CRITICAL PHENOMENA IN THE PROPAGATION OF SECOND SOUND IN He II WITH A LARGE THERMAL FLUX

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The set of equations for propagation of second sound in He II with an arbitrary difference between the normal and superfluid velocities is investigated. For internal motion velocities of the order of the second sound velocity, shock wave formation becomes possible. The temperature dependence of the critical velocities is found.

1. The problem of the effect of helium flow on the propagation of first and second sound was solved in 1951 by Khalatnikov[1] for the case of values of the normal and superfluid flow velocities that are small in comparison with the velocity of second sound (v_n, $v_s \ll u_{20}$).

In the present work, investigations are presented of linearized equations of propagation of second sound without the mentioned restriction on the velocities v_n and v_s.[2] For values $w = v_n - v_s \sim u_{20}$, critical phenomena appear that are associated with the degeneracy of the phonons of second sound.

2. The set of equations of superfluid hydrodynamics can be written in the form

$$\frac{\partial\rho}{\partial t}+\frac{\partial j_i}{\partial x_i}=0 \qquad \frac{\partial j_i}{\partial t}+\frac{\partial \Pi_{ik}}{\partial x_k}=0, \tag{1}$$

$$\frac{\partial v_{si}}{\partial t}+\frac{\partial}{\partial x_i}\left(\mu+\frac{v_s^2}{2}\right)=0, \tag{2}$$

$$\frac{\partial S}{\partial t}+\frac{\partial}{\partial x_i}(Sv_{ni})=0, \qquad S=\rho\sigma, \tag{3}$$

where

$$\rho=\rho_n+\rho_s, \qquad j_i=\rho_n v_{ni}+\rho_s v_{si}, \tag{4}$$

$$\Pi_{ik}=\rho_n v_{ni}v_{nk}+\rho_s v_{si}v_{sk}+p\delta_{ik}, \tag{5}$$

$$d\mu=-\sigma\,dT+\frac{1}{\rho}\,dp-\frac{\rho_n}{\rho}w_i\,dw_i. \tag{6}$$

Most interest attaches to the excitation of the second sound type. For their consideration we shall assume ρ = const, i.e., we shall neglect the compressibility $\partial\rho/\partial p$ and the thermal-expansion coefficient $\partial\rho/\partial T$, which is anomalously small in He II. In this case the equations become uncoupled, Eq. (1) is satisfied identically and the pressure can be expressed in terms of v_n and v_s by means of (5).

We choose a set of coordinates in which the total flow of the liquid is zero; we can then connect v_n and v_s uniquely with w. It is simpler to transform Eqs. (2) and (3) into the variables σ and v_s. By seeking a solution for all variables in the form of a simple plane wave $\sim \exp(\mathbf{k}\cdot\mathbf{r}-\omega t)$, we get, after lengthy transformations, the dispersion equation for $\omega(\mathbf{k})$:

$$\omega^2-\omega\mathrm{wk}\left[4\frac{\rho_s}{\rho}+\frac{ST}{\rho_n w^2}\frac{v_s}{T}\left(\frac{\partial T}{\partial v_s}\right)_\sigma-\frac{\sigma}{\rho_n}\left(\frac{\partial\rho_n}{\partial\sigma}\right)\right. \tag{7}$$

$$\left.-(1+\cos^2\vartheta)\frac{v_s}{\rho_n}\left(\frac{\partial\rho_n}{\partial v_s}\right)_\sigma\right]+(\mathrm{wk})^2\left[\frac{\rho_s}{\rho}-\frac{\sigma}{\rho_n}\left(\frac{\partial\rho_n}{\partial\sigma}\right)_{v_s}\right]$$

$$\times\left[3\frac{\rho_s}{\rho}+\frac{ST}{\rho_n w^2}\frac{v_s}{T}\left(\frac{\partial T}{\partial v_s}\right)_\sigma-(1+\cos^2\vartheta)\frac{v_s}{\rho_n}\left(\frac{\partial\rho_n}{\partial v_s}\right)_\sigma\right]$$

$$=\frac{\rho_s}{\rho}w^2k^2\frac{\sigma}{T}\left(\frac{\partial T}{\partial\sigma}\right)_{v_s}\left[1-\frac{\rho}{\rho_s}\cos^2\vartheta\frac{v_s}{\rho_n}\left(\frac{\partial\rho_n}{\partial v_s}\right)_\sigma\right]\cdot$$

$$\times\left[\frac{ST}{\rho_n w^2}-(1+\cos^2\vartheta)\frac{T}{\rho_n}\left(\frac{\partial\rho_n}{\partial T}\right)_{v_s}\right].$$

A quadratic equation is obtained for the phase velocity ω/k, with coefficients that are anisotropic because of the dependence on the angle ϑ between **w** and **k**.

The thermodynamic functions and their derivatives entering into the equation are transformed to the variables T and w in the usual fashion.[3] In the phonon parts of the thermodynamic quantities, one can neglect the w dependence, since the corresponding effects $\sim w^2/c^2 \ll 1$. The solutions of Eq. (7) were machine-tabulated and the diagrams $u_x = \partial\omega/\partial k_x$ and $u_y = \partial\omega/\partial k_y$ (components of the group velocity of the propagation of second sound) were constructed for various values of T and w (the x axis is drawn along the w direction).

For example, the diagrams of u(k) are given in Fig. 1 for a temperature T = 1.6°K, w = 3 m/sec and w = 18 m/sec. The presence of internal friction in He II makes the propagation of second sound in it essentially anisotropic.

Examination of the diagram for a velocity of w = 18 m/sec shows that it is critical; the Landau condition for phonons of second sound is satisfied in it: the velocity of the normal component is equal to the propagation velocity of the excitation $u_2(w_L T) = w_L$.

Two critical mechanisms are still logically possible.

The curve for the sound velocity $u = \partial\omega/\partial k$, which is symmetric relative to the x axis, can be located on one side of the origin; the minimum critical velocity w_0 for

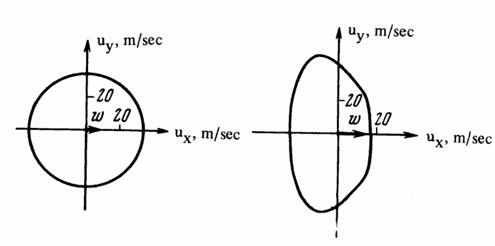

FIG. 1

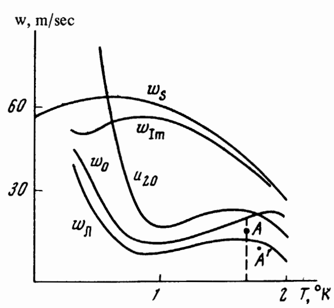

FIG. 2

which this occurs is determined from the condition $\omega(w_0, T) = 0$ for $\cos\vartheta = 1$.

Finally, complex solutions can appear in the quadratic equation (9) when its discriminant passes through zero when w increases to the corresponding critical value w_{Im}.

Figure 2 shows all three critical velocities as a function of temperature; these were obtained by computer calculation. The calculations were carried out in the temperature interval 0.2—2.0°K. At lower temperatures, the paths of the elementary excitations are too long and the macroscopic analysis is not suitable. Close to the λ point, the formulas for the thermodynamic functions for an ideal gas of excitations do not apply.

Figure 2 also gives the curves of the velocity of second sound in stationary helium and the critical temperature, which is determined from the condition of the vanishing of the density of the superfluid component $\rho_n(w_s, T) = \rho$. Thanks to the strong exponential dependence of the roton part ρ_{nr} on w, the formula for the normal density of a roton ideal gas can be used with sufficient accuracy for the determination of w:

$$\frac{\rho_{nr}}{\rho} = 13\left(\frac{\Delta}{T}\right)^{1/2} e^{-\Delta/T}\, 3\,\frac{\operatorname{ch} x - x^{-1}\operatorname{sh} x}{x^2} \quad x = \frac{p_0 w}{T},$$

whence we have, for $T \ll \Delta$,

$$w \approx w_{00}\left(1 + \frac{3T}{2\Delta}\ln\frac{\Delta}{7.2T}\right);$$

For $T = 0$, $w_s = w_{00} = \Delta/p_0$, a maximum $w_{sm} = 1.1\, w_{00}$ occurs at $T = 0.4$; it then falls off monotonically.

It is seen from Fig. 2 that all the calculated curves lie in the superfluid region ($w < w_s$). The minimum critical value gives the Landau criterion. The minimum corresponding to the curve $w_{L\,min} = 11$ m/sec is located at a temperature $T \approx 0.9$°K.

We now discuss the mechanism of the instability considered above. At a fixed temperature, the increase in w (Fig. 2) to values greater than w_L (point A) makes the flow supersonic. As also in ordinary hydrodynamics, the appearance of a shock wave of second sound is possible here, transforming the flow into subsonic (point A′) with $w < w_L$ and a different temperature. The resulting shock wave moves relative to the liquid. Only for $w = w_0$ is its velocity equal to zero.

3. The obtained critical velocities are much greater than those corresponding to the appearance of vortices. Therefore, the considered mechanism can play a decisive role near the λ point, where the corresponding critical velocities become very small. The Landau criterion suitable for the vicinity of the λ point was used for phonons of second sound by Mikeska,[4] who treated with its help the results of experiments on the critical phenomena near the λ point. However, quantitative treatment requires the consideration of the formation of shock waves in "supersonic flow," which has not yet been done. In particular, in connection with the consideration of thermal flux near the λ point, one can attempt to interpret the experiments of Peshkov[5] on the visual observation of a density jump in helium in the propagation of heat near the λ point as the observation of a stationary shock wave of second sound.

The proposed mechanism should also play a role in the determination of the structure of a vortex tube, although here the specific mechanism of the collapse of superfluidity is not completely clear. An estimate of $w_L \approx \hbar/mr_L$ for $T = 0.9$° and $w_L = 11$ m/sec gives $r_L = 1.5 \times 10^{-7}$ cm. In particular, this means that ring vortices, which form in the motion of ions in helium,[6] have a tube thickness and ring radius of the same order as the dimensions of the ions on which they form.

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