GAMMA RAYS EMITTED BY NUCLEI IN CRYSTALS

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We consider the γ decay of excited nuclei in a crystal consisting of the same nuclei but in the ground state. The character of the radiation from a thick crystal is determined by using the reciprocity theorem for absorbing media and from the previously obtained solution[1] of the stationary problem of the distribution of γ quanta in an ideal crystal in the case of resonant nuclear interaction with an individual nucleus and in the presence of an external source. It is shown that in certain angle intervals the dependence of the decay γ quanta on the thickness is not exponential but follows a power law, this being due to the suppression of the inelastic channels of the nuclear reaction[1]. The radiation from a thick crystal will then be emitted along surfaces of cones with axes along the reciprocal-lattice vector and with aperture angle $90° - \theta_B$ (θ_B is the Bragg angle). The character of the angular distribution of the intensity turns out to vary greatly, depending on whether the radiating nucleus is at a site or in an interstice. The possibility of analyzing the position of the radiating nucleus in the unit cell is demonstrated.

1. INTRODUCTION

THE purpose of the present article is to analyze the character of the radiation of the γ quanta accompanying the decay of nuclei inside a crystal. We have in mind an ideal crystal containing the same nuclei, but in the ground state. We confine ourselves here to nuclei of the Mössbauer type, for which the interaction of the γ quanta with the nuclei of the crystal has a sharply pronounced resonant character, and the ratio of the inelastic (conversion) and elastic channels of the scattering has an arbitrary value. At first glance it may seem obvious that in the case of strong resonant absorption of the γ quanta, the radiation of the nuclei situated at a sufficiently large depth should attenuate exponentially towards the surface, no matter in what direction of emergence from the crystal it is measured. Although such an assumption is natural, it turns out to be incorrect in the general case. This is due to the specific features of the motion and interaction of γ quanta with nuclei in such a crystal. As was shown in[1-3], in resonant interaction of particles having a wavelength $\lambda < a$ (a is the interatomic distance) with an individual nucleus, an interaction accompanied by the formation of a long-lived excited state that decays via both the elastic and inelastic channels, the coherent character of the scattering is fully conserved. This causes, in particular, the excitation of the nucleus to have a collective character smeared out over the crystal[3], and this changes the picture of the interaction and motion of the particles in the crystal. When the γ quanta are incident on the crystal at the Bragg angle, this leads to the appearance of the so-called effect of suppression of the inelastic channels[1,2]. The gist of this effect consists in a sharp decrease of the amplitude of formation of the excited nucleus inside the crystal, by virtue of which the yield of the nuclear reaction is partly or completely suppressed. As a result, the particles can pass through the crystal in a certain angle interval with practically no absorption. This effect has by now been observed experimentally[4,5].

It is natural that the suppression effect should appear also in the case of motion of the γ quanta that result from the decay of nuclei inside the crystal. In fact, in the case of decay in an ideal crystal, γ quanta will appear in a state that is a coherent superposition of two plane waves, with wave vectors differing by an amount equal to the reciprocal-lattice vector. But it is precisely for such states, at definite polarization, that the amplitude for the production of the excited nucleus is small. Such paired states will arise in the case of decay in directions that are close to the Bragg condition in the case of incidence on the crystal from the outside. As will be shown below, this causes the exponential character of absorption of the γ quanta in decay in thick crystals to give way to a much weaker power-law attenuation with increasing thickness.

A direct study of the radiation of a γ quantum from a nucleus inside a crystal, with allowance for the resonant interaction with the surrounding nuclei, encounters a number of peculiar difficulties. Such a problem, however, can be solved consistently by using the previously-obtained solution of the problem of the motion of ex- externally-incident γ quanta in the interior of a crystal[1], if one uses the reciprocity theorem (see, for example,[6]) generalized to include the case of an arbitrarily absorbing system. This is the method used in the present paper.

We note that in x-ray physics there is a known phenomenon, observed by Kossel[7] and consisting of the occurrence of a fine structure in the angular dependence of the intensity of the characteristic radiation of atoms emitted from a crystal near the Bragg angle. Laue[8] explained this phenomenon for a non-absorbing crystal and in the absence of vibration of the atoms, using the reciprocity theorem and considering the potential scattering by the electron shells of the atoms. The results obtained in the present paper make it possible, in particular, to find the solution of the problem for the Kossel effect in an absorbing crystal, for an arbitrary position of the radiating atom in the lattice, and with a consistent allowance for the influence of the temperature.

2. RECIPROCITY THEOREM

Assume that sources of monochromatic radiation are placed at certain points $\mathbf{r}_1$ and $\mathbf{r}_2$ in space. The field $\mathbf{E}_\xi$ ($\xi = 1, 2$) resulting from the action of one of the sources can be determined with the aid of Maxwell's equation

$$\left(k^2-\frac{\omega^2}{c^2}\right)E_\xi^i(\mathbf{k},\omega)-k^i(\mathbf{k}\mathbf{E}_\xi(\mathbf{k},\omega))=\frac{4\pi i\omega}{c^2}\left(\sum_{\mathbf{k}'}\sigma_\omega^{il}(\mathbf{k},\mathbf{k}')\times E_\xi^l(\mathbf{k}',\omega)+j_\xi^i(\mathbf{k},\omega)\right). \qquad (2.1)$$

Here $\mathbf{j}_\xi$ is the current corresponding to the source at the point $\mathbf{r}_\xi$. Let $\mathbf{r}_1$ coincide with the coordinate of the radiating nucleus inside the crystal, and let $\mathbf{r}_2$ be a point in free space, sufficiently far from the crystal. In determining the current $\mathbf{j}_1$, it is necessary to take into account the oscillation of the radiating nucleus. The currents contained in Maxwell's equation (2.1) are averages of the current operator over the quantum-mechanical state and over the statistical distribution.

Let us consider the case of a narrow resonant level, $\Gamma \ll \omega_D$ (Γ is the width of the resonant level and ω_D is the characteristic frequency of the phonon spectrum), which is typical of Mössbauer-type nuclei. We then obtain directly for the radiation in a frequency interval of the order of Γ:

$$\mathbf{j}_1(\mathbf{k},\omega)=\mathbf{j}_{01}(\mathbf{k},\omega)e^{-i\mathbf{k}\mathbf{r}_1}e^{-Z_1(\mathbf{k})/2},$$
$$Z_1(\mathbf{k})=\frac{\hbar}{2M_1N}\sum_\beta\frac{|\mathbf{k}\mathbf{V}_1(\beta)|^2}{\omega_\beta}(2\bar{n}_\beta+1). \qquad (2.2)$$

Here $\mathbf{j}_{01}$ is the current of the rigidly secured nucleus, $\mathbf{V}_1(\beta)$ is the polarization vector for the radiating nucleus in the β-th normal vibration, and $\bar{n}$ is the mean value of the occupation numbers.

Actually $\exp[-Z_1(\mathbf{k})/2]$ is the probability amplitude of radiation of a γ quantum in the direction $\mathbf{k}$ without emission or absorption of phonons. In the case of a broad resonance, when $\Gamma \gg \omega_D$, this factor is eliminated from (2.2), since emission or absorption of phonons leaves the radiation within the limits of the resonant width. We shall assume, for simplicity, that $\mathbf{j}_2$ is a rigidly secured source.

The first term on the right side of (2.1) describes the scattering of the γ quanta by the atoms of the crystal. Here σ^{il} can be represented in the form

$$\sigma_\omega^{il}(\mathbf{k},\mathbf{k}')=\sum_m\sigma_{\omega m}^{il}(\mathbf{k},\mathbf{k}')e^{-i(\mathbf{k}'-\mathbf{k})\mathbf{r}_m}, \qquad (2.3)$$

where the summation is over the atoms of the crystal. Explicit formulas for the case of electric and magnetic dipole or quadrupole resonant interactions of the γ radiation with the nuclei can be found in[1] (see also[11]). We emphasize that the nuclear vibrations in a crystal which play an important role in the scattering problem can be taken into account rigorously within the framework of Maxwell's equations, and, as shown in[1,9], the result of such an account reduces to a corresponding redefinition of the quantities $\sigma_{\omega n}^{il}$, which now depend explicitly on the temperature.

The fact of importance to us is that we are dealing with a strong resonant interaction, causing $\sigma_{\omega m}^{il}$ to be a complex quantity with an arbitrary ratio of real part to imaginary part, or, in other words, of scattering to absorption. We must obtain a reciprocity theorem for precisely this case.

We multiply (2.1) with $\xi = 1$ by $E_2^i(-\mathbf{k},\omega)$ and the equation with $\xi = 2$, in which we first make the substitution $\mathbf{k} \to -\mathbf{k}$, by $E_1^i(\mathbf{k},\omega)$. We sum the two equations over $\mathbf{k}$ and subtract one from the other. As a result we get

$$\sum_{\mathbf{k},\mathbf{k}'}[E_2^i(-\mathbf{k},\omega)\sigma_\omega^{il}(\mathbf{k},\mathbf{k}')E_1^l(\mathbf{k}',\omega)-E_1^i(\mathbf{k},\omega)\sigma_\omega^{il}(-\mathbf{k},\mathbf{k}')E_2^l(\mathbf{k}',\omega)]=-\sum_{\mathbf{k}}[\mathbf{j}_1(\mathbf{k},\omega)\mathbf{E}_2(-\mathbf{k},\omega)-\mathbf{j}_2(-\mathbf{k},\omega)\mathbf{E}_1(\mathbf{k},\omega)]. \qquad (2.4)$$

In the second term on the left side we make the substitution $\mathbf{k}' \to -\mathbf{k}'$, and then $\mathbf{k} \rightleftarrows \mathbf{k}'$ and $i \rightleftarrows l$. As a result, this term takes the form

$$-\sum_{\mathbf{k},\mathbf{k}'}E_2^i(-\mathbf{k},\omega)\sigma_\omega^{li}(-\mathbf{k}',-\mathbf{k})E_1^l(\mathbf{k}',\omega).$$

The quantity $\sigma_{\omega m}^{il}$ in (2.3) is none other than the scattering amplitude (apart from an immaterial factor). But for the scattering amplitude there is a general relation (see, for example,[10]), which in our case takes the form

$$\sigma_{\omega m}^{il}(\mathbf{k},\mathbf{k}')=\sigma_{\omega m}^{li}(-\mathbf{k}',-\mathbf{k}). \qquad (2.5)$$

We emphasize that this relation also remains valid when the spins of the ground and excited states of the nucleus differ from zero, provided only that the levels are degenerate. Otherwise the connection between the amplitudes or the corresponding $\sigma_{\omega m}$ has a more complicated character. We assume throughout that there is no hyperfine splitting.

Relation (2.5) leads directly to the vanishing of the left-hand side of (2.4). As a result we arrive at the following equation:

$$\sum_{\mathbf{k}}\mathbf{j}_{01}(\mathbf{k},\omega)\mathbf{E}_2(-\mathbf{k},\omega)e^{-i\mathbf{k}\mathbf{r}_1}e^{-Z_1(\mathbf{k})/2}=\sum_{\mathbf{k}}\mathbf{j}_{02}(-\mathbf{k},\omega)\mathbf{E}_1(\mathbf{k},\omega)e^{i\mathbf{k}\mathbf{r}_2}, \qquad (2.6)$$

which is the reciprocity theorem in the case of interest to us.

By measuring the radiation at large distances from the crystal, we separate a plane wave with a certain fixed value of $\mathbf{k}_0$. This means, in particular, that in the left side of (2.6) there is a field $\mathbf{E}_2$ produced by a source with $\mathbf{j}_{02}(-\mathbf{k}_0,\omega)$, and the polarization s of the field $\mathbf{E}_1$ corresponds to the aggregate of the Fourier components of the field in the crystal in response to radiation having the same polarity s incident on the crystal.

Let us consider an ideal crystal in which the amplitude of scattering by an individual nucleus is small compared with the interatomic distances. If radiation with a wave vector $-\mathbf{k}_0 = \boldsymbol{\kappa}_0$ is incident from the outside on the crystal, this radiation propagates in the interior of the crystal only in the same direction. If the Bragg condition is satisfied, one more wave is produced, propagating in the direction $\boldsymbol{\kappa}_1 = \boldsymbol{\kappa}_0 + \mathbf{K}$ ($\mathbf{K}$ is the reciprocal-lattice vector). We shall show that this last case is of greatest interest.

The statement that only one or two waves propagate is, strictly speaking, inaccurate. Actually, owing to the interaction with the nuclei of the medium, each wave gives way to a wave packet. The dimension of the packet in momentum space is very small and amounts to $\Delta k \sim 1/l_{coh}$, where l_{coh} is the coherence length, i.e., the length over which the initial wave experiences scattering on the order of unity. Since $l_{coh} \gg a$, the wave packets do not overlap, and the sum over **k** in the left-hand side of (2.6) breaks up into independent sums pertaining to the individual packets. Neglecting the variation of $\mathbf{j}_{01}(\mathbf{k}, \omega)$ and of $\exp[-Z_1(\mathbf{k})/2]$ in the wave-vector interval Δk, we have

$$\sum_\alpha \mathbf{j}_{01}(\varkappa_\alpha, \omega)\,\mathbf{E}_\alpha(\mathbf{r}_1)\,e^{-Z_1(\varkappa_\alpha)/2}\,e^{i\varkappa_\alpha \mathbf{r}_1} = \mathbf{j}_{02}(\varkappa_0, \omega)\,\mathbf{E}_1(-\varkappa_0, \omega)\,e^{-i\varkappa_0 \mathbf{r}_2} \quad (\alpha = 0, 1). \tag{2.7}$$

Here $\mathbf{E}_\alpha(\mathbf{r}_1)$ is the field produced at the point $\mathbf{r}_1$ by the wave propagating in the direction κ_α (we have omitted the index 2). We then have for the intensity of the γ radiation from the crystal

$$I_s(\mathbf{k}_0, \omega) = \xi \sum_{\alpha, \beta} \exp\left\{\frac{-(Z_1(\varkappa_\alpha) - Z_1(\varkappa_\beta))}{2}\right\} \mathbf{E}_{\alpha s}{}^i(\mathbf{r}_1) E_{\beta s}{}^{k*}(\mathbf{r}_1) I_{\alpha\beta}{}^{ik}, \tag{2.8}$$

$$I_{\alpha\beta}{}^{ik} = \overline{j_{01}{}^i(\varkappa_\alpha, \omega)\, j_{01}{}^{k*}(\varkappa_\beta, \omega)}; \qquad \xi = \text{const.} \tag{2.9}$$

Since we are interested in the total intensity resulting from a large number of individual decays, Eq. (2.8) will contain a certain average of the product of two transition currents; this average is in fact a sum over the spin states of the ground level and the averaged value over the spin state of the excited level. This operation is denoted by the superior bar in (2.9).

It is interesting to note that, as follows from (2.7), the radiation field at the point $\mathbf{r}_2$ is determined not by the field produced by the second source at the location of the radiating nucleus, but by the product of the field amplitude by the amplitude of the "non-excitation" of the phonons, i.e., a quantity proportional to the amplitude of formation of the excited nucleus in the case of narrow resonance. This is quite an important circumstance if we deal with an anisotropic crystal. Indeed, in this case $Z_1(\kappa)$ has different values for different directions κ, and the radiation will be emitted from the crystal even if the total field (more accurately, $\sum_\alpha \mathbf{j}_{01} \cdot \mathbf{E}_\alpha$) vanishes at the nucleus, and conversely, it may not be emitted from the crystal when the field does not vanish at the nucleus, but the total amplitude of production of the excited nucleus is equal to zero.

3. RESULTS FOR AN EXTERNAL γ-QUANTUM SOURCE

It is clear from the results of the preceding section that to describe the γ radiation of nuclei from a crystal it suffices to know the field produced by an external source at the location of the radiating nucleus. Such a problem was solved earlier in connection with an analysis of the effect of suppression of inelastic channels in resonant nuclear interaction in ideal crystals. There the problem was considered both for the Laue case (the reflected and direct waves emerge through the rear surface of the crystal), and for the Bragg case (the reflected wave emerges through the input surface). For concreteness we shall confine ourselves henceforth to crystals whose geometry corresponds to the first case.

We consider a crystal in the form of a flat plate. If a plane wave with a wave vector κ_0 close to the Bragg condition is incident on such a crystal, then the field at the point **r** in the interior of the crystal (the origin is on the input surface, the inward normal to which is the z axis) is described by a set of two waves (0, 1) having the following value (see[1], and also[11]):

$$\mathbf{E}_{0s}(\mathbf{r}) = \mathbf{e}_{0s}\mathscr{E}_{0s}(\omega)\,e^{i\varkappa_0 r}\frac{1}{2(\varepsilon_s^{(2)} - \varepsilon_s^{(1)})}\left[(2\varepsilon_s^{(2)} - g_{00})\,e^{i\varkappa_0\varepsilon_s^{(1)}z/\gamma_0} - (2\varepsilon_s^{(1)} - g_{00}) \times e^{i\varkappa_0\varepsilon_s^{(2)}z/\gamma_0}\right] \equiv \mathbf{e}_{0s}E_{0s}(\mathbf{r}),$$

$$\mathbf{E}_{1s}(\mathbf{r}) = -\,\mathbf{e}_{1s}\mathscr{E}_{0s}(\omega)\,e^{i\varkappa_1 r}\frac{g_{10}^s\beta}{2(\varepsilon_s^{(2)} - \varepsilon_s^{(1)})}\left[e^{i\varkappa_0\varepsilon_s^{(1)}z/\gamma_0} - e^{i\varkappa_0\varepsilon_s^{(2)}z/\gamma_0}\right] \equiv \mathbf{e}_{1s}E_{1s}(r), \tag{3.1}$$

Here

$$\varepsilon_s^{(1,2)} = {}^1/_4(g_{00} + \beta g_{11} - \beta\alpha) \pm {}^1/_4\left[(g_{00} + \beta g_{11} - \beta\alpha)^2 + 4\beta(g_{00}\alpha - g_{00}g_{11} + g_{01}{}^s g_{10}{}^s)\right]^{1/2}, \tag{3.2}$$

$$\gamma_\alpha = \cos\theta_\alpha, \quad \theta_\alpha = \widehat{\varkappa_\alpha \mathbf{n}}, \quad \beta = \gamma_0/\gamma_1 \qquad \alpha = \mathbf{K}(\mathbf{K} + 2\varkappa_0)/\varkappa_0^2,$$

$\mathscr{E}_{0s}(\omega)$ is the field on the input surface of the crystal.

The quantities $g_{\alpha\beta}$, which are proportional to the corresponding amplitudes for scattering by an individual atom and which play the principal role in our problem, can be represented in the case of simultaneous presence of resonant and potential electron scattering in the form

$$g_{\alpha\beta}^s = \frac{4\pi}{\varkappa_0^2\Omega_0}\sum_j e^{i(\varkappa_\alpha - \varkappa_\beta)\rho_j}\left\{e^{-(Z_j(\varkappa_\alpha) + Z_j(\varkappa_\beta))/2}\eta f_{nj}^s(\varkappa_\alpha, \varkappa_\beta) + e^{-{}^1/_2 Z_j(\varkappa_\alpha - \varkappa_\beta)} f_{ej}^s(\varkappa_\alpha, \varkappa_\beta)\right\}. \tag{3.3}$$

Here f_{nj}^s and f_{ej}^s are the coherent parts of the nuclear and electronic amplitudes for scattering by the j-th atom in the unit cell:

$$f_{nj}{}^s(\varkappa_\alpha, \varkappa_\beta) = -\frac{1}{2\varkappa_0}\frac{\Gamma_1}{\omega - \omega_0 + i\Gamma/2}\frac{2I + 1}{2I_0 + 1}p_n{}^s(\alpha, \beta)\,c_j,$$
$$f_{ej}{}^s(\varkappa_\alpha, \varkappa_\beta) = -r_0F_j(|\varkappa_\alpha - \varkappa_\beta|)\,p_e{}^s(\alpha, \beta), \tag{3.4}$$

where $c_j = 1$ for the sites of the unit cell containing nuclei of the element whose γ decay is of interest to us, but in the ground state, and $c_j = 0$ for the remaining sites; η is the concentration of the resonant isotope; ω_0, Γ_1, and Γ are the energy and elastic and total widths of the resonant level; I and I_0 are the spins of the excited and ground states of the nucleus; F_j is the atomic factor of the j-th atom in the unit cell; r_0 is the classical radius of the electron; $p_{n,e}^s(\alpha, \beta)$ are polarization factors. s = 1 stands for a polarization perpendicular to the scattering plane $(\kappa_0\kappa_1)$, and s = 2 corresponds to the case when the polarization lies in the scattering plane.

It follows from (2.8), (2.9), and (3.1) that to find the intensity of the radiation from the crystal it is necessary to know the quantity

$$B_{\alpha\beta}{}^s = I_{\alpha\beta}{}^{ik}e_{\alpha s}e_{\beta s}. \tag{3.5}$$

If we use the value of $I_{\alpha\beta}^{ik}$ obtained in[1] for the dipole and quadrupole transitions, then we obtain for $B_{\alpha\beta}^S$ directly

$$B_{\alpha\beta}{}^s = bt^s(\alpha, \beta), \qquad b = \frac{c^3}{4\omega_0}\frac{2I + 1}{2I_0 + 1}\Gamma_1 \tag{3.6}$$

with $t^S(\alpha, \beta)$ having the following values for transitions of different multipolarity:

	E1	*M1*	*E2*	
$s = 1$	1	$\cos\varphi_{\alpha\beta}$	$\cos\varphi_{\alpha\beta}$	$\varphi_{\alpha\beta} = \widehat{\varkappa_\alpha\varkappa_\beta}$. (3.7)
$s = 2$	$\cos\varphi_{\alpha\beta}$	1	$\cos 2\varphi_{\alpha\beta}$	

We note that $p_n^S(\alpha, \beta)$ coincides with $t^S(\alpha, \beta)$, and that $p_e^S(\alpha, \beta)$ coincides with $t^S(\alpha, \beta)$ for the E1 transition.

Formulas (2.8), (2.9), and (3.6) solve completely, in principle, the problem of angular distribution of the intensity of γ radiation emerging from an ideal crystal after γ decay of nuclei inside the crystal.

4. γ RADIATION OF NUCLEI FROM A CRYSTAL

Let us consider the decay of a nucleus situated at a depth l below the surface of the crystal. Far from the direction satisfying the Bragg condition $\alpha \gg |g_{\alpha\beta}|$, we have for $\epsilon_S^{(1,2)}$ the estimates $\epsilon_S^{(1)} \approx 1/2\, g_{00}$, $\epsilon_S^{(2)} \approx -1/2\, \beta\alpha$. Here $E_{1S} \sim g_{10}^S/\alpha$ and the field inside the crystal is determined in fact only by E_{0S}. Then the intensity of radiation from the crystal is

$$I_s(\varkappa_0, \omega) = \xi b |\mathscr{E}_{0s}|^2 \exp\{-Z_1(\varkappa_0) - \varkappa_0 l g_{00}''(\omega)/\gamma_0\}, \quad (4.1)$$

$$g_{00}''(\omega) \equiv \operatorname{Im} g_{00}(\omega) \approx \frac{\eta}{\Omega_0 \varkappa_0} \sigma_t(\omega) \sum_j e^{-Z_j(\varkappa_0)}, \quad (4.2)$$

σ_t is the total cross section for absorption by an individual atom.

The approximate character of (4.2) is due to the fact that in an ideal crystal the total cross section should actually contain only part of the scattering cross section (for details see[8]). We shall assume throughout, however, that the inelastic scattering by an individual atom is large compared with the elastic scattering, making it possible to disregard this circumstance (as well as the variation of the parameters of the resonant nuclear level in the crystal—see[8]).

The total intensity in the angle interval $d\Omega_{k_0}$ and in the frequency interval $d\omega$ will be designated $I_S(\mathbf{k}_0, \omega) d\omega d\Omega_{k_0}$. Then, taking into account the character of the frequency dependence of the radiation emitted following the decay, we have

$$\xi b |\mathscr{E}_{0s}(\omega)|^2 = i_s(\omega); \quad i_s(\omega) = \frac{1}{4\pi^2} \frac{\Gamma/2}{(\omega - \omega_0)^2 + \Gamma^2/4} i_{s0},$$

$$\int \frac{i_s(\omega)}{i_{s0}} d\omega\, d\Omega = 1, \quad (4.3)$$

where i_{s_0} is the total γ-quantum radiation intensity.

Assume that the radiating nuclei are at a sufficiently large depth, so that

$$\frac{\varkappa_0 l}{\gamma_0} g_{00}''(\omega) \gg 1. \quad (4.4)$$

In this case, according to (4.1), radiation will in general not emerge from the crystal in any arbitrary direction. The picture changes radically, however, if $\kappa_0 = -\mathbf{k}_0$ lies in a narrow interval near the Bragg angle. For simplicity, we confine ourselves further to the case $\beta = 1$ and assume that

$$e^{i\mathbf{K}\rho_j} = 1, \qquad Z_j(\varkappa_0) = Z_j(\varkappa_1), \quad (4.5)$$

in this case

$$g_{11} = g_{00}, \quad g_{01}^s = g_{10}^s$$

and in accordance with (3.2)

$$\varepsilon_s^{(1,2)} = \frac{1}{4}[(2g_{00} - \alpha) \pm \sqrt{\alpha^2 + 4g_{01}^{s2}}]. \quad (4.6)$$

We confine ourselves to cases in which we have at least for one polarization

$$\Delta_s = g_{00}'' - g_{01}^{s\prime\prime} \ll g_{00}''. \quad (4.7)$$

As shown in[1] for a pure nuclear scattering in the case of E1 and M1 transitions, the condition $\Delta_S = 0$ is strictly satisfied for one of the polarizations upon reflection from one family of planes. Only allowance for a sufficiently weak absorption by the electrons can lead to a certain small value of Δ_S.

If (4.4) and (4.7) are satisfied, then only the waves corresponding to one of the roots of (4.6) are retained in $E_{0S}(r)$, and the field has a noticeable value only if $\alpha < |g_{00}|$. Taking this into account and expanding $\epsilon_S^{(1,2)}$ in a series, we get

$$\varepsilon_s^{(1,2)} = -\frac{\alpha}{4} + \frac{1}{2}(g_{00} \pm g_{01}^s) \pm \frac{1}{16} \frac{\alpha^2}{g_{01}^s}, \quad (4.8)$$

We then have for the intensity of γ-quantum emission from the crystal in the considered interval of angles α, with allowance for (2.8), (3.1), (3.7), and (4.3), and for a polarization for which $t^S(\alpha, \beta) = 1$,

$$I_s = \frac{1}{4} i_s(\omega) \exp\left\{-Z_1(\varkappa_0) - \frac{\varkappa_0 l \Delta_s}{\gamma_0} - \frac{\alpha^2 g_{00}''}{8|g_{01}^s|^2}\right\} \left| 1 - \frac{\alpha}{2g_{01}^s} - e^{i\mathbf{K}r_1} \right|^2. \quad (4.9)$$

We see directly from (4.9) that γ quanta emerge from a thick crystal only in a narrow angle interval along the surfaces of cones whose axes are along the reciprocal-lattice vector $\mathbf{K}$, with an aperture angle $\chi = \widehat{\mathbf{k}_0\mathbf{K}} = 90° - \theta_B$ (θ_B is the Bragg angle). Assume that the radiating nucleus is located at a lattice site, and then

$$I_s = \frac{1}{16} i_s(\omega) \exp\left\{-Z_1(\varkappa_0) - \frac{\varkappa_0 l \Delta_s}{\gamma_0} - \frac{\alpha^2}{\alpha_{eff}^2}\right\} \frac{\alpha^2}{|g_{01}^s|^2}, \quad (4.10)$$

where

$$\alpha_{eff}^2 = \frac{|g_{01}^s|^2}{g_{01}^{s\prime\prime} \varkappa_0 l / 8\gamma_0}. \quad (4.11)$$

Recognizing that $\sin\chi d\chi = (1/4 \sin\theta_B) d\alpha$, we get for the integral intensity corresponding to each individual cone

$$J_s(\omega) = \frac{\sqrt{\pi}}{128 \sin\theta_B} i_s(\omega) \exp\left\{-Z_1(\varkappa_0) - \frac{\varkappa_0 l \Delta_s}{\gamma_0}\right\} \frac{\alpha_{eff}}{|g_{01}^s|^2} \delta\varphi, \quad (4.12)$$

where $\delta\varphi$ is the interval of the azimuthal angle φ around the axis of the cone corresponding to the receiver geometry.

Let $\Delta_S = 0$ or, in any case, $\kappa_0 l \Delta_S/\gamma_0 < 1$. It then follows from (4.12) that the integral intensity decreases with thickness not exponentially but only in a power-law fashion:

$$J_s(\omega) \sim (l/\gamma_0)^{-3/2}. \quad (4.13)$$

This is a direct consequence of the effect of suppression of the inelastic channels[1]. The scale of the angular distribution is determined, under the condition (4.4), by the value of α_{eff}.

In the case of purely nuclear interaction we have

$$\alpha_{eff}^3 = \frac{16\pi\eta\gamma_0}{(\Omega_0\varkappa_0^3)(\varkappa_0 l)} \frac{2I+1}{2I_0+1} \frac{\Gamma_1}{\Gamma} e^{-Z(\varkappa_0)}; \quad (4.11')$$

we call attention to the fact that in this case α_{eff} does not depend on the frequency at all.

It is interesting that when $\alpha = 0$ it turns out that $I_S = 0$. This is due to the vanishing of the field at the nucleus when $\alpha = 0$, and leads, in accordance with the reciprocity theorem (2.7), to vanishing of the decay intensity in this direction, in spite of the fact that the γ quanta propagate along this direction without absorption. It should be noted in this connection that this re-

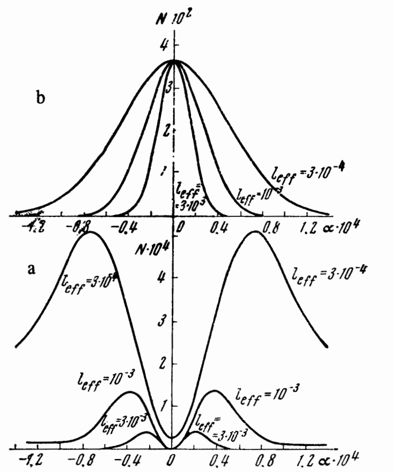

FIG. 1

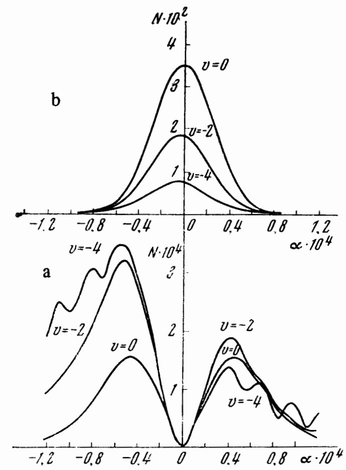

FIG. 2

sult takes place also in the absence of a complete suppression effect, i.e., that $\Delta_S \neq 0$. It is necessary here, however, to satisfy the condition (4.4), for otherwise it would be necessary to take into account all four waves in (3.1), and this would lead to $I_S \neq 0$ at $\alpha = 0$.

2. If the radiating nucleus is situated in an interstice, the picture is greatly changed. Indeed, in this case we have $\exp(i\mathbf{K}\cdot\mathbf{r}_1) \neq 1$, and recognizing that $\alpha_{eff} \ll |g_{01}|$ under the condition (4.4), we obtain

$$I_s(\varkappa_0) \approx \frac{1}{4} i_s(\omega) \exp\left\{-Z_1(\varkappa_0) - \frac{\varkappa_0 l \Delta_s}{\gamma_0} - \frac{\alpha^2}{\alpha_{eff}^2}\right\} |1 - e^{i\mathbf{K}\mathbf{r}_1}|^2. \quad (4.14)$$

Although the character of the decrease in the intensity at noticeable values of α remains the same as in the case of (4.10), the radiation intensity at $\alpha = 0$ now has a maximum. Moreover, if $\Delta_S = 0$, then there is no absorption of the decay γ quanta at all when $\alpha = 0$, a fact also connected with the effect of suppression of the inelastic channels.

Figure 1 shows plots of N against α:

$$N(\alpha) = \int I_s(\varkappa_0) \frac{\Gamma/2}{(\omega - \omega_0')^2 + \Gamma^2/4} d\omega, \quad (4.15)$$

corresponding to registration of the radiation by a tuned receiver with $\omega_0' = \omega_0$, for several values of l and for cases when the radiating nucleus is located at a site (a) or in an interstice (b). Here and henceforth we use in the calculations parameters close to the case of Fe^{57} in metallic iron (but in the absence of hyperfine splitting): $a = 2.8$ Å, $\eta = 100\%$, $E_\gamma = 14.4$ keV, $\Gamma_1/\Gamma = 0.1$, $e^{-Z(\kappa)} = 0.9$. We took into account here the exact expression (3.1) for the field and assumed that the probability of the Mössbauer effect is the same for atoms at the sites and in the interstices. The curves given for several thicknesses $l_{eff} = l/\gamma_0$ demonstrate clearly the character of the angular dependence in both cases.

If we change over from (4.14) to the integrated intensity, then we obtain for an individual cone

$$J_s(\omega) = \frac{\sqrt{\pi}}{16 \sin\theta_B} i_s(\omega) \exp\left\{-Z_1(\varkappa_0) - \frac{\varkappa_0 l \Delta_s}{\gamma_0}\right\} \alpha_{eff} |1 - e^{i\mathbf{K}\mathbf{r}_1}|^2. \quad (4.16)$$

Again, when $\Delta_S = 0$ we have a power-law rather than an exponential dependence on l:

$$J_s(\omega) = (l/\gamma_0)^{-1/2}. \quad (4.17)$$

Comparing this result with (4.13), we see that in the case of an interstitial atom, the intensity of the outgoing radiation decreases with increasing l much more slowly than in the case when the decaying atoms are at the lattice sites (this is seen directly from a comparison of the curves for different thicknesses, Figs. 1a and b), and at $l_{eff} = 10\ \mu$ the difference already amounts to two orders of magnitude.

The strong difference between the angular distributions and the thickness dependences of the intensity uncovers an interesting possibility of analyzing the position of the radiating atom in the unit cell. The change of phase $\mathbf{K}\cdot\mathbf{r}_1$ on going from one cone to the other makes it possible in this case even to determine the exact position of the atom in the interstice. It is interesting that if the position of the interstitial atom is sufficiently symmetrical, then one of the cones can correspond to the condition $\exp(i\mathbf{K}\cdot\mathbf{r}_1) = 1$, which immediately transforms case b into case a.

3. Let us analyze now the frequency dependence of the radiation emerging from the crystal. We consider first a frequency interval in which the nuclear interaction is known to prevail and where Eq. (4.4) remains in force. From (4.14) and (4.11′) we can draw in this case the interesting conclusion that the frequency spectrum of the radiation of an interstitial atom remains practically unchanged on passing through the crystal. If the radiating atom is located at a lattice site, then according to (4.10) and (4.12), a peculiar rearrangement of the frequency spectrum takes place and cancels out the initial distribution of $i_S(\omega)$ (see (4.3)). As a result, the radiation emerging from the crystal is independent of the frequency.

If the condition (4.4) is violated, then the radiation from the crystal will be determined by all four waves in (3.1). The universal character of the frequency dependence of the radiation is then lost. This transition can be traced in Fig. 2, which shows a plot of $N(\alpha)$ for different shifts of the line centers in the crystal and in the tuned receiver $v = 2(\omega_0 - \omega_0')/\Gamma$, at a fixed depth $l_{eff} = 10\ \mu$. At small α, in case (a) the intensity of the radiation actually depends weakly on v. With increasing α, the strongly-absorbing pair of waves comes into play and the curves become dependent on v. At the same time, a sharp asymmetry of the angular distribution sets in, and also oscillation of the intensity with chang-

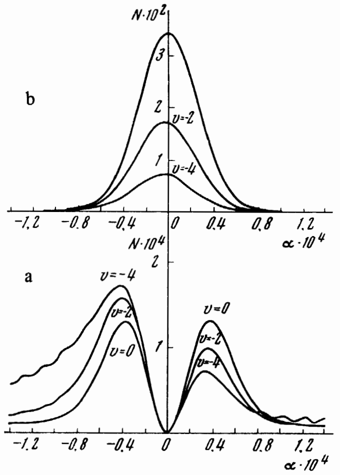

FIG. 3

ing α at noticeable values of v. (The substitution v $\rightarrow -$v corresponds to the same distribution with the substitution $\alpha \rightarrow -\alpha$.

In the case of an interstitial atom, the dependence on v is quite different. It is interesting to note that in this case the intensity decreases sharply with increasing v, whereas in case (a), at least for limited values of v, the total intensity increases. This is connected with the already noted difference in the character of the frequency dependence.

If the frequency interval is increased, an ever-increasing role is assumed by scattering and absorption by the electrons. The frequency dependence of the ra radiation is then significantly altered. In particular, interference between the nuclear-resonance and electron-resonance scattering appears, due primarily to the character of the behavior of $g'_{\alpha\beta}$ (see, for example, (4.11) and (3.3)).

Figure 3 shows the same series of curves as Fig. 2, but with allowance for the interaction with electrons. In the calculation we assumed for the zero-angle scattering amplitude $f'_e = 0.1 f''_n$ ($\omega = \omega_0$) and $f''_e = 0.01 f''_n$ ($\omega = \omega_0$). We see that allowance for the interaction with the electrons greatly changes the angular distribution, particularly at noticeable values of $|v|$, at which the resonant nuclear interaction decreases appreciably.

4. Let us discuss briefly the character of the temperature dependence of the radiation emerging from a thick crystal. If the approximation that leads to (4.9) is valid, then the main effect connected with the oscillations of the nuclei consist in the narrowing of the angular distribution of the decay γ quanta, which is determined by the dependence of α_{eff} (4.11′) on the probability of the Mössbauer effect

$$\Delta\alpha \sim e^{-Z(\varkappa_0)/2}. \tag{4.18}$$

Here $\Delta\alpha$ is the effective width of the angular distribution, and $Z(\kappa_0)$ is the argument of the exponential in the probability of the Mössbauer effect (see (2.2)).

It is of interest to note the different temperature dependences of the intensity for atoms at the sites and in the interstices. In the former case the intensity at the maximum does not change at all with temperature (see (4.10), (4.11′), and (3.3)) if the interaction is predominantly nuclear and the condition (4.4) is satisfied.

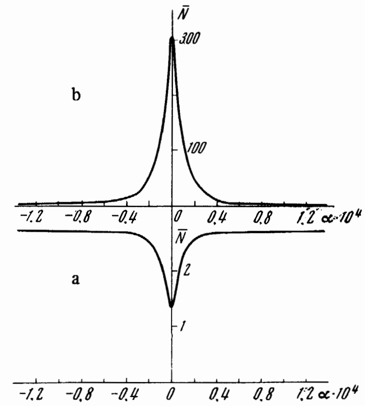

FIG. 4

In the latter, the intensity at the maximum decreases with increasing temperature simply like the Mössbauer-effect probability.

The integrated radiation intensity from the crystal decreases in both cases, quite slowly if the radiating nucleus is at a site ($\exp[-Z(\kappa_0)/2]$) and much faster ($\exp[-3Z(\kappa_0)/2]$) for a radiating nucleus at an interstice.

There is also a peculiar dependence on the concentration of the resonant isotope. In the case of purely nuclear interaction, the width of the angular distribution within each cone decreases with decreasing η like $\eta^{1/2}$ (see (4.11′)). If the radiating nucleus is in the interstice, then a similar dependence on the concentration takes place for the integral intensity, whereas for a nucleus at a site, at fixed l, an increase of the integrated intensity should be observed with decreasing η ($\sim \eta^{-1/2}$).

5. SOME EXPERIMENTAL ASPECTS

An important question for the observation of the γ decay of nuclei of an ideal crystal is the arrangement of the active nuclei inside the crystal. The simplest solution, which retains all the above-described aspects of the problem, is to introduce the active nuclei into a narrow layer near one of the surfaces of a crystalline plate.

The effect can also be observed if the radiating atoms are uniformly distributed through the crystal thickness. The curves corresponding to cases (a) and (b) are shown in Fig. 4. The ordinates represent the quantity $\bar{N}(\alpha) = \int_0^\infty N_l(\alpha)dl$ in arbitrary units. We used the same parameters in the calculation as in the preceding figures, and the ratio of the nuclear to electronic scattering amplitudes is the same as in Fig. 3.

So far we have considered only the case with a resonant analyzer of the radiation. Naturally, it is much simpler to measure the intensity of the γ radiation without a frequency analysis. In this case, at large values of α, a noticeable role is assumed by the wings of the energy distribution in the radiation of γ quanta from the nuclei, and consequently the nonresonant interaction with the electrons becomes significant. At a limited thickness, all four waves then begin to play a role. This leads, for example, to oscillations of the intensity with change in the angle α, and also to an increase of

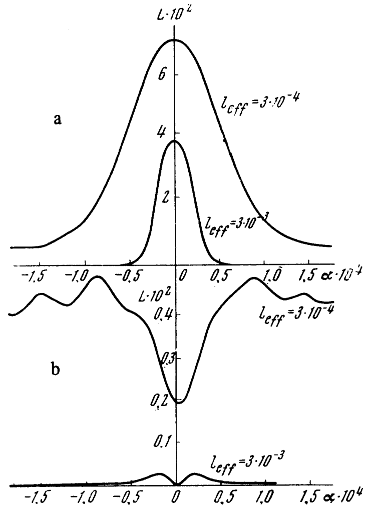

FIG. 5

the total intensity as compared with the tuned receiver.

Figure 5 shows curves demonstrating the angular distribution of the radiation emitted from the crystal (near an individual cone). The ordinates represent the quantity (compare with (4.15)) $L(\alpha) = \int I_S(\kappa_0)d\omega$, and the parameters are the same as in the preceding figures (the interaction with the electrons is the same as in Fig. 3). One can see clearly the anomalous angular distribution, although on the whole it is less pronounced than the case of a tuned receiver. Thus, an investigation of the angular distribution of γ quanta from an ideal crystal is possible, in principle, also in a very simple formulation without a tuned receiver.

[1] A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. **48**, 327 (1965) [Sov. Phys.-JETP **21**, 215 (1965)].
[2] Yu. Kagan and A. M. Afanas'ev, ibid. **49**, 1504 (1965) [**22**, 1032 (1966)].
[3] Yu. Kagan and A. M. Afanas'ev, ibid. **50**, 271 (1966) [**23**, 178 (1966)]; ZhETF Pis. Red. **2**, 130 (1965) [JETP Lett. **2**, 81 (1965)].
[4] V. K. Voĭtovetskiĭ, I. L. Korsunskiĭ, A. I. Novikov, and Yu. F. Pazhin, ZhETF Pis. Red. **11**, 149 (1970) [JETP Lett. **11**, 91 (1970)].
[5] G. V. Smirnov, V. V. Sklyarevskiĭ, and A. N. Artem'ev, ibid. **11**, 579 (1970) [**11**, 400 (1970)].
[6] L. D. Landau and E. M. Lifshitz, Élektrodinamika sploshnykh sred (Electrodynamics of Continuous Media), Gostekhizdat, 1957 [Addison-Wesley, 1959].
[7] W. Kossel, V. Loeck and H. Vages, Zs. Phys., **94**, 139, 1935.
[8] M. Laue, Röntgenstrahlen-interferenzen, Braunschweig, 1960.
[9] A. M. Afanas'ev and Yu. Kagan, Zh. Eksp. Teor. Fiz. **52**, 191 (1967) [Sov. Phys.-JETP **25**, 124 (1967)].
[10] L. D. Landau and E. M. Lifshitz, Kvantovaya Mekhanika (Quantum Mechanics), Fizmatgiz, 1963 [Addison-Wesley, 1965].
[11] Yu. Kagan, A. M. Afanas'ev, and I. P. Perstnev, Zh. Eksp. Teor. Fiz. **54**, 1530 (1968) [Sov. Phys.-JETP **27**, 819 (1968)].

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