves lies in a plane perpendicular to the stationary magnetic field. In the case of an isotropic electric fluid, this mode is called the "extraordinary" or plasma wave. We consider the cyclotron wave that is defined by Eq. (3.15). We shall assume the coefficients α_n to be small in comparison with unity, and also limit ourselves to the long wave limit $k_{\text{eff}}R_{\text{eff}} \ll 1$. We can then use the formula for the frequency of the ordinary wave in an isotropic electron fluid, which was obtained in[10,15]. In the case of an ellipsoidal Fermi surface, we find the following expression for the frequency:

$$\omega = m\Omega^*(\theta, \varphi)\{1 + \alpha_2 - {}^1/_{10}k_{\text{eff}}^2(\theta, \varphi)R_{\text{eff}}^2(\theta, \varphi)[1 + {}^{20}/_7\alpha_2]\}. \quad (3.17)$$

We shall not limit now the number of Legendre polynomials in the expansion (3.7). In the long-wave limit ($k_{\text{eff}}R_{\text{eff}} \ll 1$), we can use the expressions for the resonance frequencies of cyclotron waves propagating in an isotropic electron fluid transversely to the magnetic field, expressions found in[12,14]. According to the results of these researches we get, for the case of an ellipsoidal Fermi surface,

$$\omega_{2m} = \Omega^*(\theta, \varphi)(1 + \alpha_2)\left\{m + \frac{1 + \alpha_3}{140}k_{\text{eff}}^2(\theta, \varphi)R_{\text{eff}}^2(\theta, \varphi) \times \left[\frac{(3 - m)(4 - m)}{1 + m\alpha_2 - (m - 1)\alpha_3} - \frac{(3 + m)(4 + m)}{1 - m\alpha_2 + (m + 1)\alpha_3}\right]\right\};$$

$$\omega_{lm} = \Omega^*(\theta, \varphi)(1 + \alpha_l)\left\{m + \frac{k_{\text{eff}}^2(\theta, \varphi)R_{\text{eff}}^2(\theta, \varphi)}{4(2l - 1)} \times \left(\frac{1 + \alpha_{l-1}}{2l - 1}\left[\frac{(l - 1 + m)(l + m)}{1 + m\alpha_l - (m - 1)\alpha_{l-1}} - \frac{(l - 1 - m)(l - m)}{1 - m\alpha_l + (m + 1)\alpha_{l-1}}\right] + \frac{1 + \alpha_{l+1}}{2l + 3}\left[\frac{(l + 1 - m)(l + 2 - m)}{1 + m\alpha_l - (m - 1)\alpha_{l+1}} - \frac{(l + 1 + m)(l + 2 + m)}{1 - m\alpha_l + (m + 1)\alpha_{l+1}}\right]\right)\right\}, \quad (3.18)$$

Thus, in the case of an electron fluid with an ellipsoidal Fermi surface, the spectrum of possible frequencies of both spin and cyclotron waves becomes dependent on the orientation of the stationary magnetic field $\mathbf{B}$ relative to the axes of the crystal. No restrictions are added here to the form of the correlation functions $\psi(\mathbf{p}, \mathbf{p}')$ and $\varphi(\mathbf{p}, \mathbf{p}')$. Expressions were obtained for the frequencies of the cyclotron waves propagating at such an angle to the stationary magnetic field that $k_{\text{eff}} = 0$. In this case, the maximum possible value for k_{eff} is established. The spectra of cyclotron waves propagating transversely to the magnetic field are also found. For $m_1 = m_2 = m_3$, all the results obtained go over into the well known expression for the spectra of waves in an isotropic fluid.

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[1] I. M. Lifshitz and M. I. Kaganov, Usp. Fiz. Nauk **69**, 419 (1959) [Soviet Phys.-Uspekhi **2**, 831 (1960)].

[2] W. A. Harrison, Pseudopotentials in the Theory of Metals, New York, 1966 (Russian translation, Mir, 1968).

[3] F. S. Ham, Phys. Rev. **128**, 2524 (1962).

[4] V. P. Silin, Zh. Eksp. Teor. Fiz. **35**, 1243 (1958) [Soviet Phys.-JETP **8**, 870 (1959)].

[5] V. P. Silin, Zh. Eksp. Teor. Fiz. **33**, 495 (1957) [Soviet Phys.-JETP **6**, 387 (1958)].

[6] V. P. Silin, Supplement to the book of V. I. Akhiezer, V. G. Bar'yakhtar and S. V. Peletminskiĭ, Spinovye volny (Spin Waves) (Nauka, 1967).

[7] P. M. Platzman and P. A. Wolff, Phys. Rev. Lett. **18**, 280 (1967).

[8] D. R. Fredkin and A. R. Wilson, Preprint No. 92038, Physics Department, University of California, San Diego, La Jolla, Calif. 1967.

[9] S. C. Ying and J. J. Quinn, Phys. Rev. **180**, 241 (1969).

[10] V. P. Silin, Zh. Eksp. Teor. Fiz. **54**, 1016 (1968) [Soviet Phys.-JETP **27**, 541 (1968)].

[11] P. M. Platmann, W. M. Walsh and E-Ni Foo, Phys. Rev. **172**, 689 (1968).

[12] N. D. Mermin and Y. C. Cheng, Phys. Rev. Lett. **20**, 839 (1968).

[13] Y. C. Cheng, J. S. Clarke and N. D. Mermin, Phys. Rev. Lett. **20**, 1486 (1968).

[14] S. C. Ying and J. J. Quinn, Phys. Rev. Lett. **20**, 1007 (1968).

[15] S. C. Ying and J. J. Quinn, Phys. Rev. **173**, 473 (1968).

[16] S. C. Ying and J. J. Quinn, Phys. Rev. **180**, 193 (1969).

[17] V. P. Silin, Zh. Eksp. Teor. Fiz. **55**, 697 (1968) [Soviet Phys.-JETP **28**, 363 (1969)].

[18] G. P. Alodzhants, Fizika metallov i metallovedenie, (in press).

[19] F. S. Ham and D. C. Mattis, I. B. M. J. Res. Develop. **4**, 143 (1960).

[20] H. J. Mackey and J. R. Sybert, Phys. Rev. **172**, 603 (1968).

[21] V. P. Silin, Fiz. Metal. Metallov. (in press).

[22] V. P. Silin, Zh. Eksp. Teor. Fiz. **34**, 707 (1958) [Soviet Phys.-JETP **7**, 486 (1958)].

[23] E-Ni Foo, Phys. Rev. **182**, 674 (1969).

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