MULTIVALUED SASAKI EFFECT IN MANY-VALLEY SEMICONDUCTORS

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The existence of several stable nonequilibrium distributions of the electrons in the valleys turns out to be possible in the case of a rapid decrease of the intervalley transition time with an increase of the heating electric field in many-valley semiconductors in a certain range of electric field strengths; these correspond to different values of the conductivity tensor of the semiconductor—the multivalued Sasaki effect. In this connection it turns out that when the direction of the current is along the principal axes, relative to which the valleys are equivalently arranged (for example, the [100] axis in germanium or the [111] axis in silicon), the state with the highest symmetry, in which all of the valleys are identically populated, is unstable, but states having a preferential population of one of the valleys are stable. In connection with a variation of the electric field, two types of transitions are possible from isotropic or slightly anisotropic states to strongly anisotropic states: 1) gradual transitions, 2) abrupt transitions. In the latter case of an abrupt transition the current through the sample changes and current hysteresis exists in a certain range of the field strengths. A magnetic field may stimulate abrupt transitions (jumps) from one stable state to another; here the transverse e.m.f. in the crystal is changed by the jump. The two-valley model (which can be realized in germanium by achieving the regime of short-circuiting along the [001] direction) is analyzed in detail in this work, and also results are presented for germanium when the current is directed along the [100] and [110] axes, and for silicon when the current is along the [111] axis. The estimates which are made indicate that the multivalued sasaki effect should occur in pure germanium at low temperatures.

INTRODUCTION

A heating electric field destroys the equilibrium distribution of the electrons with respect to the equivalent valleys in a many-valley semiconductor, leading to a new distribution:

$$n_\alpha = \tau_\alpha n \Big/ \sum_{\beta=1}^{\nu} \tau_\beta \qquad (1)$$

(here n denotes the total concentration of electrons, τ_α is the time for the drift of an electron from the valley labelled α—an α-electron—to any other valley). The different values of τ_α, leading to unequal values of n_α, are associated with the different values of the mobility tensors $\mu_{ik}^{(\alpha)}$ and consequently with unequal heating of the electrons from different valleys. The latter property is responsible for the appearance in heating fields of an anisotropy in the electrical conductivity of homogeneous cubic many-valley semiconductors—the Sasaki effect (see the review article[1] where a list of literature references is given).

In order to obtain the maximum repopulation of the electrons, other conditions being equal, the following are required: in the first place, the maximum sharp monotonic dependence of τ_α on the heating power, and in the second place, the absence of energy exchange between the valleys (independent energy balance of the valleys). The second requirement is satisfied if the times τ_α substantially exceed the intravalley energy relaxation times of the electrons and provided that there is no intensive electron-electron exchange of energy between valleys during collisions without intervalley transitions.

In the majority of situations the Sasaki effect is single-valued, that is, for a given orientation of the sample, to each value of the applied electric field there corresponds a unique redistribution of the electrons between the valleys and, thus, a unique magnitude and direction of the transverse (anisotropic) electric field. However, situations are possible when in a certain range of applied electric field strengths the Sasaki effect is not single-valued, that is, several stationary distributions n_α appear. It was first possible to perceive this from figures representing the results of numerical calculations in [2]. Shyam and Kroemer[3] called attention to this possibility, which indicated the identical nature of the multivalued Sasaki effect and the negative transverse differential conductivity observed in [4] (see also the review article [5]).

The possibility of a multivalued Sasaki effect is completely related to the existence of a transverse (anisotropic) electric field. If a transverse current is not present, then this field as a whole does not do any work. However, owing to the presence of transverse currents of electrons in different valleys, the transverse field transfers the energy of the electrons from one valley to the others; the energy transfer due to the dependence $\tau_\alpha(\epsilon)$ leads to a redistribution of the carriers.

A detailed qualitative theory of the multivalued Sasaki effect is developed below. The simplest two-valley model is considered in Sec. 1; in Sec. 2 the stability of the stationary distributions obtained in Sec. 1 is investigated; the influence of a magnetic field (the Hall effect) is considered in Sec. 3 under the conditions for the multivalued Sasaki effect; Sec. 4 is devoted to the multivalued Sasaki effect in actual energy structures of the type n-Ge and n-Si; estimates of the dependence of τ_α on the heating power are made in Sec. 5, making it possible to reach conclusions about certain real experimental situations.

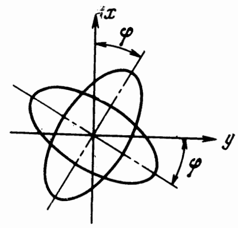

FIG. 1. Two-valley model.

1. DISTRIBUTION OF THE ELECTRONS IN THE VALLEYS IN A TWO-VALLEY SEMICONDUCTOR

Let us consider a two-valley semiconductor with the valleys, lying in the xy plane along mutually perpendicular axes, arbitrarily oriented relative to the x axis, which is chosen along the direction of current flow in the sample (see Fig. 1):

$$i_y = 0. \tag{2}$$

In particular, this situation is realized in n-Ge if the z axis coincides with one of the fourth-order axes and if the short-circuiting regime is realized in the direction of this axis (for example, if the size of the sample in the z direction greatly exceeds the size in the x direction[1]). (In passing we note that if a short-circuiting regime exists in the y or z direction, then the effect we are interested in will not exist: for all values of E_x and φ the unique distribution of the electrons between the valleys is always stable.)

Let us assume that independent energy balance of the valleys, which was mentioned in the Introduction, is realized. Then the average power absorbed by a single electron in the field **E**,

$$p_\alpha = \mathbf{j}_\alpha \mathbf{E} / n_\alpha \tag{3}$$

completely determines[2] the distribution function of the α-electrons and consequently all of their parameters: the drift time

$$\tau_\alpha = \tau(p_\alpha) \tag{4}$$

and the principal values of the mobility tensor

$$\begin{aligned} \mu_{xx}^{(1,2)} &= \mu(p_{1,2})(1 \mp a\cos 2\varphi),\\ \mu_{yy}^{(1,2)} &= \mu(p_{1,2})(1 \pm a\cos 2\varphi),\\ \mu_{xy}^{(1,2)} &= \mu_{yx}^{(1,2)} = \mp a\mu(p_{1,2})\sin 2\varphi, \end{aligned} \tag{5}$$

where the angle φ is defined in Fig. 1: $0 \le \varphi \le \pi/2$, $0 < a < 1$. (In the general case the anisotropy parameter a also depends on $p_{1,2}$; however if a single scattering mechanism dominates for all values of p or if mechanisms having identical anisotropies of their relaxation times give comparable contributions, then a ceases to depend on p.) Let us introduce the angle of anisotropy θ:

$$\operatorname{tg}\theta \equiv E_y / E_x; \tag{6}$$

then from Eqs. (3) and (5) it follows that

$$\Pi_{1,2} \equiv \frac{p_{1,2}}{e\mu(p_{1,2})} = E_x^2 \sec^2\theta\,[1 \mp a\cos 2(\varphi - \theta)]. \tag{7}$$

The parameters $\Pi_{1,2}$, just like $p_{1,2}$ (within the limits of a single-valued relation between Π and p) completely determine $\tau_{1,2}$ and $\mu^{(1,2)}$. Determining the field E_y from the condition (2), we obtain the following transcendental equation for θ:

$$\frac{\Phi\{E_x^2 \sec^2\theta\,[1 - a\cos 2(\varphi-\theta)]\}}{\Phi\{E_x^2 \sec^2\theta\,[1 + a\cos 2(\varphi-\theta)]\}} = \frac{a\sin(2\varphi-\theta)+\sin\theta}{a\sin(2\varphi-\theta)-\sin\theta}, \tag{8}$$

where $\Phi\{\Pi\} \equiv \tau(\Pi)\,\mu(\Pi)$.

One can reach certain conclusions about the roots of Eq. (8) without specifying the form of $\Phi\{\Pi\}$. For this purpose let us rewrite Eq. (8) in the form

$$a\,\frac{\Phi(\Pi_1) - \Phi(\Pi_2)}{\Phi(\Pi_1) + \Phi(\Pi_2)} = \frac{\sin\theta}{\sin(2\varphi - \theta)} \tag{8'}$$

and let us investigate this equation.

1. A root of Eq. (8) occurs only in the interval (θ_2, θ_1), where

$$\operatorname{tg}\theta_{1,2} = \pm\frac{a\sin 2\varphi}{1 \pm a\cos 2\varphi}; \tag{9}$$

this follows from the fact that the left-hand side of Eq. (8′), $L(\vartheta)$, lies within the limits $-a < L(\vartheta) < a$; here the quantity $\vartheta = \tan\theta$ has been introduced.

2. It is easy to verify that

$$\theta(E_x^2, \varphi) = -\theta(E_x^2, \pi/2 - \varphi); \tag{10}$$

from here it follows that it is sufficient to consider the interval $0 < \varphi \le \pi/4$.

3. If $\Phi\{\Pi\}$ is a monotonic function of Π, then $L(\vartheta)$ is a monotonic function of ϑ in the interval $(\tan\theta_2, \tan\theta_1)$; this follows from the fact that $\Pi_1(\vartheta)$ has a minimum at $\vartheta = \tan\theta_1$, and $\Pi_2(\vartheta)$ has a minimum at $\vartheta = \tan\theta_2$ $(R(\vartheta) = -\Pi_2'/\Pi_1')$. If $\Phi(\Pi')$ is a monotonically increasing function, then $L(\vartheta)$ decreases monotonically in the indicated interval, and since the right-hand side of Eq. (8′), $R(\vartheta)$, increases monotonically in this interval from $-a$ to a, there is a single root of Eq. (8) (see Fig. 2a, curve 1′). However, if $\Phi(\Pi)$ is a monotonically decreasing function, then $L(\vartheta)$ monotonically increases in the interval $(\tan\theta_2, \tan\theta_1)$, so that the number of roots of Eq. (8) can be expressed in general as an odd number greater than unity.

4. If $\Phi(\Pi)$ is a monotonically increasing function, then the only root of (8) is negative for $\varphi < \pi/4$, equal to zero for $\varphi = \pi/4$, and positive for $\varphi > \pi/4$ (this follows from the fact that $R(0) = 0$ and $L(\vartheta) = 0$ for $\theta = \varphi - (\pi/4)$). Thus, in the case of the "anomalous"

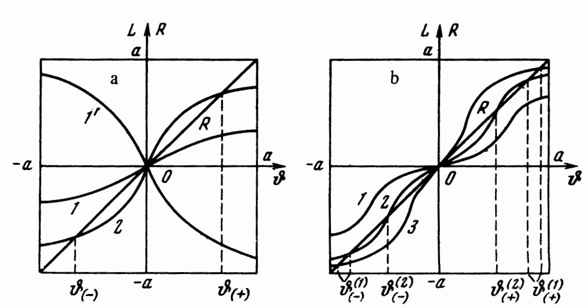

FIG. 2. Dependence of the left-hand (L) and right-hand (R) sides of Eq. (8′) on ϑ for $\varphi = \pi/4$ and for different values of E_x.

[1] In this connection, however, the danger arises of a decomposition of the sample in the z direction into domains having fields E_z which are different in magnitude and oppositely directed.

[2] It is assumed that in each valley the distribution function is close to its isotropic component.

Sakata effect[6] there exists a unique and, as shown below (Sec. 3), stable solution of Eq. (8).

Let us return to the case of a monotonically decreasing $\Phi(\Pi)$: $\Phi'(\Pi) \equiv d\Phi/d\Pi < 0$ (which corresponds to the "normal" Sasaki effect). Let us consider the cases $\varphi = \pi/4$ and $\varphi < \pi/4$ separately.

1) $\varphi = \pi/4$ (Fig. 2). In this case $R(\vartheta) = \vartheta$, $L(0) = 0$ so that the root is always $\vartheta = 0$, which is an inflection point of $L(\vartheta)$. The condition for the tangency of $L(\vartheta)$ and $R(\vartheta)$ at $\vartheta = 0$,

$$-2a^2E_x^2(\Phi'/\Phi)_{\Pi=E_x^2} = 1, \tag{11}$$

determines a certain critical field E_c at which a change in the number of solutions of Eq. (8) by two occurs. Two types of these changes are possible.

Type I occurs for

$$A(E_c) \equiv [(\Phi')^2 - \Phi\Phi'' + 2a^2E_c^2(\Phi'\Phi'' - \Phi\Phi'''/3)]_{\Pi=E_c^2} < 0, \tag{12}$$

when at the point $\vartheta = 0$ the curvature of $L(\vartheta)$ changes from positive for $\vartheta < 0$ to negative for $\vartheta > 0$ (Fig. 2a). In this case for

$$-2a^2E_x^2(\Phi'/\Phi)_{\Pi=E_x^2} > 1 \tag{13}$$

two more solutions of Eq. (8), $\vartheta_{(-)} = \vartheta_{(+)}$ $(\theta_{(-)} = -\theta_{(+)})$ supplement the solution $\vartheta = 0$ in the neighborhood of this point, where these solutions are not present for

$$-2a^2E_x^2(\Phi'/\Phi)_{\Pi=E_x^2} < 1. \tag{14}$$

We have the following expressions for these roots for small deviations of $|E_x|$ from E_c (as long as $|\vartheta| \ll a$)

$$\theta_{(\pm)}^2 = R(E_c)(|E_x| - E_c), \tag{15}$$

where

$$R(E_c) = -\frac{2\Phi^2(E_c^2)[\alpha(E_c^2) + E_c^2(d\alpha/d\Pi)_{\Pi=E_c^2}]}{A(E_c)E_c^3}, \tag{16}$$

$$\alpha \equiv -\Phi'/\Phi.$$

Type II occurs for

$$A(E_c) > 0. \tag{12'}$$

when at the point $\vartheta = 0$ the curvature of $L(\vartheta)$ changes from negative (for $\vartheta < 0$) to positive (Fig. 2b). In this case for field strengths close to E_c, one solution exists in the vicinity of the point $\vartheta = 0$ if (13) is fulfilled, and three solutions exist if (14) is satisfied.

For the actual dependences of Φ on Π, several solutions of Eq. (11) exist, that is, several critical fields. We shall call the one for which

$$\left[\alpha(\Pi) + \Pi\frac{d\alpha}{d\Pi}\right]_{\Pi=E_c^2} > 0, \tag{17}$$

the lower critical field $E_c^{(l)}$, and the field for which the opposite inequality (opposite to (17)) holds will be called the higher critical field $E_c^{(h)}$. As long as one excludes the situation when there are no critical fields at all, i.e., the case when Eq. (1) does not have any solutions, then the simplest and most realistic situation is the case when only two critical fields exist—one lower and one upper, and also, as it is not difficult to verify, $E_c^{(h)} > E_c^{(l)}$ (here the case of such idealized dependences $\Phi(\Pi)$ for which only one critical field $E_c^{(l)}$ exists is included because one can assume $E_c^{(h)} \to \infty$). We shall only discuss this situation. The following special cases are possible.

a) The point $\vartheta = 0$ is the only inflection point for $L(\vartheta)$, where $A(E_x) < 0$ everywhere (Fig. 2a). Then for both critical fields there is a change of the number of solutions of type I, and the dependence $\theta(|E_x|)$ has the form shown in Fig. 3a.

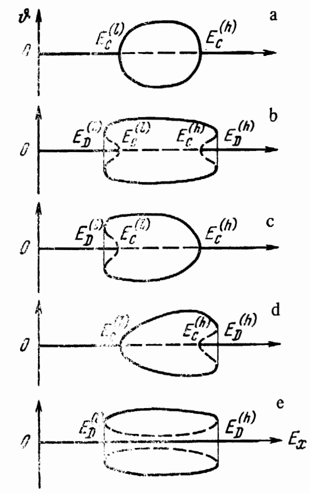

FIG. 3. Possible dependences of ϑ on E_x for $\varphi = \pi/4$.

b) In addition to the inflection point associated with $\vartheta = 0$ there are two more inflection points, where $A(E_x) > 0$ everywhere (Fig. 2b). In this case at both critical fields a change in the number of solutions of type II occurs, and in addition to the critical fields $E_c^{(l,h)}$ two more characteristic fields $E_D^{(l,h)}$ exist, in which the number of solutions changes by 4, where necessarily $E_D^{(l)} < E_c^{(l)} < E_c^{(h)} < E_D^{(h)}$. The dependence of θ on $|E_x|$ for this case is shown in Fig. 3b.

c) The sign of $A(E_x)$ changes with increasing magnitude of the field E_x so that the signs of $A(E_c^{(l)})$ and $A(E_c^{(h)})$ do not agree. In this connection the dependences have the "composite" form shown in Figs. 3c and 3d.

It is also necessary to include among the actual situations those such that (for $A(E_x) > 0$ and in the presence of three inflection points) no critical fields of the type E_c exist inside the interval of field strengths $(E_D^{(l)}, E_D^{(h)})$, i.e., Eq. (11) does not have any solutions. The case is illustrated in Fig. 3e. The situations shown in Figs. 3a–3e completely exhaust all possible forms of the dependence of ϑ on E_x (except for trivial cases) under the assumption that the dependence $L(\vartheta)$ is given only by curves of the type shown in Figs. 2a and 2b, and under the assumption that the number of critical fields E_c does not exceed two. For more complicated dependences $L(\vartheta)$ even richer $\vartheta(E_x)$ can be obtained; however, it is necessary to attribute them to unrealistic situations.

From Fig. 3 it is clear that even in the simple situations under consideration, up to five different stationary values of $\vartheta(E_x)$ may exist. As is shown in Sec. 2, in sufficiently thick samples and under the assumption that $\mu(\Pi)$ does not fall off faster than $\Pi^{-1/2}$ with increasing Π, then those solutions for which

$$\operatorname{tg}\Omega_L < \operatorname{tg}\Omega_R, \tag{18}$$

are stable with respect to quasineutral perturbations, where $\Omega_L(\vartheta)$ denotes the angle of inclination of $L(\vartheta)$ to the axis of abscissas, and $\Omega_R(\vartheta)$ denotes the angle of inclination of $R(\vartheta)$. On Fig. 3 and everywhere in what follows below, the solutions which are stable in the sense of the criterion (18) are plotted as solid lines, and the unstable solutions are plotted as dotted lines. Thus, in the interval of field strengths $(E_c^{(l)}, E_c^{(h)})$ the solution $\vartheta = 0$, corresponding to the state with uniformly populated valleys, is always unstable, but anisotropic states having a preferential population of one of the valleys are stable. In the case indicated in Fig. 3a (henceforth referred to simply as case a), the transition from the isotropic state to an anisotropic state occurs continuously: the angle θ increases smoothly from zero. In cases b, c, and d the transitions occur abruptly, where the sudden establishment (or disappearance) of the anisotropic state is accompanied by a sudden change in the current through the sample (and also by a sudden change of certain other properties of the sample). In the intervals of field strengths $(E_c^{(l)}, E_D^{(l)})$ and (or) $(E_c^{(h)}, E_D^{(h)})$ there exists hysteresis of the current and in the anisotropy angle θ as functions of the field $|E_x|$; thus, in cases c and d there should be single-valued hysteresis and in case b a twofold hysteresis on the current-voltage characteristics (IVC).

It is not difficult to investigate the IVC of a sample near the points $E_c^{(l)}$ and $E_c^{(h)}$ in case a (and near the points associated with a change in the number of solutions of type I in cases c and d) when the angle θ is changing continuously. For arbitrary values of φ the IVC is given by the formula

$$J = eE_x\bar{\mu}\left(1 - \frac{\operatorname{tg}\theta}{\operatorname{tg}(2\varphi-\theta)}\right), \quad \bar{\mu} = \frac{\Phi(\Pi_1)+\Phi(\Pi_2)}{\tau(\Pi_1)+\tau(\Pi_2)}. \tag{19}$$

For $\varphi = \pi/4$ near the critical field strengths

$$\frac{J-J_c}{J_c} = \left[\frac{1}{E_c} + \frac{1}{\bar{\mu}}\left(\frac{d\bar{\mu}}{dE_x}\right)_{E_x=E_c} - R(E_c)\right](E_x - E_c); \quad J_c = J(E_c). \tag{20}$$

If the coefficient inside the square brackets is negative, then a region of negative conductivity is formed near the point E_c (for $E_x > E_c^{(l)}$ or (and) for $E_x < E_c^{(h)}$), and the formation of this region may be associated with the appearance of an additional instability which is not considered here. The region of negative conductivity appears for sufficiently large values of a and apparently occurs in a small (in comparison with $E_c^{(l)}$) interval of field strengths. (For example, for $\Phi(\Pi) \sim \exp - \gamma\Pi$ and for an unessential dependence of $\bar{\mu}$ on Π, this region appears near $E_c^{(l)} = 1/a\sqrt{2\gamma}$ for $6a^2 > 1$.)

2) $\varphi < \pi/4$ (Fig. 4). In this case, as before $R(0) = 0$, but $|\tan\theta_2| > \tan\theta_1$ and $L(\vartheta) = 0$ for $\theta = -\psi \equiv -(\pi/4) + \varphi$. Therefore negative roots of Eq. (8) are conceivable only for $\theta_2 < -\psi$, i.e., in the following range of the angles φ:

$$\pi/4 - \operatorname{arctg} a < \varphi \leqslant \pi/4. \tag{21}$$

(The regions corresponding to the possible roots of Eq. (8) for $\Phi' < 0$ and for $\Phi' > 0$ are indicated below on Fig. 6.) If $L(\vartheta)$ has a single inflection point corresponding to a change of the curvature with increasing ϑ from positive to negative (Fig. 4a), then the existence of a region of field strengths $(E_c^{(l)}(\psi), E_c^{(h)}(\psi))$ is possible in which Eq. (8) has two "unusual" negative roots $\theta_{(-)}^{(1)} < \theta_{(-)}^{(2)}$ (of which only $\theta_{(-)}^{(1)}$ is stable) in addition to the "usual" positive root $\theta_{(+)}$. The interval $(E_c^{(l)}(\psi), E_c^{(h)}(\psi))$ lies inside the interval $(E_c^{(l)}, E_c^{(h)})$ considered above. The qualitative dependence of $\vartheta(E_x)$ for this case is shown in Fig. 5a, and a chart of the roots $\theta(\varphi)$ for different values of E_x is shown in Fig. 6a. The part of this chart for which the conditions

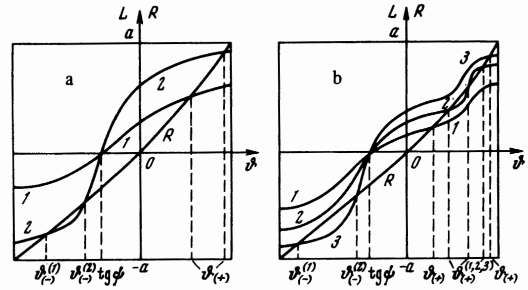

FIG. 4. Dependences of the left-hand and right-hand sides of Eq. (8′) on ϑ for $\varphi < \pi/4$ and for different values of E_x.

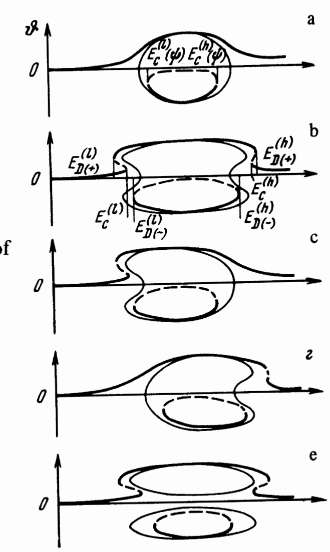

FIG. 5. Possible dependence of ϑ on E_x for $\varphi < \pi/4$.

$$|\psi|, \ |\theta| \leqslant a, \tag{22}$$

are satisfied can be calculated by solving the cubic equation:

$$A\theta^3 + B\theta - C\psi = 0, \tag{23}$$

which approximately represents Eq. (8) upon fulfillment of the conditions (22); here A is determined by formula (12):

$$B = -\frac{\Phi\Phi'}{E_x^2}\left(1 + \frac{\Phi}{2a^2E_x^2\Phi'}\right), \quad C = \frac{\Phi\Phi'}{E_x^2},$$

where the function Φ and its derivatives are evaluated at the point $\Pi = E_x^2$. The condition for the existence of three real roots has the form

$$B^3/AC^2 < -{}^{27}/_4\psi^2. \tag{24}$$

For $\psi = 0$ this condition takes the form (13) if $A < 0$, and it takes the form (14) if $A > 0$.

If $L(\vartheta)$ has three inflection points (see Fig. 4b) then, as is indicated in Fig. 5b and Fig. 6b, the interval between the critical fields $E_c^{(l)}(\psi)$ and $E_c^{(h)}(\psi)$ is expanded in comparison with the case $\psi = 0$ (this is evident from inequality (24) for $A > 0$). The fields $E_D^{(l,h)}$ are now

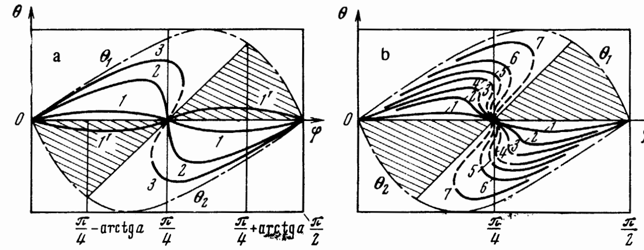

FIG. 6. Dependences of the anisotropy angle θ on φ for different values of E_X.

different for the positive and negative roots, where

$$(E_{D(+)}^{(l)}(\psi), E_{D(+)}^{(h)}(\psi)) \supset (E_D^{(l)}, E_D^{(h)}) \supset (E_{D(-)}^{(l)}(\psi), E_{D(-)}^{(h)}(\psi)).$$

The intermediate situations corresponding to cases 3c–3e for $\varphi = \pi/4$ are indicated, respectively, by Figs. 5c–5e.

The value of the stationary values of θ makes it possible to calculate the values of $\Pi_{1,2}$ from formulas (7), and from these one can determine all parameters of the valleys, including $\tau(\Pi_{1,2})$ and $\Phi(\Pi_{1,2})$.

2. STABILITY OF STATIONARY TWO-VALLEY DISTRIBUTIONS WITH RESPECT TO SMALL QUASINEUTRAL PERTURBATIONS

In Sec. 1 some of the obtained stationary distributions were called stable, and the others were called unstable. On Figs. 3, 5, and 6 the stable values of the anisotropy angle θ are indicated by solid lines, and the unstable values by dotted lines. In all of these cases one has in mind stability (or instability) with respect to small quasineutral redistributions of the carriers relative to the stationary distributions, whose Fourier components we denote by

$$\delta n_1(\mathbf{k}) = -\delta n_2(\mathbf{k}) = \nu(\mathbf{k}). \qquad (25)$$

The time evolution of $\nu(\mathbf{k})$ (and also the field $\mathscr{E}(\mathbf{k})$ and the currents $\mathbf{i}_{1,2}(\mathbf{k})$ in each of the valleys associated with these deviations) is determined by the following equations:

$$i[\mathbf{k} + \vec{\mathscr{E}}] = 0, \qquad (26)$$

$$\pm ik\mathbf{i}_{1,2} = -\nu(\tau_1^{-1} + \tau_2^{-1}) - \frac{n_1}{\tau_1}(\eta_1 - \eta_2) + i\omega\nu, \qquad (27)$$

$$\mathbf{i}_{1,2} = -\hat{\mu}^{(1,2)}[n_{1,2}(\vec{\mathscr{E}} + \mathbf{E}\varkappa_{1,2}) \pm \nu\mathbf{E} \pm i\beta_{1,2}\mathbf{k}\nu], \qquad (28)$$

where $\beta_{1,2}\hat{\mu}^{(1,2)} \equiv \hat{D}_{1,2}$; $\hat{D}_{1,2}$ are the coefficients of the diffusion tensor; $\eta_{1,2}$ and $\kappa_{1,2}$ are determined by the equations $\delta\hat{\mu}^{(1,2)} = \hat{\mu}^{(1,2)}\kappa_{1,2}\,\delta(\tau_{1,2}^{-1}) = -\tau_{1,2}^{-1}\eta_{1,2}$. Since $\hat{\mu}^{(1,2)}$ and $\tau_{1,2}$ are entirely determined by the quantities $\Pi_{1,2}$, then

$$\varkappa_{1,2} = \zeta_{1,2}\delta\Pi_{1,2}, \quad \eta_{1,2} = \xi_{1,2}\delta\Pi_{1,2}, \qquad (29)$$

where

$$\zeta_{1,2} = \frac{1}{\mu^{(1,2)}}\frac{d\mu^{(1,2)}}{d\Pi_{1,2}}, \quad \xi_{1,2} = -\frac{1}{\tau_{1,2}}\frac{d\tau_{1,2}}{d\Pi_{1,2}},$$

and also from Eqs. (7) and (3)

$$n_{1,2}\mu^{(1,2)}(1 + \zeta_{1,2}\Pi_{1,2})\delta\Pi_{1,2} \pm \mu^{(1,2)}\nu\Pi_{1,2} = -(\mathbf{j}_{1,2}\vec{\mathscr{E}} + \mathbf{i}_{1,2}\mathrm{E}). \qquad (30)$$

Let us assume that the field E_X is stringently given, i.e.,

$$\mathscr{E}_x = 0. \qquad (31)$$

(In the case of a negative differential resistivity when a domain instability of the field E_X may occur, condition (31) may not be fulfilled.) In the presence of condition (31) $k_X = 0$ and $i_{1X} = -i_{2y}$. From expressions (26) through (30) we obtain the equations which determine $\delta\Pi_{1,2}$ and $\nu' \equiv \nu(n_1 + n_2)/n_1n_2$:

$$\Phi_1\delta\Pi_1 + \Phi_2\delta\Pi_2 + i\beta k\lambda\nu' = 0; \qquad (32)$$

$$\left(\xi_1 + ik\lambda\zeta_1\frac{\alpha_2\Phi_2}{\alpha_1\Phi_1 + \alpha_2\Phi_2}\right)\delta\Pi_1 - \left(\xi_2 - ik\lambda\zeta_2\frac{\alpha_1\Phi_1}{\alpha_1\Phi_1 + \alpha_2\Phi_2}\right)\delta\Pi_2 + \left(1 - i\omega\tau_0 + ik\lambda\frac{\alpha_2 n_2\Phi_2 - \alpha_1 n_1\Phi_1}{(n_1 + n_2)(\alpha_1\Phi_1 + \alpha_2\Phi_2)} + \beta k^2\frac{\alpha_1\alpha_2\Phi_1\Phi_2}{\alpha_1\Phi_1 + \alpha_2\Phi_2}\right)\nu' = 0, \qquad (33)$$

$$\left[\zeta_1 + \frac{\Phi_1}{4\lambda^2}(\alpha_1\Phi_1 + \alpha_2\Phi_2)\right]\delta\Pi_1 - \left[\zeta_2 + \frac{\Phi_2}{4\lambda^2}(\alpha_1\Phi_1 + \alpha_2\Phi_2)\right]\delta\Pi_2 + \left[1 - \frac{i\beta k}{4\lambda}\frac{\beta_2 n_1 - \beta_1 n_2}{\beta_2 n_1 + \beta_1 n_2}(\alpha_1\Phi_1 + \alpha_2\Phi_2) - \frac{i\beta k}{\lambda}\frac{\beta_1\alpha_1\Phi_1 n_2 - \beta_2\alpha_2 n_1\Phi_2}{\beta_2 n_1 + \beta_1 n_2}\right]\nu' = 0,$$

where

$$\tau_0 = \frac{\tau_1\tau_2}{\tau_1 + \tau_2}, \quad \lambda = j_{1\nu}\frac{\tau_1}{n_1}, \quad \alpha_{1,2} = \frac{\mu_{\nu\nu}^{(1,2)}}{\mu^{(1,2)}}, \quad \beta = \frac{\beta_2 n_1 + \beta_1 n_2}{n_1 + n_2} \qquad (34)$$

Setting the determinant of Eqs. (32)–(34) equal to zero determines $\omega(k)$, and also the quantity of interest to us, $\mathrm{Im}\,\omega$, is obtained in the form

$$\mathrm{Im}\,\omega(k) = -\frac{B_0 + B_1 k^2}{B_2}, \qquad (35)$$

where

$$B_0 = \frac{\Phi_1}{\Phi_2}\frac{d\Phi_2}{d\Pi_2} + \frac{\Phi_2}{\Phi_1}\frac{d\Phi_1}{d\Pi_1} + \frac{\Phi_1\Phi_2}{2\lambda^2}(\alpha_1\Phi_1 + \alpha_2\Phi_2),$$

$$B_1 = \beta\left(\lambda^2\zeta_1\zeta_2 + \frac{\alpha_1\alpha_2\Phi_1^2\Phi_2^2}{2\lambda^2} + \frac{\zeta_1\Phi_2^2\alpha_2 + \zeta_2\Phi_1^2\alpha_1}{2} + \frac{\zeta_2\tau_2\beta_1\alpha_1\Phi_1^2 + \zeta_1\tau_1\beta_2\alpha_2\Phi_2^2}{2(\beta_2\tau_1 + \beta_1\tau_2)}\right),$$

$$B_2 = \zeta_1\Phi_2 + \zeta_2\Phi_1 + \frac{\Phi_1\Phi_2}{2\lambda^2}(\alpha_1\Phi_1 + \alpha_2\Phi_2).$$

The condition $\mathrm{Im}\,\omega(k) < 0$, whose fulfillment is necessary for stability of the stationary solutions, has the form

$$(B_0 + B_1 k^2)/B_2 > 0. \qquad (36)$$

First let us consider the special case when μ does not depend on Π ($\zeta_1 = \zeta_2 = 0$). Then in B_1 and B_2 there is only a single term, which is actually positive, which survives, and condition (36) reduces to

$$B_0 > -\beta\frac{\alpha_1\alpha_2\Phi_1^2\Phi_2^2}{2\lambda^2}k^2. \qquad (37)$$

It is not difficult to see that

$$B_0 = -\mathfrak{G}^2(\theta)(\mathrm{tg}\,\Omega_L - \mathrm{tg}\,\Omega_R), \qquad (38)$$

where Ω_L is the angle of inclination of the left-hand side of Eq. (8′) and Ω_R is the angle of inclination of its right-hand side to the axis of abscissas on Figs. 2 and 4 at the point of intersection of L and R. In Eq. (34) the right-hand side has a maximum at $k^2 = 0$; thus, in very thick samples the criterion for stability (37) reduces to the condition (18).

In thin samples, where a minimum value $k_{min}^2 \sim 1/d^2$ exists (d denotes the thickness), condition (37) reduces to

$$\mathrm{tg}\Omega_R - \mathrm{tg}\Omega_L > -\beta k_{min}^2\frac{a(1 - a^2\cos^2 2\varphi)(\Phi_1\Phi_2)^{3/2}}{(\Phi_2 + \Phi_2)^2\cos\theta\sqrt{a^2\sin^2(2\varphi - \theta) - \sin^2\theta}}, \qquad (39)$$

so that the solutions corresponding to certain parts of the "dotted" sections on Figs. 3, 5, and 6 adjoining the "solid" sections become stable. As is evident from expression (39), the characteristic length with which one must compare d is the length for intervalley scattering, $\mathscr{L} = \sqrt{D\tau}$, so that criterion (18) is valid for $d \gg \mathscr{L}$.

If the mobility depends on Π, then a new situation arises only in the case when μ decreases with heating ($\zeta_{1,2} < 0$). We note that for

$$\frac{\varkappa_1}{Y_1} + \frac{\varkappa_2}{Y_2} < -1, \tag{40}$$

where

$$Y_{1,2} = 1 + \frac{2\lambda^2}{\alpha_{1,2}\Phi_{1,2}^2}\zeta_{1,2}, \quad \varkappa_1 = \frac{\beta_1\tau_2}{\beta_1\tau_2 + \beta_2\tau_1}, \quad \varkappa_2 = 1 - \varkappa_1,$$

B_1 becomes negative, and for

$$\alpha_1 Y_1 \Phi_1 + \alpha_2 Y_2 \Phi_2 < 0 \tag{41}$$

B_2 is negative.

If $B_1, B_2 < 0$, then just as in the case $\zeta_1, \zeta_2 \geq 0$) the right-hand side of Eq. (35) has a maximum for $k^2 = 0$, and the criterion for the stability of the stationary solutions in thick samples has the form of the inequality opposite to (18), i.e., the solutions corresponding to the dotted sections on Figs. 3, 5, and 6 are stable, and the solutions corresponding to the solid sections are unstable.

If the quantities B_1 and B_2 have opposite signs, then for large values of k^2 the right-hand side of (35) may become positive for all reasonable values of B_0, so that all stationary solutions found in Sec. 1 (both the ones corresponding to solid lines as well as the ones corresponding to dotted lines) are unstable. Apparently in this case the stable stationary distributions are inhomogeneous in y.

Let us consider the signs of B_1 and B_2 associated with $\varphi = \pi/4$ for the trivial solution $\theta = 0$. In this connection $\Pi_1\zeta_1 = \Pi_2\zeta_2 = -m$ (in the case of a power-law dependence of μ on Π one has $\mu \sim \Pi^{-m}$), and the criteria (40) and (41) have, respectively, the forms

$$1 < 2a^2 m < 2, \tag{40'}$$

$$2a^2 m > 1. \tag{41'}$$

Thus, for $a^2 m > 1$ the quantities B_1 and B_2 have opposite signs whereas upon fulfillment of (40′) both of these quantities are negative. We note that condition (41′) means that $\mu(E_x)$ must decrease with the field more rapidly than E_x^{-1} so that in this connection negative differential conductivity of the sample will exist. Thus, our analysis breaks down only in the case $B_1, B_2 > 0$.

3. THE HALL EFFECT UNDER THE CONDITIONS FOR THE MULTIVALUED SASAKI EFFECT (TWO-VALLEY MODEL)

If in our arrangement it were possible to continuously vary the angle φ near the value $\pi/4$, then as it is not difficult to see from Fig. 6 the dependence of E_y on ψ would have the form of one of the curves shown in Fig. 7. It is possible to obtain a similar possibility with a known approximation by using a magnetic field directed along the z axis ($H_z = H$). Such a magnetic field, as long as it is small, leads to a small Hall e.m.f.,

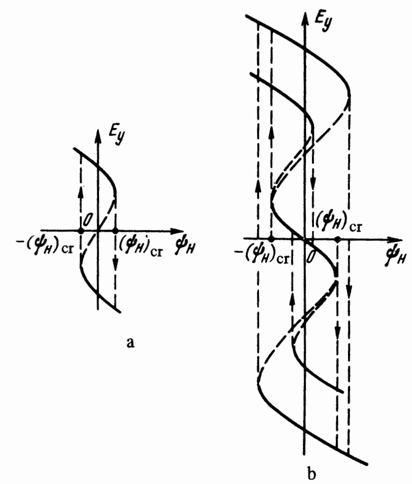

FIG. 7. Possible dependences of the transverse field E_y on the Hall angle ψ_H for $\varphi = \pi/4$.

which is superimposed on the Sasaki e.m.f. However, with an increase of H in one of the directions up to a certain critical value, a jump occurs from one stable Sasaki state to another, which becomes apparent in a discontinuous increase of the Hall e.m.f.

Without specifying the form of $\Phi(\Pi)$ one can compute the value of the critical angle only near the critical field E_c. In a small (in the sense of the magnitude of the Hall angle in each of the valleys) magnetic field Eq. (8) is replaced by the equation

$$\frac{\overline{\Phi}\{E_x^2 \sec^2\theta[1 - a\cos 2(\varphi - \theta)]\}}{\overline{\Phi}\{E_x^2 \sec^2\theta[1 + a\cos 2(\varphi - \theta)]\}} = \frac{a\sin(2\varphi - \theta) + \sin\theta - \psi_H^{(2)}\cos\theta}{a\sin(2\varphi - \theta) - \sin\theta + \psi_H^{(1)}\cos\theta}, \tag{42}$$

where $\overline{\Phi} = \pi(1 - \Lambda H^2)\tau$, Λ denotes the coefficient of magneto-conductivity, $\psi_H^{(1,2)} \equiv \psi_H(\Pi_{1,2}) = \rho_{1,2}H$ are the Hall angles in each of the valleys. For $\varphi = \pi/4$ and $|\theta| \ll a$ the angle θ is determined by Eq. (23), in which it is necessary to replace ψ by ψ_H. The condition

$$(\psi_H)_{cr}^2 = \frac{4B^3}{27AC^2} \sim \| E_x| - E_c|^3 \tag{43}$$

determines the critical value of the magnetic field, at which the field E_y changes discontinuously. In the case $A < 0$ (Fig. 7a) the angle $(\psi_H)_{cr}$, which is determined by condition (43), qualitatively completely characterizes the behavior of a semiconductor in a magnetic field. For $A > 0$ with the aid of (43) one can only find the magnetic field determining the transitions from states with small θ to states with large θ (Fig. 7b), so that a complete qualitative picture can be obtained only for specific dependences $\Phi(\Pi)$.

One may obtain analogous discontinuous transitions between the stable Sasaki states by deforming the semiconductor in such a manner that an energy gap appears between the bottom of the first and the bottom of the second valleys.

4. DISTRIBUTION OF THE ELECTRONS OVER THE VALLEYS IN n-Ge AND n-Si

In a real, many-valley cubic crystal, the sample of which is cut out such that

$$i_y = i_z = 0, \tag{44}$$

both components of the transverse field, E_y and E_z, will generally be different from zero. In addition, in the general case it is necessary to deal with more than two valleys: with four valleys in germanium and with three pairs (each of the pairs may be considered as a single valley) in silicon. Therefore a complete analysis of the intervalley redistribution, similar to the one carried out in Secs. 1 and 2 for the two-valley model, becomes difficult. Here we confine our attention to only the cases when in n-Ge the current is directed along the crystallographic axes of fourth and second orders, and also the case when the current is along the axis of third-order in n-Si.

1. Germanium, [100] direction. Let us line up the direction of the x axis with the [100] direction, and the y and z axes with the two other directions of the fourth-order axes ([010], [001]) so that the mobility tensors of the electrons in each of the four valleys turn out to be the following:[3)]

$$\hat{\mu}_{1,2,3,4} = \mu_{1,2,3,4}\begin{vmatrix} 1 & \substack{\mp\\ +} a & \substack{-\\ -\\ +\\ +} a \\ \substack{-\\ +\\ +} a & 1 & \substack{\mp\\ \pm} a \\ \substack{-\\ -\\ +\\ +} a & \substack{\mp\\ \pm} a & 1 \end{vmatrix}, \tag{45}$$

where (the choice of signs from above downwards corresponds to the indices 1, 2, 3, 4)

$$\mu_{1,2,3,4} = \mu(\Pi_{1,2,3,4}), \quad \mu = \frac{2\mu_t + \mu_l}{3}, \quad a = \frac{\mu_t - \mu_l}{2\mu_t + \mu_l},$$

$\mu_l(\Pi)$ and $\mu_t(\Pi)$ are the longitudinal and transverse mobilities in the valley, and just as in Sec. 1 it is assumed that a does not depend on $\Pi_\alpha = p_\alpha/e\mu_\alpha$ $(0 < a < 1/2)$. We note that in n-Ge a is always close to $1/2$.

From conditions (44) it follows that

$$E_y = aE_x\frac{K + aK_1K_2}{1 - a^2K_2^2}, \quad E_z = aE_x\frac{K_1 + aK_2K}{1 - a^2K_2^2} \tag{46}$$

where

$$K \equiv K_{1,3;2,4} \equiv \frac{\Phi_1 + \Phi_3 - \Phi_2 - \Phi_4}{\Phi_1 + \Phi_2 + \Phi_3 + \Phi_4},$$

$K_1 \equiv K_{1,2;3,4}$; $K_2 \equiv K_{1,4;3;2}$; as before $\Phi_\alpha \equiv \tau_\alpha\mu_\alpha$, and also $\Phi_\alpha = \Phi(\Pi_\alpha)$, where

$$\Pi_{1,2,3,4} = E_x^2\left[1 \substack{\mp\\ +} 2a\frac{E_y}{E_x} \substack{=\\ \mp\\ +} 2a\frac{E_z}{E_x} + \frac{E_y^2 + E_z^2}{E_x^2} \substack{\mp\\ \pm} \frac{E_yE_z}{E_x^2}\right]. \tag{47}$$

The unknown values of E_y and E_z are determined from Eqs. (46). There are three types of solutions for these equations:

a) the trivial solution:

$$E_y = E_z = 0 \quad (\Pi_1 = \Pi_2 = \Pi_3 = \Pi_4); \tag{48}$$

b) fourfold degenerate solutions of the two-valley type:

$$\begin{aligned} &1, 2\colon\ E_y = \pm a|K|, \quad E_z = 0 \quad (\Pi_1 = \Pi_3 \lesseqgtr \Pi_2 = \Pi_4);\\ &3, 4\colon\ E_z = \pm a|K|, \quad E_y = 0 \quad (\Pi_1 = \Pi_2 \lesseqgtr \Pi_3 = \Pi_4); \end{aligned} \tag{49}$$

c) fourfold degenerate solutions of the four-valley type:

$$\begin{aligned} &1, 2\colon\ E_y = E_z \gtrless 0 \quad (\Pi_1 \lesseqgtr \Pi_2 = \Pi_3 \lesseqgtr \Pi_4);\\ &3, 4\colon\ E_y = -E_z \gtrless 0 \quad (\Pi_2 \gtreqless \Pi_1 = \Pi_4 \gtreqless \Pi_3). \end{aligned} \tag{50}$$

In order to investigate the conditions for the origin of distributions of a nontrivial type, let us assume that the anisotropic component of the field is small:

$$E_\perp^2 \equiv E_y^2 + E_z^2 \ll E_x^2. \tag{51}$$

Then from (46) for $\vartheta^2 = E_\perp^2/E_X^2$ we have (compare with (15))

$$\vartheta^2 = R_{1,2}(E_c^{(l,h)})(|E_x| - E_c^{(l,h)}), \tag{52}$$

where as before E_c is determined by Eq. (11); in the case of the solutions of two-valley type it is necessary to use the coefficient $R_1(E_c)$ given by the right-hand side of Eq. (16), and in the case of solutions of the four-valley type—the coefficient $R_2(E_c)$ which differs from $R_1(E_c)$ by the replacement in the denominator of $A(E_c)$ by

$$A^0(E_c) = [(\Phi')^2(1 + a^2) + 2a\Phi\Phi'' + 4a^2E_c^2(\Phi'\Phi'' - \Phi\Phi'''/3)]_{\Pi=E_c^2}. \tag{53}$$

It is not difficult to see that the sign of $A^0(E_c)$, which determines the behavior of the four-valley solutions, may differ from the sign of $A(E_c)$. For example, for $\Phi \sim \exp -\gamma\Pi$ we have $A(E_c^{(l)}) = -2/3\,\gamma^2 \exp(-2\gamma\Pi_c)$ but $A^0(E_c^{(l)}) = \gamma^2[(1 + a)^2 - 4/3]\exp(-2\gamma\Pi_c)$, i.e., $A^0 > 0$ for $a > (2/\sqrt{3}) - 1$.

In the plane of $|\vartheta|$ and E_x the two-valley and four-valley type solutions are given by curves which start at the point $[0;\ E_c^{(l)}]$ and end at the point $[0,\ E_c^{(h)}]$, similar to one of the graphs in the upper half-plane in Figs. 3a–3e (depending on the signs of $A(E_c^{(l,h)})$ and $A^0(E_c^{(l,h)})$).

The analysis, which is similar to the one carried out in Sec. 2, shows that in thick samples ($d_y, d_z \gg \mathscr{L}$ the solutions of two-valley type are always unstable, and the trivial solution is unstable in the region of field strengths $(E_c^{(l)}, E_c^{(h)})$ and stable outside of this interval. As far as the solutions of the four-valley type are concerned, they behave like the anisotropic solutions in the two-valley model, i.e., the states having the largest values of $|\vartheta|$ are stable, those which are represented by solid lines in Fig. 3.

2. Germanium, [110] direction. It is precisely this situation which was considered in [2,3]. Let us keep the z axis, as before, directed along the [001] axis; then the direction of the y axis is lined up with the [110] direction. From symmetry considerations it follows that

$$E_y = 0, \tag{54}$$

so that, having introduced the notation $E_z/E_x = \tan\theta = \vartheta$, we have

$$\Pi_2 = \Pi_4 = E_x^2(a + \sec^2\theta),$$

$$\Pi_{1,3} = E_x^2(\sec^2\theta - a \mp 2\sqrt{2}\,a\,\mathrm{tg}\,\theta), \tag{55}$$

since in the xz plane we have

$$\hat{\mu}_{1,3} = \mu_{1,3}\begin{vmatrix} 1 - a & \mp a\sqrt{2} \\ \mp a\sqrt{2} & 1 \end{vmatrix}, \quad \hat{\mu}_{2,4} = \mu_{2,4}\begin{vmatrix} 1 + a & 0 \\ 0 & 1 \end{vmatrix}. \tag{56}$$

The stationary values of ϑ are determined from the equation

$$a\sqrt{2}\frac{\Phi(\Pi_1) - \Phi(\Pi_3)}{\Phi(\Pi_1) + \Phi(\Pi_3) + 2\Phi(\Pi_2)} = \vartheta,$$

[3)]The following numbering is adopted for the valleys: 1–[111], 2–[1$\bar{1}$1], 3–[11$\bar{1}$], 4–[1$\bar{1}\bar{1}$].

which has, in addition to the trivial solution of two-valley type $\vartheta = 0$ ($\Pi_1 = \Pi_3 < \Pi_2 = \Pi_4$), doubly degenerate solutions of the four-valley type: $\vartheta_{(-)} = -\vartheta_{(+)}$. The critical fields $E_c^{(l,h)}$, determining the instability region of the trivial solution, are found from the condition

$$-4a^2E_x^2\left(\frac{\Phi'}{\Phi}\right)_{\Pi=E_x^2(1-a)} = 1 + \frac{\Phi\{E_x^2(1+a)\}}{\Phi\{E_x^2(1-a)\}}, \qquad (57)$$

which is the analog of condition (11).

If the dependence of $\theta_\pm^2$ on $|E_x|$ near the critical field is written in the form (15), then we have the following expression for $R(E_c)$:

$$R(E_c) = -\frac{2[\Phi\{E_c^2(1+a)\} + \Phi\{E_c^2(1-a)\}]^2[\alpha_1(E_c^2) + E_c^2(d\alpha_1/dE_x^2)_{E_x=E_c}]}{E_c^3A^{(1)}(E_c)} \qquad (58)$$

where

$$\alpha_1(E_x^2) = -\frac{(\Phi')_{\Pi=E_x^2(1-a)}}{\Phi\{E_x^2(1+a)\} + \Phi\{E_x^2(1-a)\}}, \qquad (59)$$

$$A^{(1)}(E_c) = (\Phi')_{\Pi=E_c^2(1-a)}[(\Phi')_{\Pi=E_c^2(1+a)} + (\Phi' + 8a^2E_c^2\Phi'' + {}^{16}/_3a^4E_c^4\Phi''')_{\Pi=E_c^2(1-a)}]$$

and where the sign of $A^{(1)}(E_c)$, just like in the case of the two-valley model, determines the type of change in the number of solutions at the critical field strength. For $\Phi \sim \exp -\gamma\Pi$ the critical field E is almost $\sqrt{2}$ times larger than in the case when the current is directed along the [100] axis, and also the type of change in the number of solutions should be different since $A^{(1)}(E_c^{(l)}) \approx -{}^2/_3\gamma^2 \exp(-2\gamma\Pi_c(1-n)) < 0$ (whereas $A^0(E_c^{(l)}) > 0$).

3. Silicon, [111] direction. Having directed the x axis and the current along the crystallographic direction [111], and the y axis along the [0$\bar{1}$1] direction, in the xz plane we obtain

$$\hat{\mu}_{1,2} = \mu_{1,2}\begin{vmatrix} 1 & -a/\sqrt{2} \\ -a/\sqrt{2} & 1+a/2 \end{vmatrix}; \quad \hat{\mu}_3 = \mu_3\begin{vmatrix} 1 & \sqrt{2}a \\ \sqrt{2}a & 1-a \end{vmatrix}, \qquad (60)$$

where we define μ and a in the same way as for germanium.

Besides the trivial solution corresponding to a uniform population of all three pairs of valleys, there exist two more types of triply degenerate solutions corresponding to an identical population of any two pairs of valleys at the expense of enriching (type I) or depleting (type II) the population of the remaining third pair of valleys. Let us select as an example one solution of each type, for which $E_y = 0$, and also

$$\Pi_1 = \Pi_2 \neq \Pi_3, \qquad (61)$$

where

$$\Pi_{1,2} = E_x^2[1 - a\sqrt{2}\vartheta + (1 + a/2)\vartheta^2],$$
$$\Pi_3 = E_x^2[1 + 2a\sqrt{2}\vartheta + (1-a)\vartheta^2];$$

as before $\vartheta = \tan\theta = E_z/E_x$. The equation for ϑ has the form

$$\vartheta = \sqrt{2}\frac{aM}{1+aM}, \quad M = \frac{\Phi(\Pi_1) - \Phi(\Pi_3)}{2\Phi(\Pi_1) + \Phi(\Pi_3)}. \qquad (62)$$

The solutions of Eq. (62) are disposed in the interval $(-a\sqrt{2}/(1-a),\ a\sqrt{2}/(2+a))$, over which the right-hand side of (62) varies monotonically. Carrying out the analysis, which is analogous to the one carried out in Sec. 1 for the simplest model, we arrive at the conclusion concerning the existence of an interval of field strengths ($E_D^{(l)}$, $E_D^{(h)}$) in which two other (see Fig. 8) solutions exist in addition to the trivial solution $\vartheta = 0$. in this connection, inside the interval ($E_D^{(l)}$, $E_D^{(h)}$) there may exist an interval of field strengths ($E_c^{(l)}$, $E_c^{(h)}$) in which the trivial solution is unstable (Fig. 8a), so that the semiconductor necessarily is found in one of the stable anisotropic states. If the condition $E_y = 0$ is rigorously ensured (i.e., if a short-circuiting regime is created in the y direction, like the regime in the z direction created in Sec. 1), then both sections represented by the solid line on Fig. 8a are stable. However, if the appearance of a fluctuating field E_y is possible (i.e., $i_y = 0$), then the solution depicted by the curve in the upper half-plane in Fig. 8a turns out to be unstable and goes over into one of the states where the population of only one valley is enriched. If in order to calculate the fields E_D it is necessary to specify the form of $\Phi(\Pi)$, then just as before, expressions for the fields E_c can be obtained in general form after an expansion of $\Phi_{1,2}$ in terms of small ϑ on E_x near the critical field is given by the expression

$$\vartheta = 4\sqrt{2}\,\frac{\alpha(\Pi_c) + \Pi_c(d\alpha/d\Pi)_{\Pi=\Pi_c}}{3\alpha(\Pi_c) + 2a\Pi_c(\Phi''/\Phi)_{\Pi=\Pi_c}}\,\frac{|E_x| - E_c}{E_c}, \qquad (63)$$

where $\Pi_c = E_c^2$. Since the denominator of expression (63) in real situations is apparently always positive, in the interval ($E_c^{(l)}$, $E_c^{(h)}$) a semiconductor smoothly (without any discontinuity) changes into the state with $\vartheta > 0$, which corresponds to an identical enrichment of the first and second pairs of valleys at the expense of a depletion of the third pair, and which is stable only upon a stringent guaranty that $E_y = 0$. In the state of the opposite type—with a strong enrichment of the third pair at the expense of the first two pairs, only a discontinuous transition is possible, as is clear from Fig. 8a.

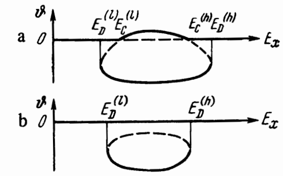

FIG. 8. Possible dependences of ϑ on E_x for the current along the [111] axis in silicon.

5. ESTIMATES OF THE DEPENDENCE OF THE INTERVALLEY TRANSITION TIMES ON THE HEATING POWER

The intervalley redistribution was investigated theoretically[2] and experimentally[3] in appreciable electric fields ($E_x > 1$ kV/cm), when the average energy of the carrier exceeds the energy of an intervalley phonon ϵ_0 (in germanium, according to [7], the intervalley transitions correspond to a phonon with an energy $T_0 = 316°$ K), so that the τ_α depend on the heating power comparatively weakly. Some "acceleration" of this dependence may occur, as noted in [3], due to transitions between equivalent valleys by means of higher minima of the conduction band.

We consider that a more favorable region for the observation of the multivalued Sasaki effect and the special properties of the electrical conductivity and of the

Hall effect associated with it is the region of extremely low lattice temperatures ($kT \ll \epsilon_0$) and of such field strengths E_x that the average energy of a carrier remains below ϵ_0, so that the dependence of τ_α on the heating power is close to exponential. For this region of field strengths and temperatures, let us estimate the function $\Phi(\Pi)$ which determines the field E_c and the entire behavior of the semiconductor.

Let us assume (since ϵ_0 is smaller than the energy of an optical phonon) that in the case under consideration the only mechanism of intravalley scattering of electrons is scattering by acoustic phonons (for which extremely pure crystals are required; an experimental situation close to the one being considered was studied by Kastal'skiĭ and Ryvkin [8]). Such a situation was considered in the article by Gantsevich, [9] and from his formulas one can obtain the following expression for the drift time of electrons from the valley labelled α:

$$\frac{\tau_0}{\tau}=\left\{\int_0^\infty R(x)\left(1+\frac{x}{\sigma}\right)^{g_\alpha}dx+\int_0^\infty R(x)\left(1+\frac{x+x_0}{g_\alpha}\right)^{g_\alpha}dx\right\} \quad (64)$$

$$\times\left\{\frac{4}{\sqrt{\pi}}\left(\int_0^\infty R(x)\,dx\right)\left(\int_0^\infty \sqrt{x}\,e^{-x}\left(1+\frac{x}{g_\alpha}\right)^{g_\alpha}dx\right\}^{-1},$$

where

$$x_0=\varepsilon_0/kT,\quad R(x)=e^{-x}\sqrt{x(x+x_0)},\quad g_\alpha=\frac{e\tau_i\mu(0)}{kT}\Pi_\alpha,$$

and the τ_i are energy relaxation times. For $x_0 \gg 1$, g_α from Eq. (64) it follows that

for $g_\alpha \ll 1$

$$\frac{\tau_0}{\tau_\alpha}=\frac{1}{2}\left[1+\left(\frac{x_0}{g_\alpha}\right)^{g_\alpha}\right];$$

for $x_0 \gg g_\alpha \gg 1$

$$\frac{\tau_0}{\tau_\alpha}=\left(\frac{x_0}{g_\alpha}\right)^{g_\alpha}\frac{\sqrt{\pi}}{2^{3/4}\,\Gamma(3/4)\,g_\alpha^{3/4}}.$$

The quantity Φ'/Φ entering into the expression for the critical field is equal to the sum $(\tau'/\tau)+(\mu'/\mu)$, where in our case $0 > \mu'/\mu > -1/4\Pi$, and $\tau'/\tau = -g\kappa(g)/\Pi$, where $\kappa(g)$ is a slowly varying factor:

$$\varkappa(g)=\frac{\ln(x_0/g)-1}{1+(g/x_0)^g}\quad \text{for}\quad g\ll 1,$$

$$\varkappa(g)=\ln(eg/x_0)\quad \text{for}\quad g\gg 1$$

(here e is the base of the natural logarithms). Thus, the critical field is determined from the condition

$$2a^2 g_c \varkappa(g_c) \approx 1. \quad (65)$$

One root of Eq. (65) is located near $g_c \approx 1/2a^2 \ln x_0$ so that the field $E_c^{(l)}$ approximately coincides with the field at the beginning of the heating up or is even somewhat smaller than it ($g_c < 1$). The second root of (65) is located at $g_c \gg 1$ when $\kappa(g_c) \approx 0$, so that the field $E_c^{(h)}$ corresponds to a situation when the average energy approaches ϵ_0.

We further note that near the root $E_c^{(h)}$ the function $\Phi \approx x_0^{-g} = \exp - \gamma\Pi$, where

$$\gamma=\frac{e\tau_i\mu(0)}{kT}\ln x_0.$$

The estimates of A, A^0, and $A^{(1)}$ made above using such a $\Phi(\Pi)$ indicated that in germanium, for the direction of the current along the [110] axis and also along the [100] axis, upon provision that $E_z = 0$ there is a change in the number of solutions at the points $E_c^{(l)}$ of type I, but if the current is along the [100] axis and $E_z \neq 0$ then changes of type II may occur. The latter property justifies the detailed analysis of this type of changes, which was carried out in Sec. 1.

[1] V. I. Denis, In: Aktual'nye voprosy fiziki poluprovodnikov i poluprovodnikovykh priborov (Current Problems in the Physics of Semiconductors and Semiconducting Devices), Vil'nyus, 1969, p. 50.

[2] H. G. Reik and H. Risken, Phys. Rev. **126**, 1737 (1962).

[3] M. Shyam and H. Kroemer, Appl. Phys. Lett. **12**, 283 (1968).

[4] E. Erlbach, Phys. Rev. **132**, 1976 (1963).

[5] J. C. McGroddy, M. I. Nathan, and J. E. Smith, IBM J. Res. Develop. **13**, 543 (1969).

[6] V. A. Kochelap and V. V. Mitin, Fiz. Tekh. Poluprov. **4**, 1051 (1970) [Sov. Phys.-Semicond. **4**, 896 (1970)].

[7] Gabriel Weinreich, T. M. Sanders, Jr., and H. G. White, Phys. Rev. **114**, 33 (1959).

[8] A. A. Kastal'skiĭ, Fiz. Tekh. Poluprov. **2**, 653 (1968) [Sov. Phys.-Semicond. **2**, 546 (1968)]; A. A. Kastal'skiĭ and S. M. Ryvkin, ZhETF Pis. Red. **7**, 446 (1968) [JETP Lett. **7**, 350 (1968)].

[9] S. V. Gantsevich, Fiz. Tverd. Tela **9**, 909 (1967) [Sov. Phys.-Solid State **9**, 707 (1967)].

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